

# Nambu-Goldstone fermion in a Bose-Fermi mixture with an explicitly broken supersymmetry

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# Outline

- Introduction
- Formalism
- Results
- Summary

# What is supersymmetry?

- Symmetry associated with interchange between fermions and bosons

Particles confirmed in our world

Fermions <b>(Half-integer spin)</b>	Bosons <b>(Integer spin)</b>
Electron	
	Photon
Quark	
	Higgs boson

# What is supersymmetry?

- Symmetry associated with interchange between fermions and bosons

Particles confirmed in our world

Predicted supersymmetric partners

Fermions (Half-integer spin)	Bosons (Integer spin)
Electron	Selectron
Photino	Photon
Quark	Squark
Higgsino	Higgs boson

# What is supersymmetry?

- Symmetry associated with interchange between fermions and bosons

Particles confirmed in our world

Predicted supersymmetric partners

Supersymmetry is a strong candidate for going beyond the standard model. **HOWEVER**, there is no evidence for supersymmetry in our world.

Quark

Squark

Higgsino

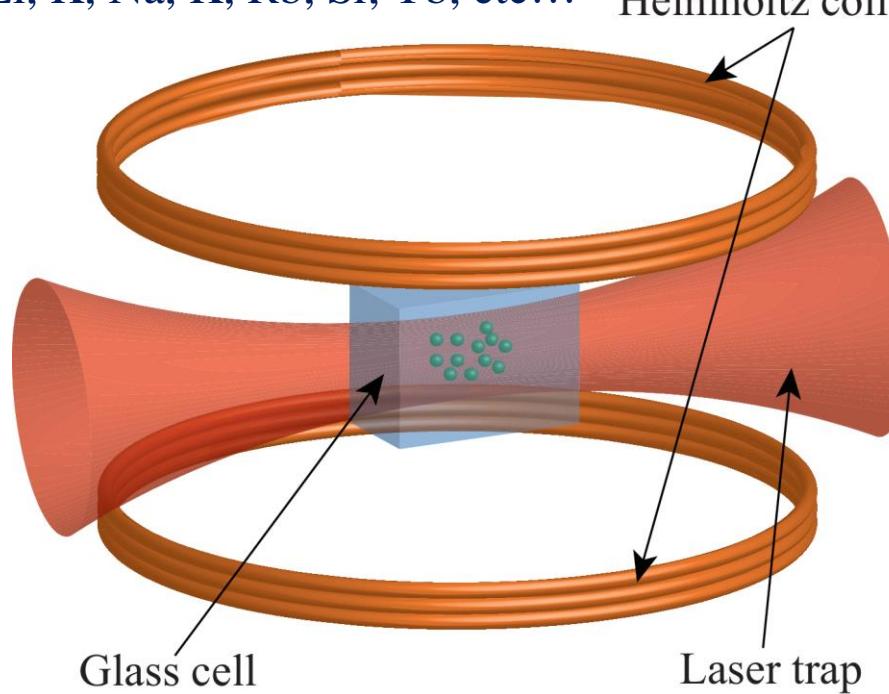
Higgs boson

# Ultracold atomic gases as “Quantum simulator”

Schematic figure of cold atom experiments

Atomic species

Li, K, Na, K, Rb, Sr, Yb, etc...



Temperature:  $T \sim O(10^{1\sim 2})$  nK

Density:  $n \sim O(10^{15})$  cm<sup>-3</sup>

Tunable “parameters”

