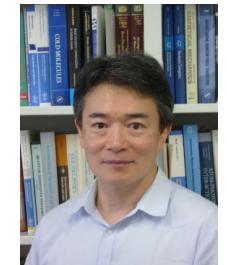
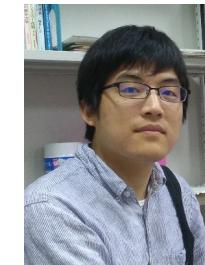


Localization and universality in non-Hermitian many-body systems

Ryusuke Hamazaki

R.H., K. Kawabata and M. Ueda.
Phys. Rev. Lett. 123, 090603 (2019)

R.H., K. Kawabata, N. Kura and M. Ueda.
Phys. Rev. Research, to appear
[arXiv:1904.13082]



Kohei

Naoto

Prof. Ueda

iTHEMS QFT Seminar

Sorry, this talk is
*mainly based on numerical results
*not about quantum field theory

Outline

❖ Introduction

- Many-body localization in Hermitian systems
- Non-Hermitian Hamiltonians

❖ Non-Hermitian many-body localization

R.H., K. Kawabata and M. Ueda.
Phys. Rev. Lett. 123, 090603 (2019)

❖ Universality in non-Hermitian random matrices

R.H., K. Kawabata, N. Kura and M. Ueda.
arXiv:1904.13082

Outline

❖ Introduction

- Many-body localization in Hermitian systems
- Non-Hermitian Hamiltonians

❖ Non-Hermitian many-body localization

R.H., K. Kawabata and M. Ueda.
Phys. Rev. Lett. 123, 090603 (2019)

❖ Universality in non-Hermitian random matrices

R.H., K. Kawabata, N. Kura and M. Ueda.
arXiv:1904.13082

Thermalization of isolated quantum systems

- ❖ Deriving stat. mech. from quant. mech.

unitary time evolution

$$\langle \psi(0) | \hat{O} | \psi(0) \rangle \longrightarrow \text{Tr}[\hat{\rho}_{\text{mic}} \hat{O}] ?$$

microcanonical ensemble

$$\hat{\rho}_{\text{mic}} = \frac{1}{d_{E,\Delta E}} \sum_{|E_\alpha - E| < \Delta E} |E_\alpha\rangle \langle E_\alpha|$$

Dating back to J. von Neumann in 1929

Rapid development in these two decades

Observation of thermalization dynamics
with ultra cold atomic gases
(almost isolated, highly tunable systems)

cf)

S. Trotzky et al., Nature Physics (2012)

Localization transition in quantum many-body systems

- ❖ Generic translation invariant systems thermalize

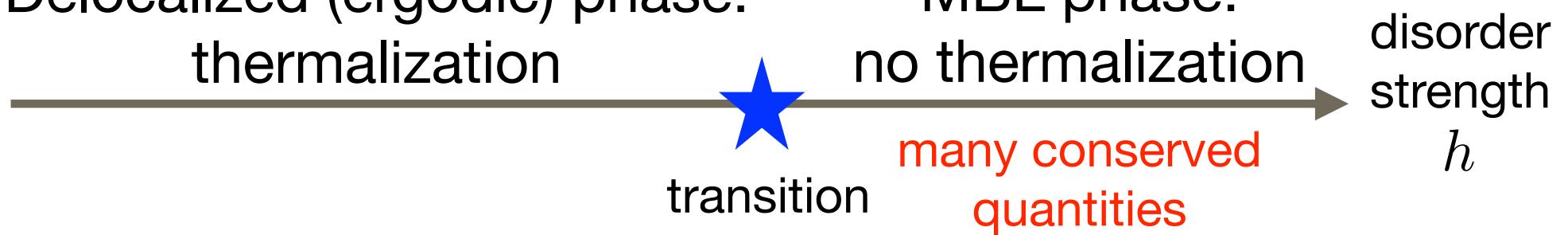
$$\langle \psi(0) | \hat{O} | \psi(0) \rangle \xrightarrow{\text{unitary time evolution}} \text{Tr}[\hat{\rho}_{\text{mic}} \hat{O}]$$

- ❖ Possibility of two phases in disordered systems

e.g.) $H = \sum_{i=1}^L [h_i \hat{S}_i^z + J \vec{\hat{S}}_i \cdot \vec{\hat{S}}_{i+1}] \quad h_i \in [-h, h]$

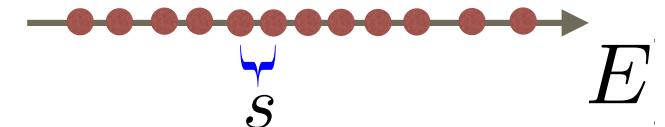
(many-body localized)

Delocalized (ergodic) phase: MBL phase:
thermalization no thermalization



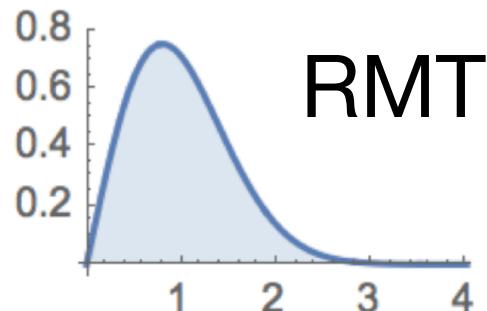
Theory of many-body localization: level-spacing statistics

- ❖ Level spacing distributions $P(s)$ many-body eigenvalues



Delocalized phase:
thermalization

$$p_{\text{AI}}(s) \simeq \frac{\pi s}{2} e^{-\frac{\pi s^2}{4}}$$



MBL phase:

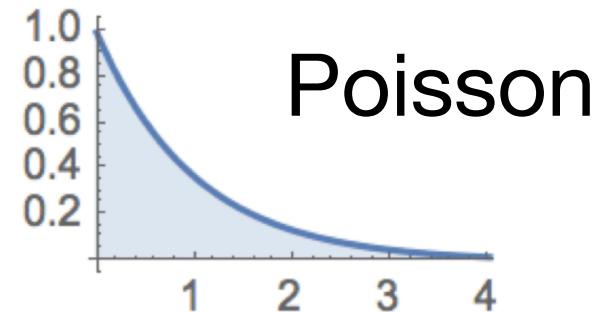
no thermalization

disorder strength



transition

$$P_{\text{P}}(s) = e^{-s}$$



Universality of random matrices:
complexity of eigenstates

uncorrelated eigenstates:
many symmetry sectors

Outline

❖ Introduction

- Many-body localization in Hermitian systems
- Non-Hermitian Hamiltonians

❖ Non-Hermitian many-body localization

R.H., K. Kawabata and M. Ueda.
Phys. Rev. Lett. 123, 090603 (2019)

❖ Threefold way in non-Hermitian random matrices

R.H., K. Kawabata, N. Kura and M. Ueda.
arXiv:1904.13082

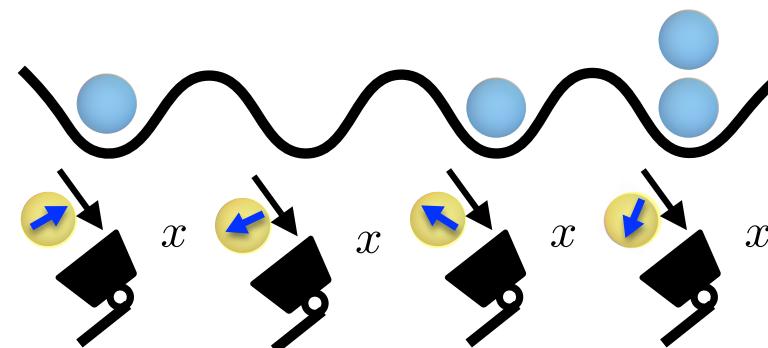
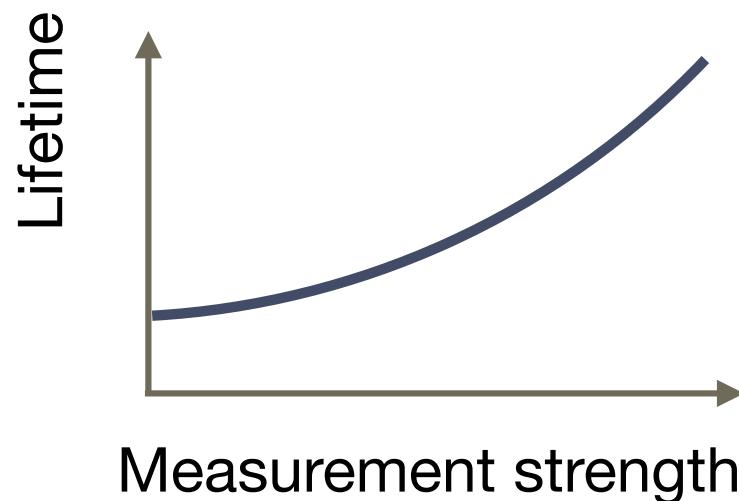
Dynamics of open quantum systems

Real systems are attached to external environment

How does it affect stat. mech. of many-body dynamics?

State-of-the-art experiments in e.g., cold atoms

Controllable dissipation, real-time measurement



Increase of particle lifetime with measurement (quantum Zeno effect)

cf) YS. Patil et al., PRL (2015)

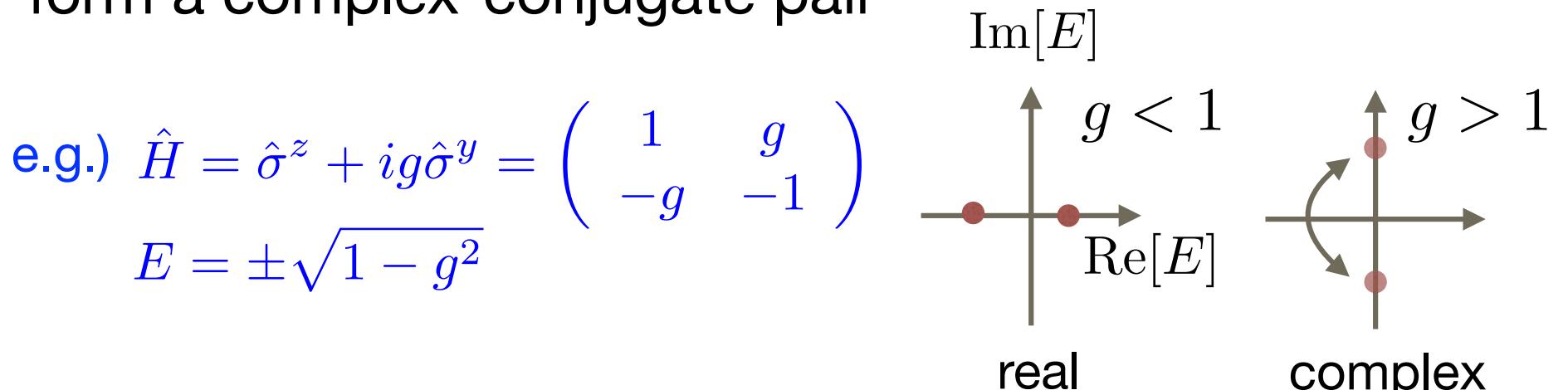
A class of open systems is described by non-Hermitian Hamiltonian

Time-reversal symmetry and real-complex transition

- ❖ $\hat{H} \neq \hat{H}^\dagger$ $\hat{H} = \sum E_\alpha |E_\alpha^R\rangle \langle E_\alpha^L|$
 E_α : complex in general $|E_\alpha^{R/L}\rangle$: right/left eigenstate

- ❖ Time-reversal operator \hat{T} with $\hat{T}^2 = 1$

If $[\hat{T}, \hat{H}] = 0$, eigenvalues of \hat{H} are either real or form a complex-conjugate pair



Two real eigenvalues collide and become imaginary (real-complex transition)

Outline

❖ Introduction

- Many-body localization in Hermitian systems
- Non-Hermitian Hamiltonians

❖ Non-Hermitian many-body localization

R.H., K. Kawabata and M. Ueda.
Phys. Rev. Lett. 123, 090603 (2019)

❖ Universality in non-Hermitian random matrices

R.H., K. Kawabata, N. Kura and M. Ueda.
arXiv:1904.13082

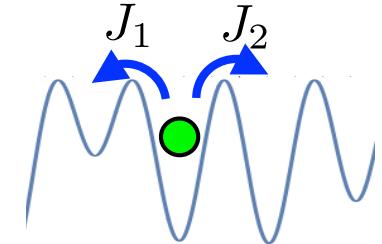
Review: Hatano-Nelson model

N. Hatano and D. R. Nelson, Phys. Rev. Lett., 77, 570 (1996)

❖ Localization & time-reversal symmetry

$$\hat{H} = - \sum_{i=1}^L \left(e^{-g} \hat{b}_{i+1}^\dagger \hat{b}_i + e^g \hat{b}_i^\dagger \hat{b}_{i+1} \right) + \sum_{i=1}^L h_i \hat{n}_i$$

a particle with asymmetric hopping on a disordered potential
non-Hermiticity localization



❖ Localization & real-complex transitions occur for the single-particle level

weak disorder: delocalized \leftrightarrow complex eigenvalues

strong disorder: localized \leftrightarrow real eigenvalues

❖ Single-particle problem

- One-particle spectra
- No interaction effect

Aim of this study

What physics emerges if all of non-Hermiticity, disorder and interactions are present?

Our finding

Novel real-complex transition appears,
which we argue is triggered by
non-Hermitian extension of the MBL

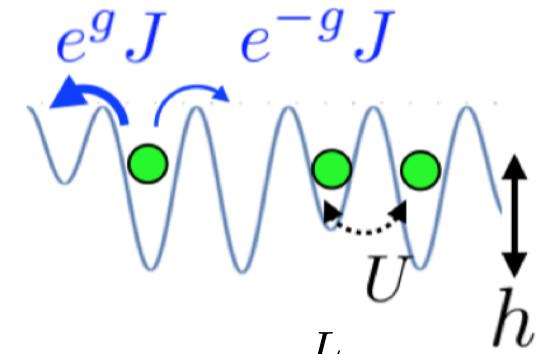
R.H., K. Kawabata and M. Ueda. Phys. Rev. Lett. 123, 090603 (2019)

Interacting model with asymmetric hopping and disorder

[Phys. Rev. Lett. 123, 090603 (2019)]

- ❖ Interacting hardcore bosons with disorder and asymmetric hopping terms (1D, PBC)

$$\hat{H} = -J \sum_{i=1}^L \left(e^{-g} \hat{b}_{i+1}^\dagger \hat{b}_i + e^g \hat{b}_i^\dagger \hat{b}_{i+1} \right) + U \sum_{i=1}^L \hat{n}_i \hat{n}_{i+1} + \sum_{i=1}^L h_i \hat{n}_i$$