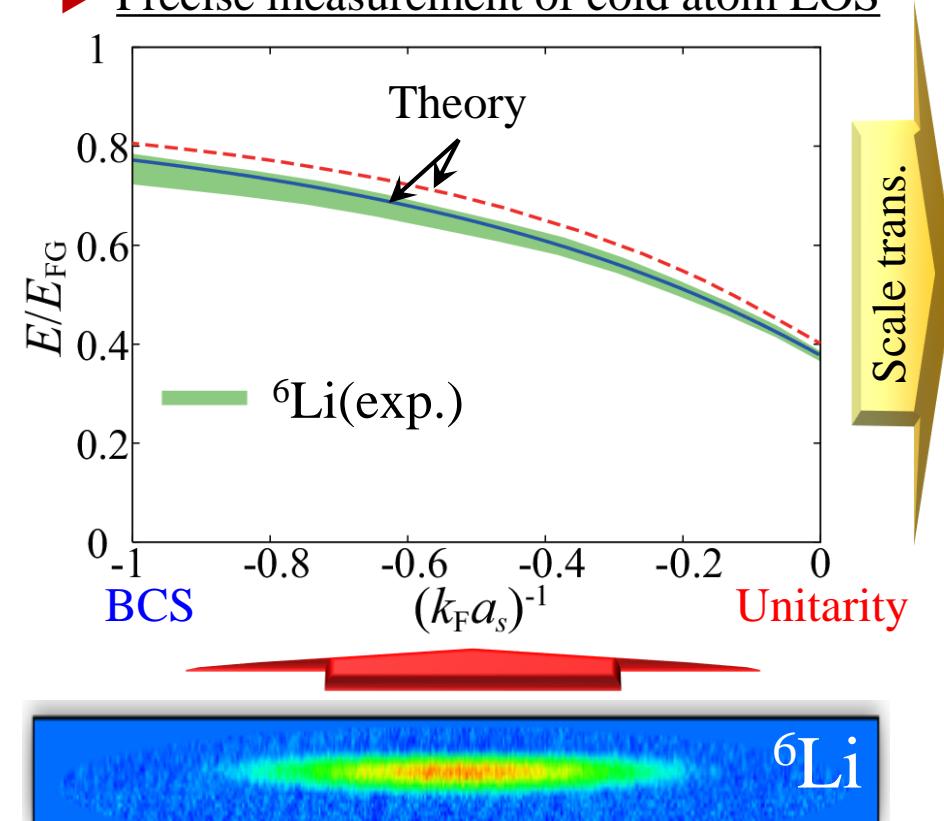
1. **Quantum statistics**  
(boson or fermion)
2. **Interaction**  
(Feshbach resonance)
3. **Spin**  
(hyperfine states called pseudospin)
4. **Thermodynamics**  
(density and temperature)
5. **Lattice geometry**  
(optical lattice)
6. **Dimension**  
(3D to 1D, synthetic dim...)etc...

# Simulating neutron matter with an ultracold Fermi atomic gas

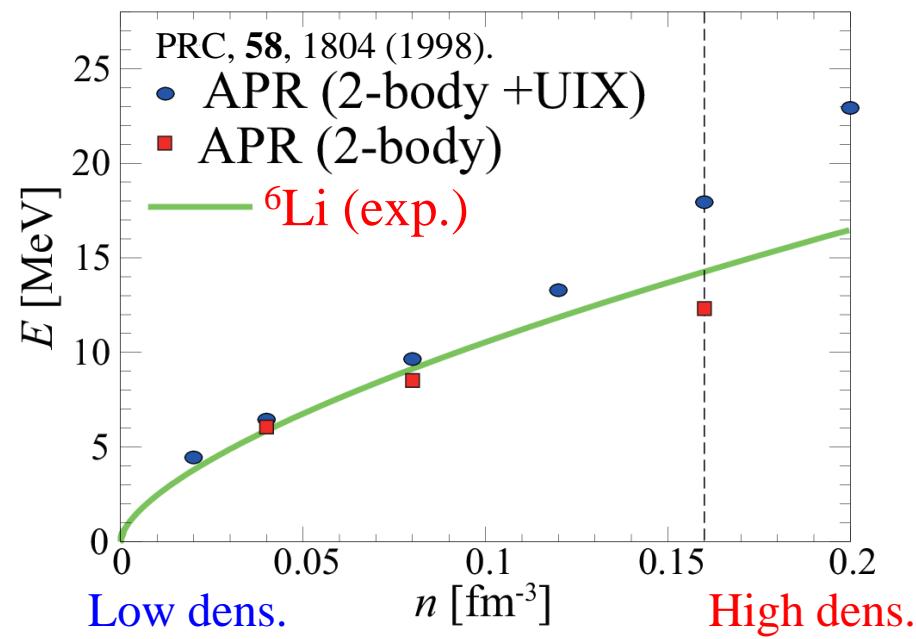
- Ultracold Fermi atomic gases can be used as a quantum simulator of dilute neutron matter