← Time-reversal symmetry $\hat{H} = \hat{H}^*$

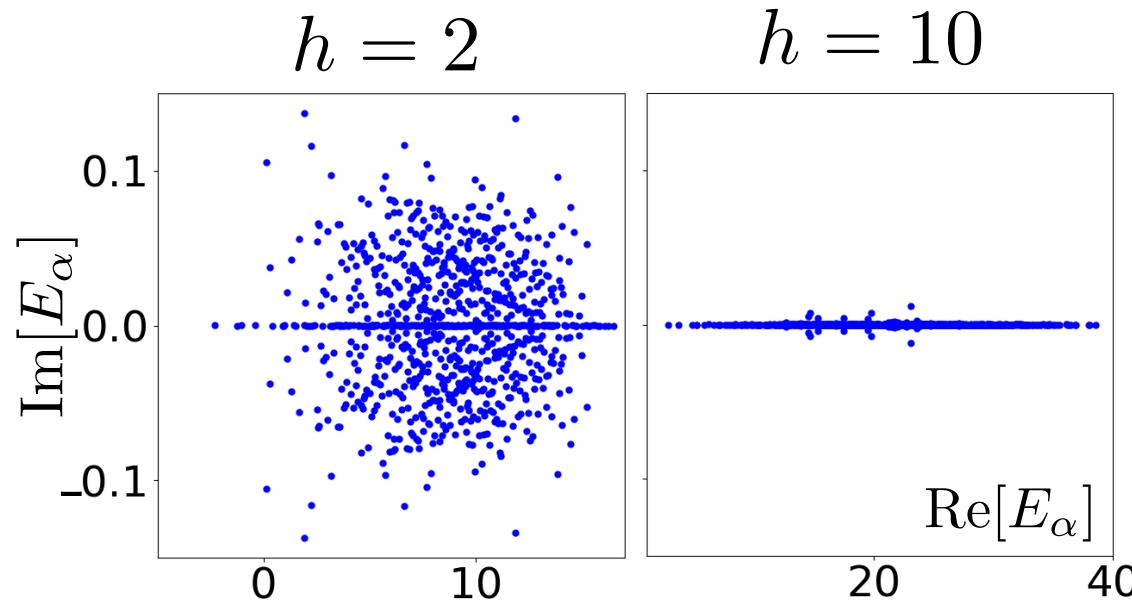
- For $g = 0$, disordered XXZ model (MBL when $7 \lesssim h$)
- For $U = 0$, Hatano-Nelson model

Spectra of the model

[Phys. Rev. Lett. 123, 090603 (2019)]

❖ Spectra

$g = 0.1$
 $L = 12$



cf) Eigenvalues are either real or form complex-conjugate pairs
due to time-reversal symmetry

Complex many-body eigenvalues are suppressed
as we increase disorder strength!

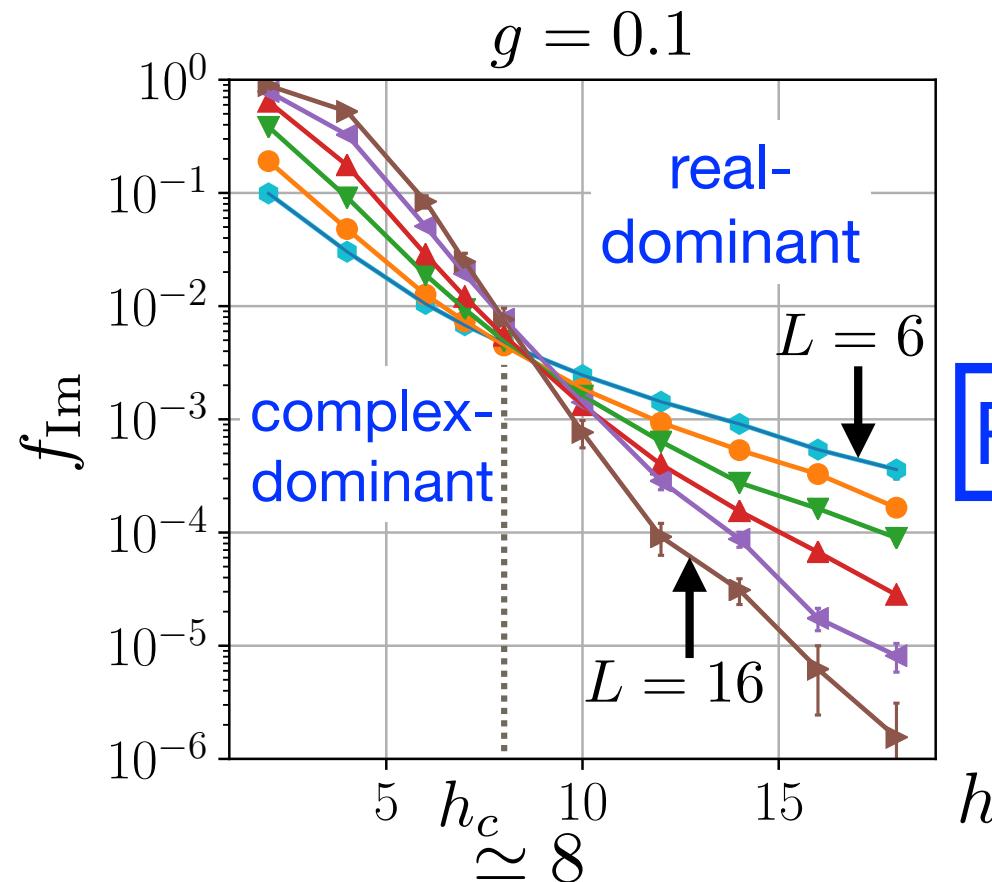
Real-complex transition of the many-body eigenvalues

[Phys. Rev. Lett. 123, 090603 (2019)]

❖ Ratio of eigenvalues with nonzero imaginary parts

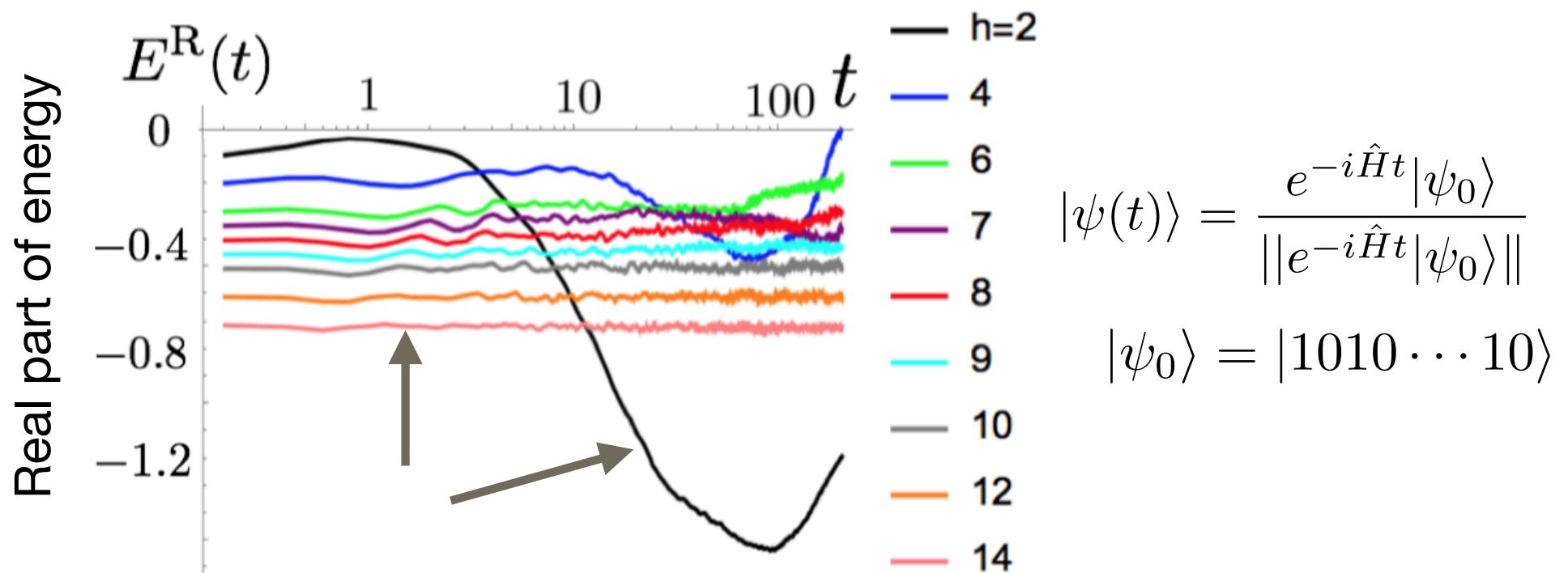
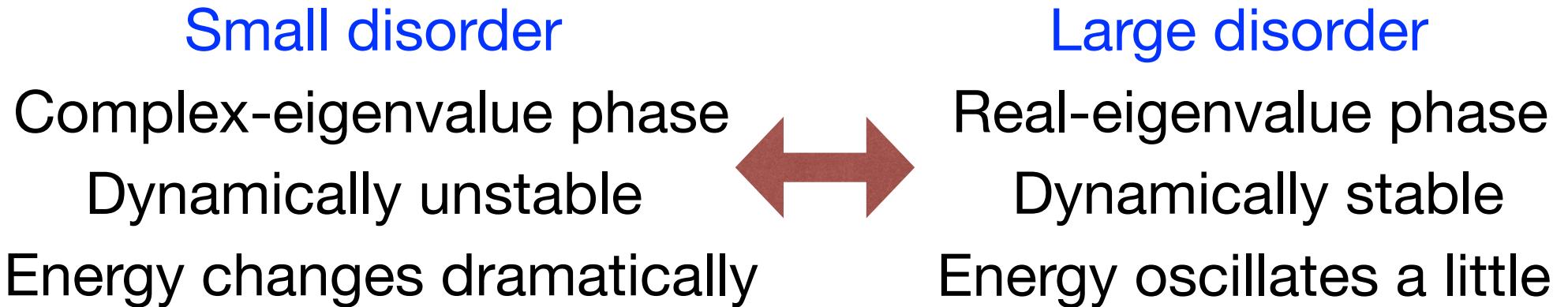
$$f_{\text{Im}} = \frac{(\# \text{ of eigenvalues with nonzero imaginary parts})}{(\# \text{ of all eigenvalues})}$$