M. Horikoshi, M. Koashi, [HT](#), Y. Ohashi, and M. Kuwata-Gonokami, PRX, 7, 041004 (2017).

## ► Precise measurement of cold atom EOS



## ► EOS of neutron matter and cold atom



Agreement in the low density region

# Can we simulate supersymmetry (SUSY) in ultracold atoms?

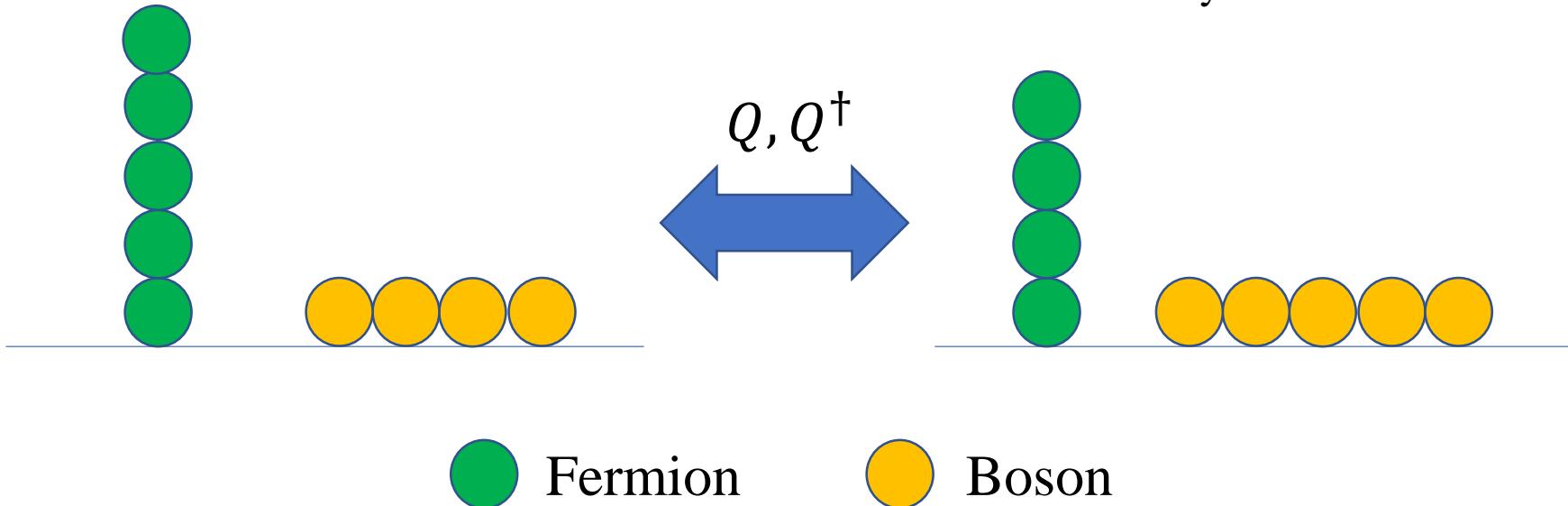
- In principle, it is possible

$$\text{Supercharge} : Q = b^\dagger f$$

Removing one fermion and adding one boson

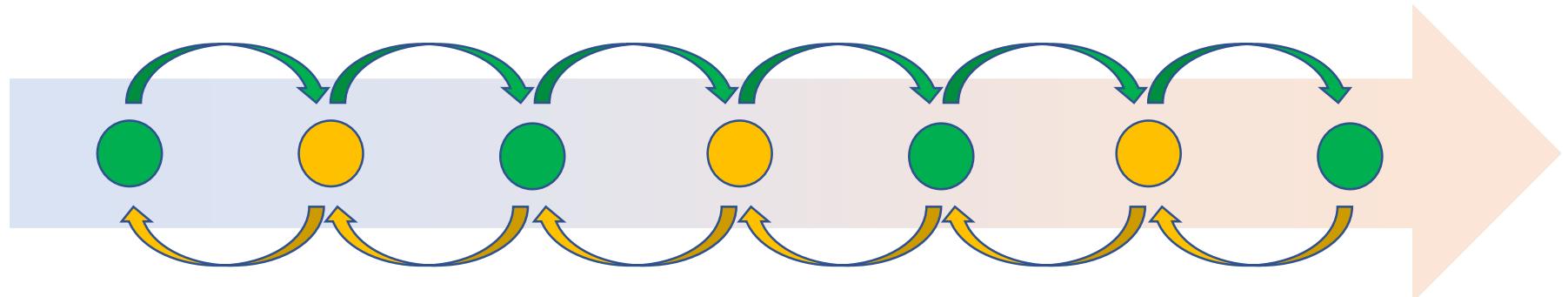
Supersymmetric when  $[Q, H] = 0$

$H$ : Hamiltonian of system



# Nambu-Goldstone fermion

- Zero-energy collective mode associated with spontaneous supersymmetry breaking



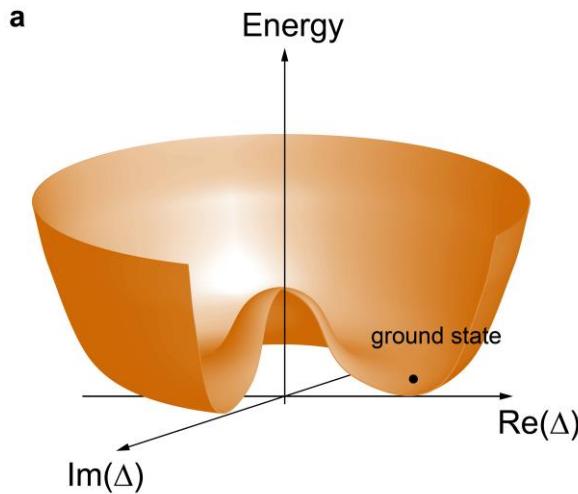
“Goldstino”

Type-II Nambu Goldstone mode

$$\text{Dispersion relation : } \omega = \omega_G + p^2/(2m_G)$$

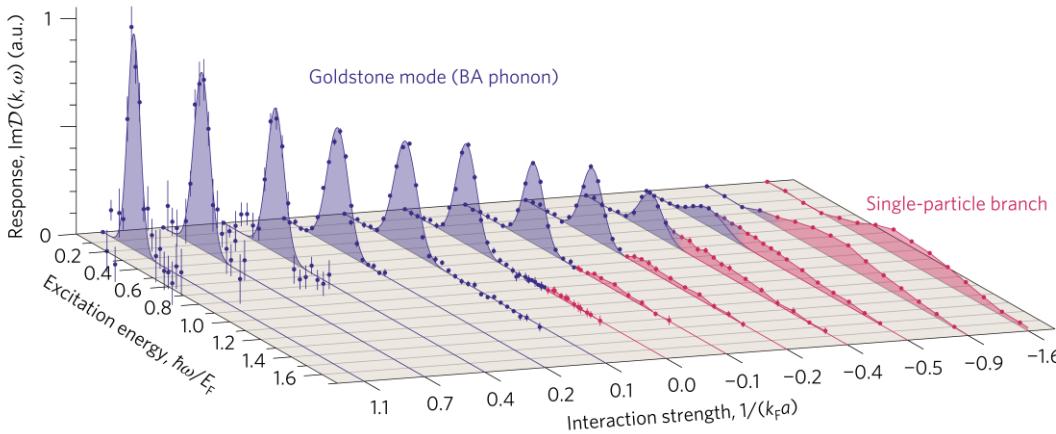
Goldstino gap  $\omega_G \rightarrow 0$  when the Hamiltonian is supersymmetric