average
over
disorder



Consequence: dynamical stability transition

[Phys. Rev. Lett. 123, 090603 (2019)]



Non-Hermitian MBL: level-spacing statistics

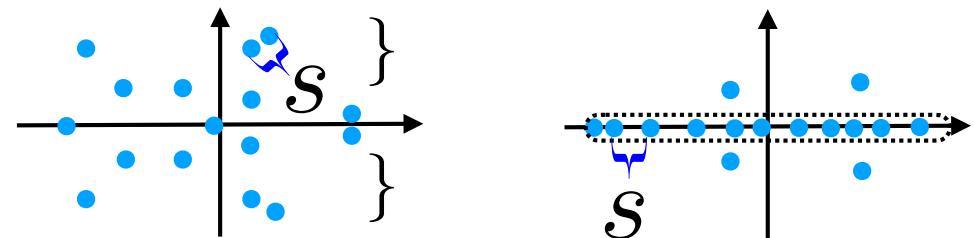
[Phys. Rev. Lett. 123, 090603 (2019)]

❖ Level-spacing statistics:

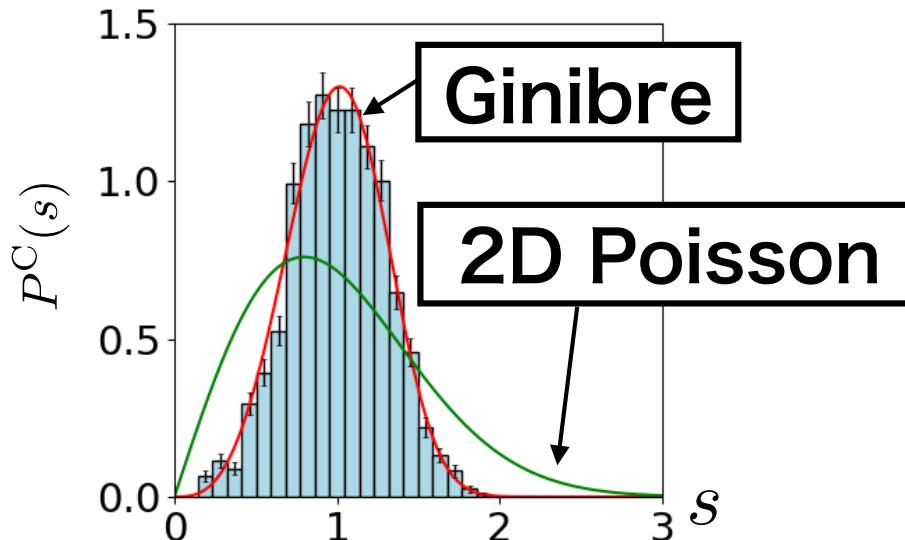
random matrix ensemble
for non-Hermitian matrices

Natural extension of
the Hermitian case!

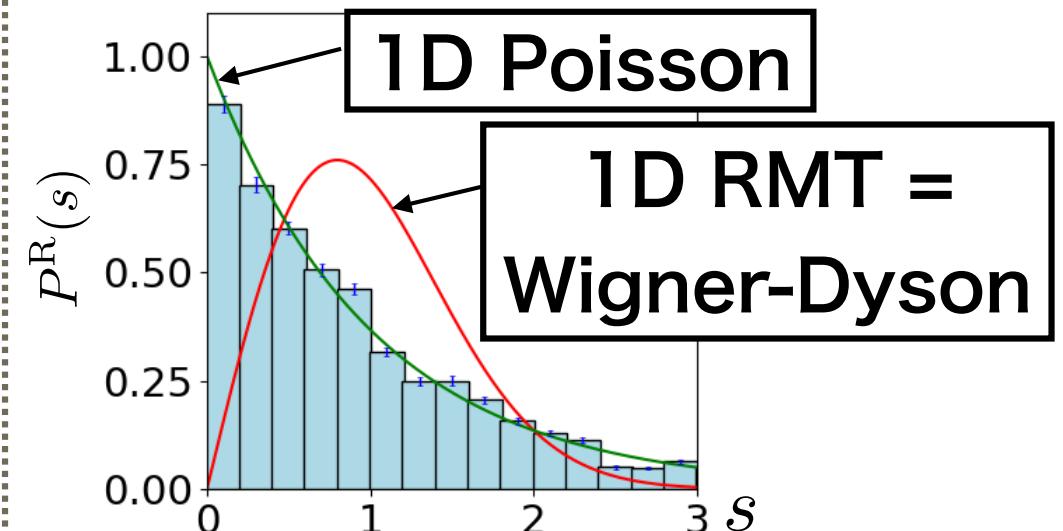
Ginibre $\xrightarrow[\text{large } h]{}$ Poisson
(in complex plane) (in real axis)



$h = 2$ (delocalized phase)



$h = 14$ (localized phase)



Coincidence of real-complex and MBL transitions?

Non-Hermitian MBL: entanglement entropy

[Phys. Rev. Lett. 123, 090603 (2019)]

❖ Entanglement entropy of eigenstates

- Consider right eigenstates

$$S^R = \left[\overline{\text{Tr}[\hat{\rho}_{L/2}^\alpha \ln \hat{\rho}_{L/2}^\alpha]} \right]_\alpha$$

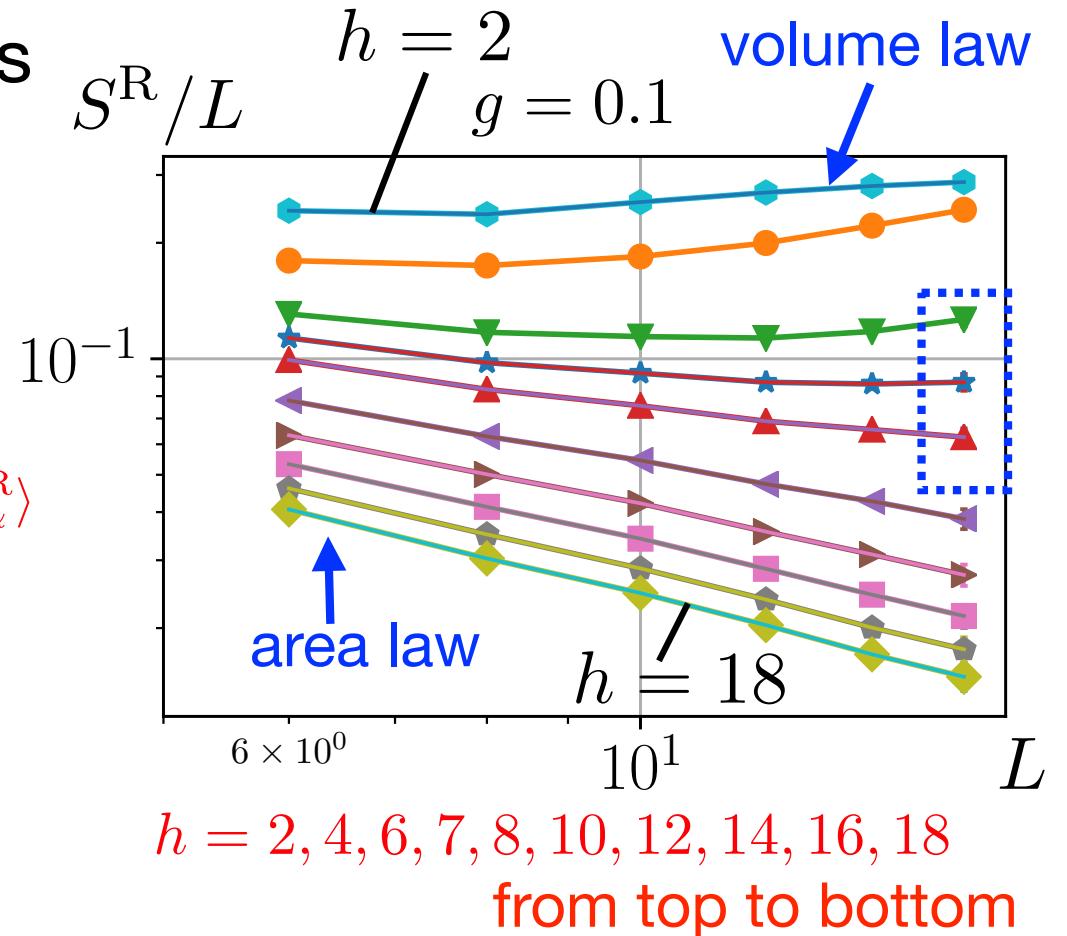
↑
average over eigenstates

$\hat{\rho}_{L/2}^\alpha$: reduced density matrix for $|E_\alpha^R\rangle$

Volume to area law

Non-Hermitian MBL

transition at $h_c = 7 \pm 1$



Coincidence of real-complex and MBL transitions?

Coincidence of two transition points

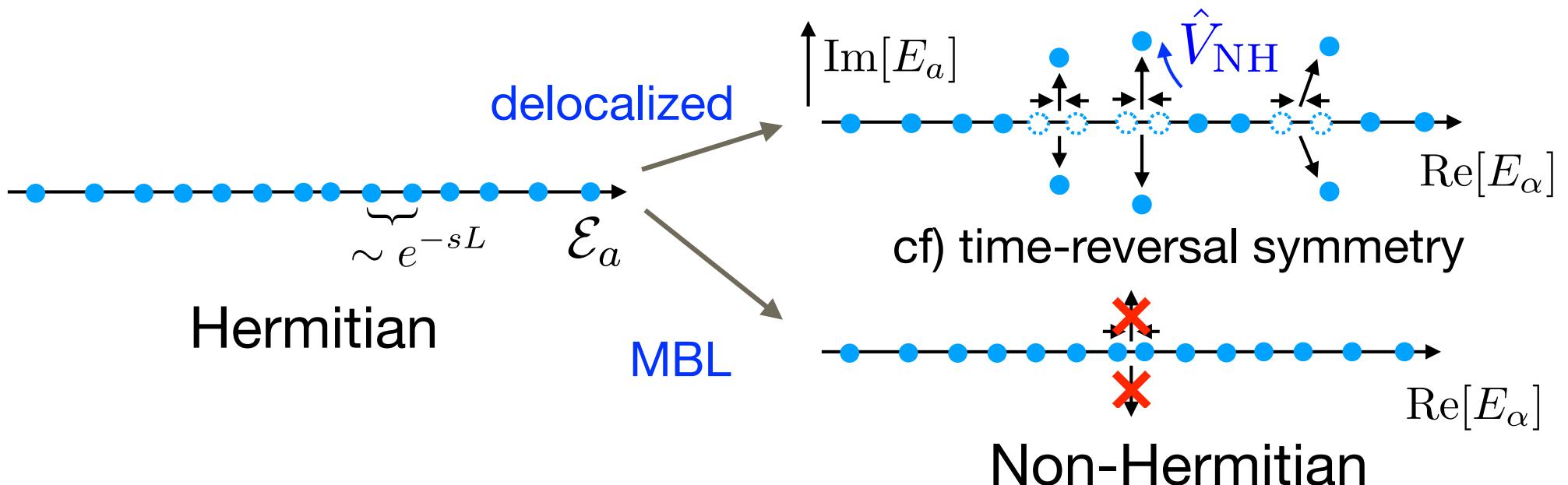
[Phys. Rev. Lett. 123, 090603 (2019)]

- Adjacent eigenstates do not mix by non-Hermitian perturbations only in MBL phase

$$\hat{H} = \sum_a \mathcal{E}_a |\mathcal{E}_a\rangle \langle \mathcal{E}_a| + \hat{V}_{\text{NH}} \quad \frac{|\langle \mathcal{E}_{a+1} | \hat{V}_{\text{NH}} | \mathcal{E}_a \rangle|}{|\mathcal{E}_{a+1} - \mathcal{E}_a|} \begin{cases} \gg 1 & \text{(delocalized)} \\ \ll 1 & \text{(MBL)} \end{cases}$$

cf) M. Serbyn et al., PRX (2015)

→ Coalescence process is suppressed in the MBL phase



Outline

❖ Introduction

- Many-body localization in Hermitian systems
- Non-Hermitian Hamiltonians

❖ Non-Hermitian many-body localization

R.H., K. Kawabata and M. Ueda.
Phys. Rev. Lett. 123, 090603 (2019)

❖ Universality in non-Hermitian random matrices

R.H., K. Kawabata, N. Kura and M. Ueda.
arXiv:1904.13082

Universality of Hermitian random matrices

- ❖ Universal statistics of random matrices

$$H = \begin{pmatrix} H_{11} & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ \vdots & \cdots & H_{NN} \end{pmatrix}$$

random variables taken from $Q(H_{ij})$

e.g.) level-spacing distributions $p(s)$, $s \propto E_{\alpha+1} - E_{\alpha}$ eigenvalue