# Observation of collective modes in superfluid Fermi gases



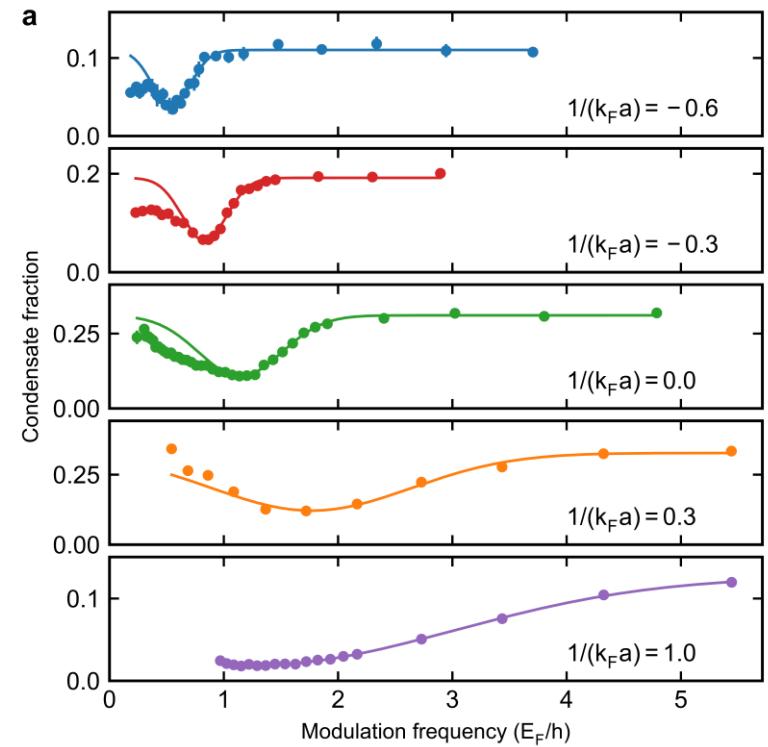
## Nambu-Goldstone (phase) mode

S. Hoinka, *et al.*, Nat. Phys. **13**, 943 (2017).



## Higgs (amplitude) mode

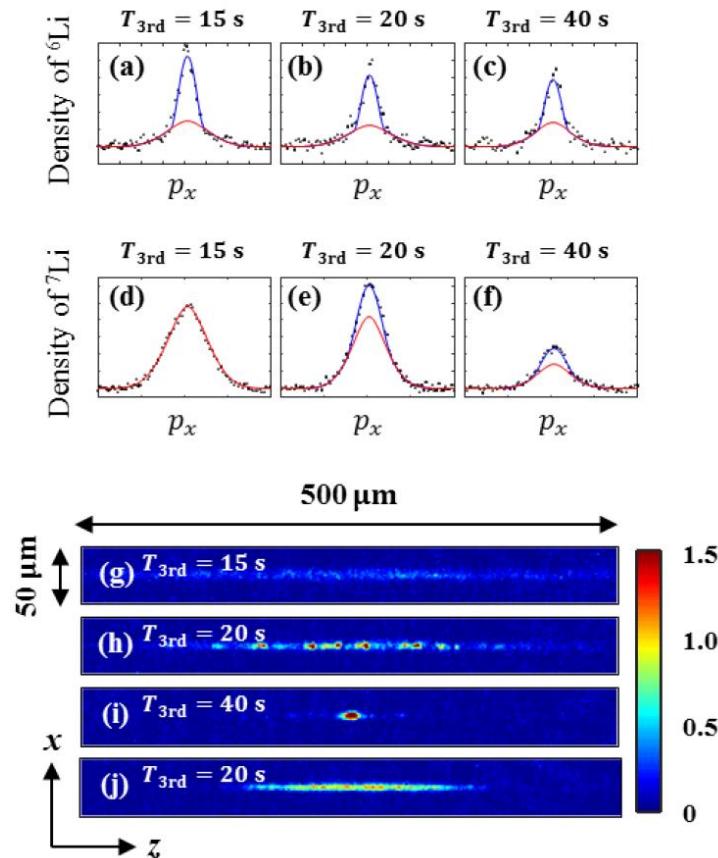
A. Behle, *et al.*, Nat. Phys. **14**, 781 (2018).



# Possible candidates for Simulating SUSY in Ultracold Bose-Fermi mixtures

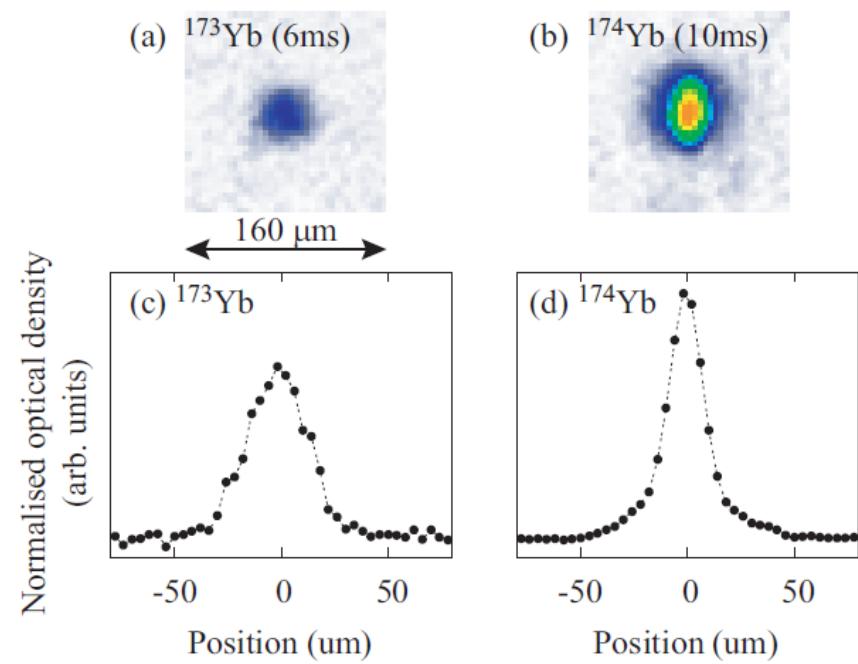
## ${}^6\text{Li}$ (Fermi)- ${}^7\text{Li}$ (Bose)

T. Ikemachi, *et al.*, J. Phys. B **50**, 01LT01 (2016).



## ${}^{173}\text{Yb}$ (Fermi)- ${}^{174}\text{Yb}$ (Bose)

T. Fukuhara, *et al.*, Phys. Rev. A **79**, 021601(R) (2009).



# Possible candidates for Simulating SUSY in Ultracold Bose-Fermi mixtures

${}^6\text{Li}$  (Fermi)- ${}^7\text{Li}$  (Bose)

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${}^{173}\text{Yb}$  (Fermi)- ${}^{174}\text{Yb}$  (Bose)

T. Fukuhara, *et al.*, Phys. Rev. A **79**, 021601(R) (2009).

**It is not completely supersymmetric  
→ Goldstino gap due to explicit breaking?**

Explicit breakings	${}^6\text{Li}$ - ${}^7\text{Li}$ mixture	${}^{173}\text{Yb}$ - ${}^{174}\text{Yb}$ mixture
Mass ratio $m_b/m_f$	7/6	174/173
Chemical potential diff. $\Delta\mu = \mu_f - \mu_b$	Controllable	Controllable
Interaction diff. $\Delta U = U_{bb} - U_{bf}$	$U_{bf}$ cannot be changed $U_{bb}$ → Feshbach resonance	$U_{bf}$ and $U_{bb}$ are fixed

$U_{bf}$ : boson-fermion interaction,  $U_{bb}$ : boson-boson interaction

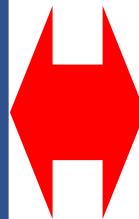
# Goal of this talk

- We theoretically investigate how the Goldstino acquires the energy gap in the presence of explicitly supersymmetry breakings in ultracold Bose-Fermi mixture.
- We also discuss effects of Goldstino on the fermionic spectral function which can be observed by spectroscopic measurements.

Nambu-Goldstone boson  
in high-energy physics

Pion (gapped mode)

With explicit chiral symmetry breaking



Nambu-Goldstone fermion  
in condensed matter physics

“Goldstino” (gapped mode)

With explicit **supersymmetry** breaking

# Outline

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- Formalism
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# Model Hamiltonian

We consider a repulsively interacting non-relativistic Bose-Fermi mixture

$$\begin{aligned} H &= H_{0f} + H_{0b} + V_{bb} + V_{bf} + V_{ff} \\ &= \int d^d \mathbf{r} \psi_f^\dagger(\mathbf{r}) \left( \frac{-\nabla^2}{2m_f} - \mu_f \right) \psi_f(\mathbf{r}) + \int d^d \mathbf{r} \psi_b^\dagger(\mathbf{r}) \left( \frac{-\nabla^2}{2m_b} - \mu_b \right) \psi_b(\mathbf{r}) \\ &\quad + \frac{1}{2} \int d^d \mathbf{r} d^d \mathbf{r}' \psi_b^\dagger(\mathbf{r}) \psi_b^\dagger(\mathbf{r}') U_{bb}(\mathbf{r} - \mathbf{r}') \psi_b(\mathbf{r}') \psi_b(\mathbf{r}) \\ &\quad + \int d^d \mathbf{r} d^d \mathbf{r}' \psi_b^\dagger(\mathbf{r}) \psi_f^\dagger(\mathbf{r}') U_{bf}(\mathbf{r} - \mathbf{r}') \psi_f(\mathbf{r}') \psi_b(\mathbf{r}) \\ &\quad + \frac{1}{2} \int d^d \mathbf{r} d^d \mathbf{r}' \psi_f^\dagger(\mathbf{r}) \psi_f^\dagger(\mathbf{r}') U_{ff}(\mathbf{r} - \mathbf{r}') \psi_f(\mathbf{r}') \psi_f(\mathbf{r}) \end{aligned}$$