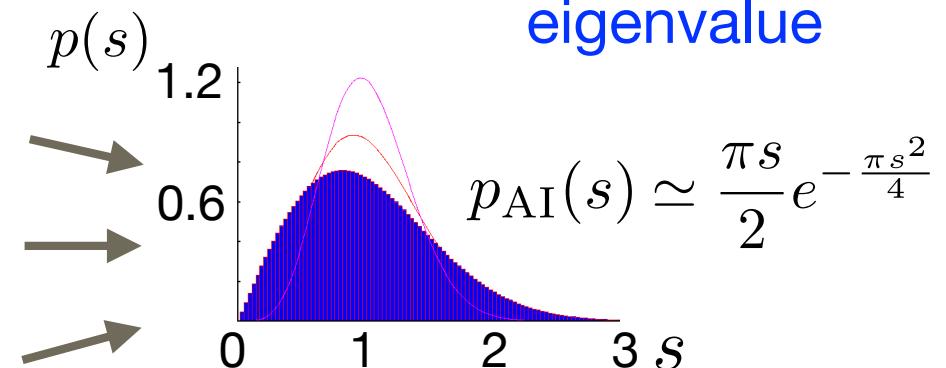
$$Q(H_{ij}) \propto e^{-\beta H_{ij}^2}$$

(real Gaussian)

$$Q(H_{ij}) \propto \delta(H_{ij} - 1) + \delta(H_{ij} + 1)$$

(real Bernoulli)

:



- ❖ Non-random but complicated systems may obey the same universality

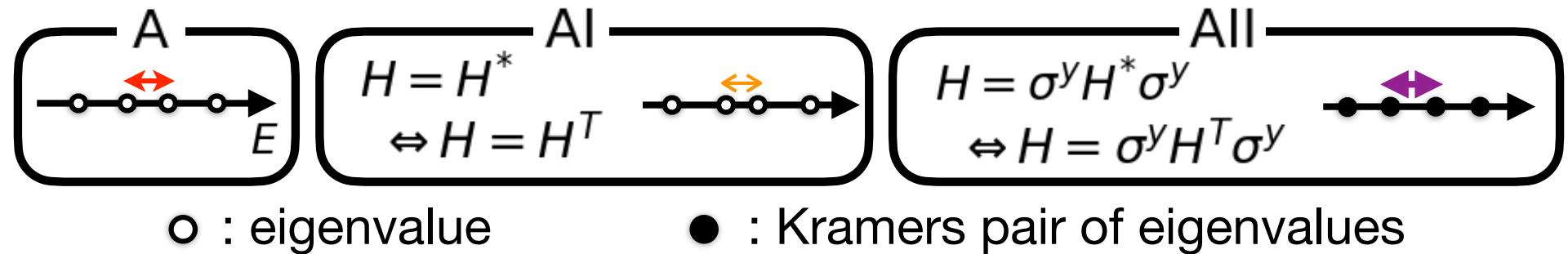
Nuclei

Quantum
chaos

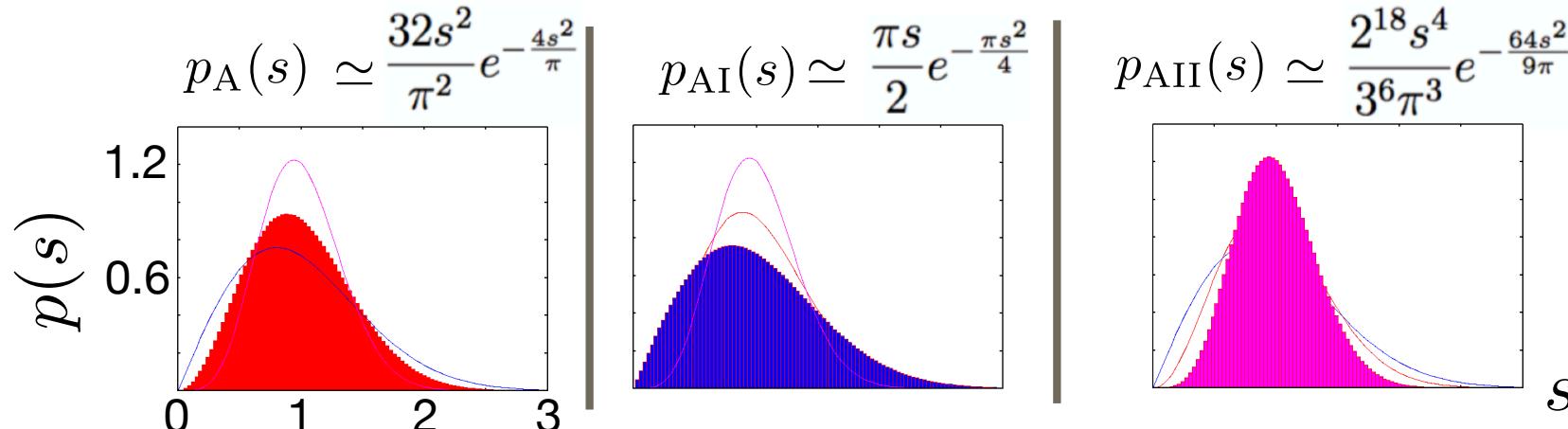
Mesoscopic
systems

Random matrices and symmetry

- ❖ Dyson's threefold way: time-reversal symmetry



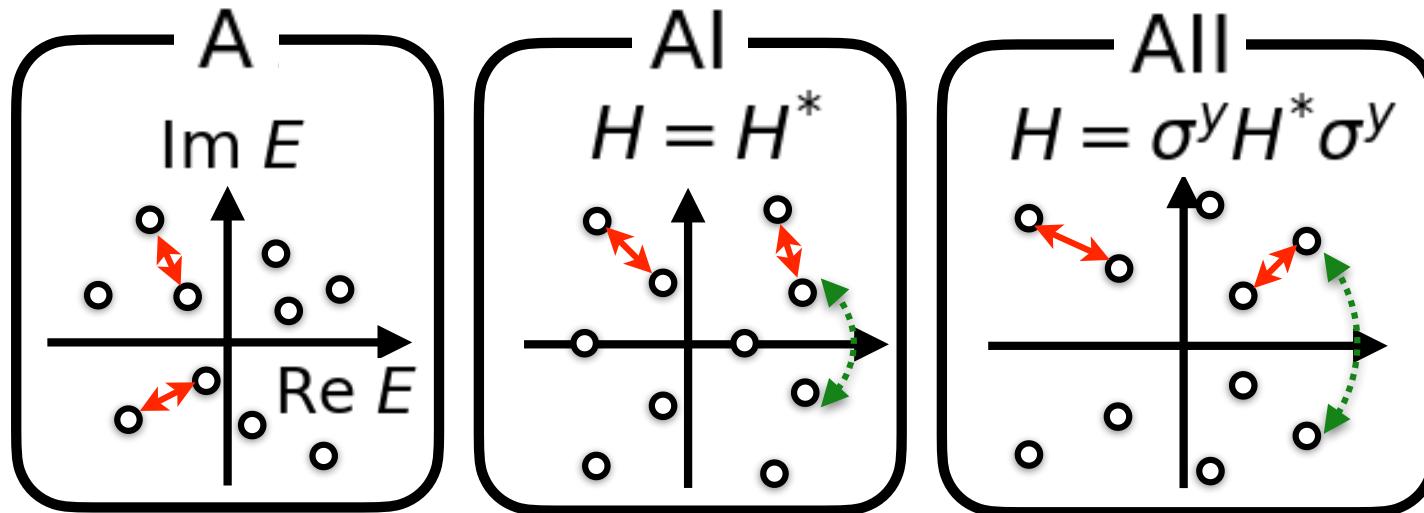
Different level-spacing distributions (e.g., peak structure)



- ❖ Also appear in e.g. chaotic systems with the same symmetry

Symmetry classification and universality in non-Hermitian random matrices

- ❖ Ginibre's threefold way: time-reversal symmetry



level-spacing distributions in the complex plane

$$p(s), \quad s \propto \min_{\beta} |E_{\alpha} - E_{\beta}|$$

- ← Three classes lead to the **same** universality unlike the Hermitian threefold way
(symmetry only creates nonlocal correlations)

Motivation of our work

Does the threefold **different** universality appear even for non-Hermitian random matrices?

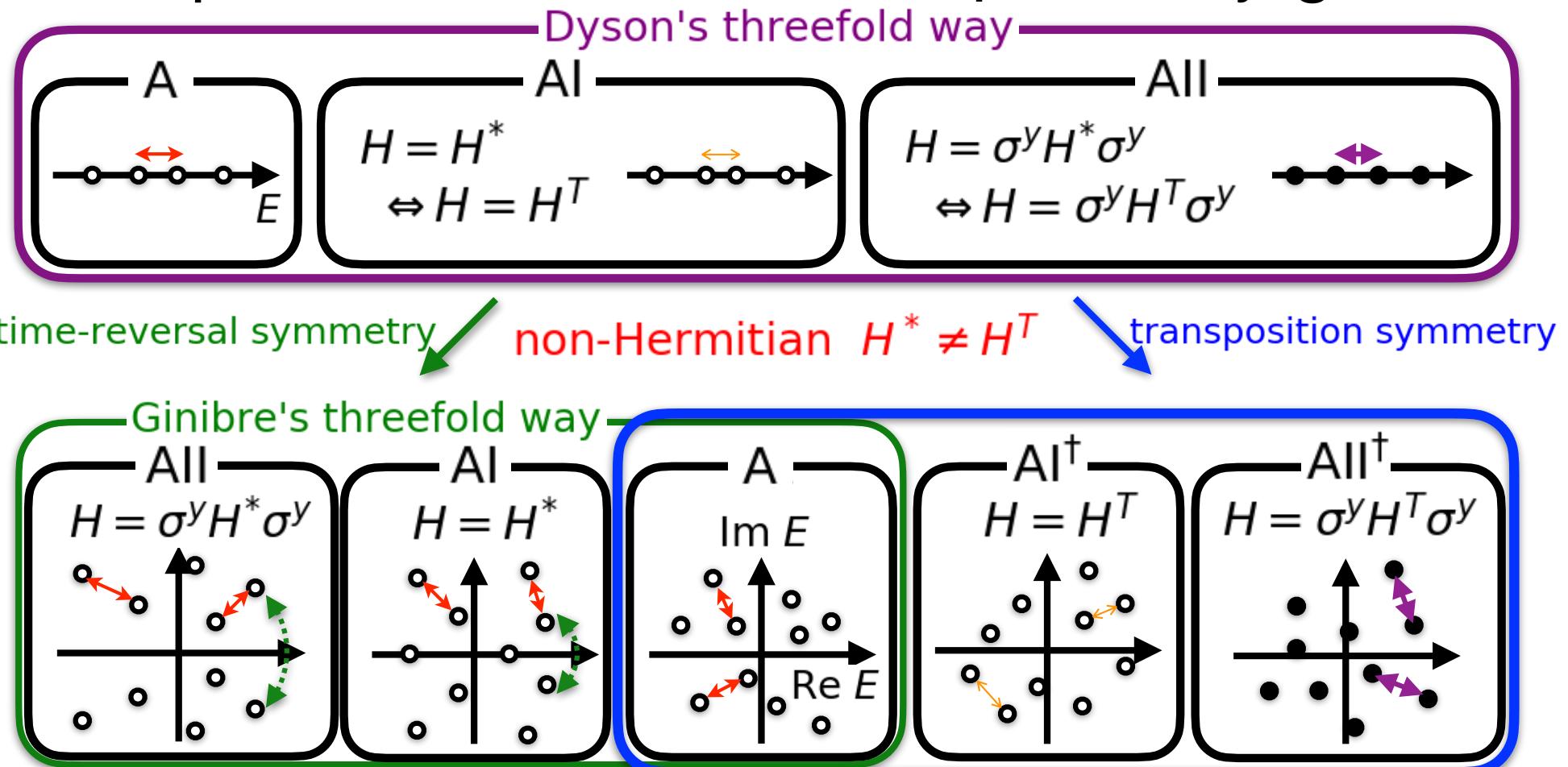
Our finding

Yes,
consider transposition symmetry
instead of time-reversal symmetry!

R.H., K. Kawabata, N. Kura and M. Ueda. arXiv:1904.13082

Role of transposition symmetry

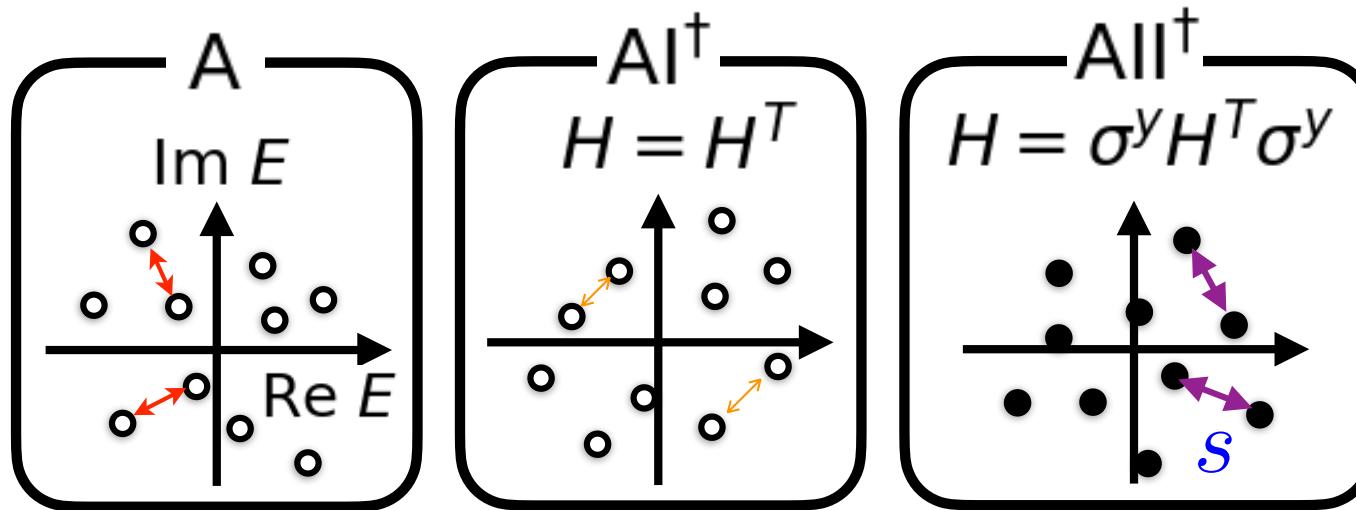
❖ Transposition differs from complex conjugation!



[D. Bernard and A. LeClair (2002); U. Magnea (2008); Kawabata et al. (2018)]

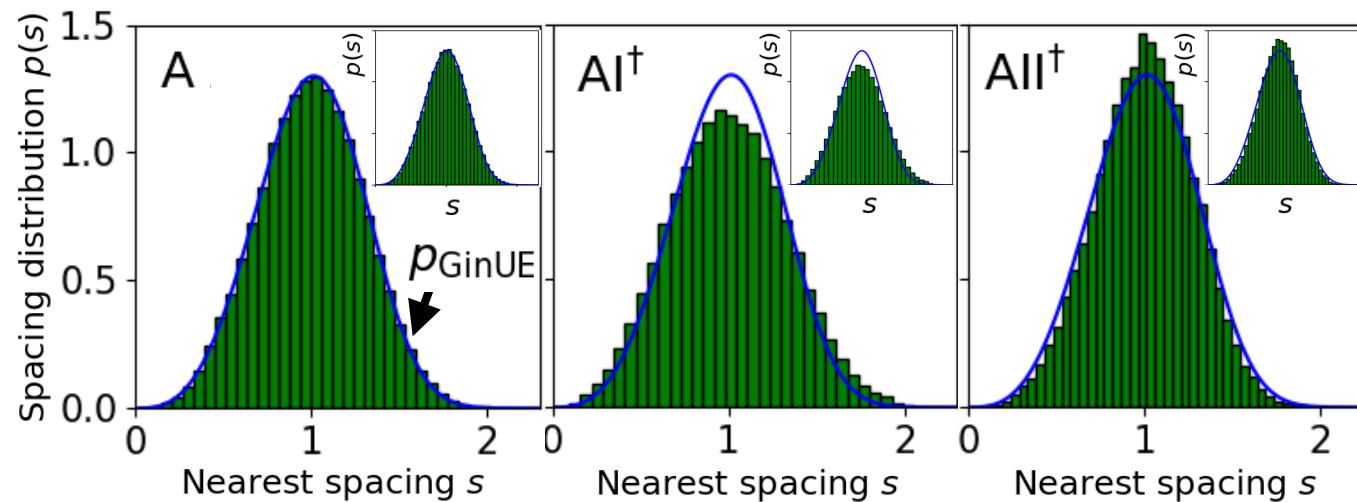
Threefold way in non-Hermitian random matrices

R.H. et al., arXiv:1904.13082



Transposition alters interactions between eigenvalues, just as the Hermitian case

❖ Universal level-spacing distributions



(main) Gaussian
(inset) Bernoulli
cf) peak structure

Other non-Hermitian symmetries

R.H. et al., arXiv:1904.13082

- ❖ Only transposition symmetry alters the level repulsions, leading to distinct universality of $p(s)$

Class	Symmetry	nonlocal correlations	GinUE for $p(s)$?
A	None		Yes (12)
AI (D^\dagger)	TRS, + (PHS † , +)	$H = H^*$	(E, E^*) pair
AII (C^\dagger)	TRS, - (PHS † , -)	$H = \sigma^y H^* \sigma^y$	(E, E^*) pair
AI †	TRS † , +	$H = H^T$	-
AII †	TRS † , -	$H = \sigma^y H^T \sigma^y$	Kramers pair
D	PHS, +	$H = -H^T$	($E, -E$) pair
C	PHS, -	$H = -\sigma^y H^T \sigma^y$	($E, -E$) pair
AIII	CS (pH)	$H = -\sigma^z H^\dagger \sigma^z$	($E, -E^*$) pair
AIII †	SLS (CS †)	$H = -\sigma^z H \sigma^z$	($E, -E$) pair

TRS: time-reversal symmetry, PHS: particle-hole symmetry
 CS: chiral symmetry, SLS: sub lattice symmetry

Other non-Hermitian symmetries

R.H. et al., arXiv:1904.13082

- ❖ Only transposition symmetry alters the level repulsions, leading to distinct universality of $p(s)$

Only three universal level-spacing statistics exist among all non-Hermitian symmetry classes
(38 types)

C	PHS, -	$H = -\sigma^y H^\dagger \sigma^y$	$(E, -E)$ pair	Yes
AIII	CS (pH)	$H = -\sigma^z H^\dagger \sigma^z$	$(E, -E^*)$ pair	Yes
AIII [†]	SLS (CS [†])	$H = -\sigma^z H \sigma^z$	$(E, -E)$ pair	Yes

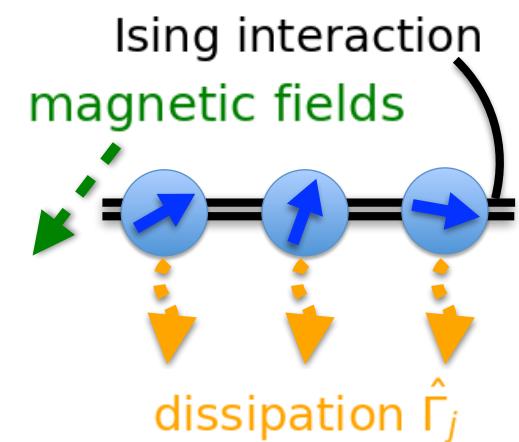
TRS: time-reversal symmetry, PHS: particle-hole symmetry
CS: chiral symmetry, SLS: sub lattice symmetry

Spectra of the Lindblad many-body operator

R.H. et al., arXiv:1904.13082

$$\frac{d\hat{\rho}}{dt} = \mathcal{L}[\hat{\rho}] = -i[\hat{H}, \hat{\rho}] + \sum_{j=1}^L \gamma \left[\hat{\Gamma}_j \hat{\rho} \hat{\Gamma}_j^\dagger - \frac{1}{2} \{ \hat{\Gamma}_j^\dagger \hat{\Gamma}_j, \hat{\rho} \} \right]$$

$$\begin{cases} \hat{H} = - \sum_{j=1}^{L-1} (1 + \epsilon_j) \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z - \sum_{j=1}^L (-1.05 \hat{\sigma}_j^x + 0.2 \hat{\sigma}_j^z) \\ \hat{\Gamma}_j = \text{(i)} \hat{\sigma}_j^z \text{ (dephasing)} \quad \text{or} \quad \text{(ii)} \hat{\sigma}_j^- \text{ (damping)} \end{cases}$$

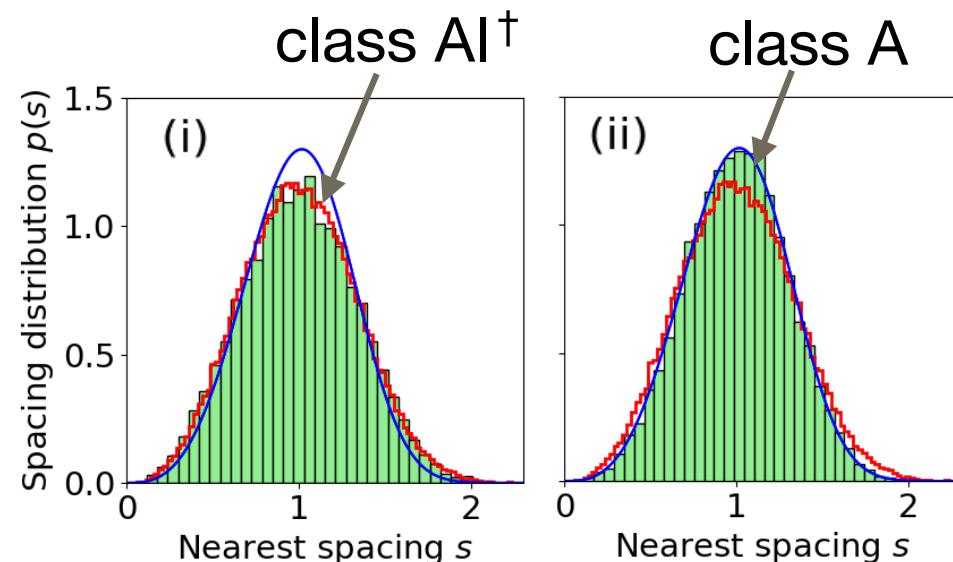


Only (i) keeps the transposition symmetry

Liouvillian spectra $\{\lambda_\alpha\}$

$$\mathcal{L}[\hat{\Lambda}_\alpha] = \lambda_\alpha \hat{\Lambda}_\alpha$$

The universality appears!



Conclusion

- ❖ Real-complex transition can occur due to non-Hermitian extension of the MBL transition, characterized by the level-spacing statistics

R.H., K. Kawabata and M. Ueda. PRL (2019)

- ❖ Threefold universality classes for non-Hermitian random matrices are found by considering transposition symmetry, not complex-conjugation as Ginibre's classes

R.H., K. Kawabata, N. Kura and M. Ueda. arXiv:1904.13082