$\psi_f$ : annihilation operator of a fermion

$\mu_f$ : chemical potential of a fermion

$\psi_b$ : annihilation operator of a boson

$\mu_b$ : chemical potential of a boson

$m_f$ : mass of a fermion

$U_{bf}$ : boson-fermion repulsion

$m_b$ : mass of a boson

$U_{bb(f)}: U_{bb} + U_{ff}$ : boson-boson (fermion-fermion) repulsion

# Model Hamiltonian

We assume that all interactions are weak s-wave scatterings.

$$\begin{aligned} H &= H_{0f} + H_{0b} + V_{bb} + V_{bf} \\ &= \int d^d \mathbf{r} \psi_f^\dagger(\mathbf{r}) \left( \frac{-\nabla^2}{2m_f} - \mu_f \right) \psi_f(\mathbf{r}) + \int d^d \mathbf{r} \psi_b^\dagger(\mathbf{r}) \left( \frac{-\nabla^2}{2m_b} - \mu_b \right) \psi_b(\mathbf{r}) \\ &\quad + \frac{U_{bb}}{2} \int d^d \mathbf{r} \psi_b^\dagger(\mathbf{r}) \psi_b^\dagger(\mathbf{r}) \psi_b(\mathbf{r}) \psi_b(\mathbf{r}) \\ &\quad + U_{bf} \int d^d \mathbf{r} \psi_b^\dagger(\mathbf{r}) \psi_f^\dagger(\mathbf{r}) \psi_f(\mathbf{r}) \psi_b(\mathbf{r}) \end{aligned}$$

$U_{bb} = \frac{4\pi a_{bb}}{m_b}$	$U_{bf} = \frac{2\pi a_{bf}}{m_r}$	$a_{bb(bf)}$ : s-wave scattering length $U_{ff} = 0$ due to Pauli principle $m_r^{-1} = m_f^{-1} + m_b^{-1}$ : reduced mass
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$\psi_f$ : annihilation operator of a fermion     $\mu_f$ : chemical potential of a fermion

$\psi_b$ : annihilation operator of a boson     $\mu_b$ : chemical potential of a boson

$m_f$ : mass of a fermion

$U_{bf}$ : boson-fermion repulsion

$m_b$ : mass of a boson

$U_{bb}$ : boson-boson repulsion

# Supercharge

$$Q = \int d^d \mathbf{r} q(\mathbf{r}) = \int d^d \mathbf{r} \psi_b^\dagger(\mathbf{r}) \psi_f(\mathbf{r})$$

Commutation relation with  $H$    Supersymmetric when  $[H, Q] = 0$

$$[H, Q] = \chi \int d^d \mathbf{r} \psi_b^\dagger(\mathbf{r}) \frac{\nabla^2}{2m_r} \psi_f(\mathbf{r}) + \Delta\mu Q$$

$$-\Delta U \int d^d \mathbf{r} \psi_b^\dagger(\mathbf{r}) \psi_b^\dagger(\mathbf{r}) \psi_b(\mathbf{r}) \psi_f(\mathbf{r})$$

Explicitly supersymmetry breaking parameters

Mass

$$\chi = \frac{m_b - m_f}{m_b + m_f}$$

Chemical potential

$$\Delta\mu = \mu_f - \mu_b$$

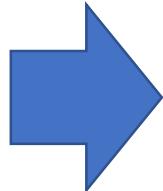
Interaction

$$\Delta U = U_{bf} - U_{bb}$$

# Gell-Mann-Oakes-Renner relation

- The relation for the pion mass (NG boson mass) and the current quark mass (explicitly breaks the chiral symmetry) in QCD can be utilized for the **Goldstino gap**.

$$M_\pi^2 \propto m_q$$



$$\begin{aligned}\omega_G &= -\frac{1}{N} \langle \{[H, Q], q^\dagger(\mathbf{0}, 0)\} \rangle \\ &= -\Delta\mu + \chi \langle \mathcal{E} \rangle + \frac{2\Delta U}{U} \langle V \rangle\end{aligned}$$

Total density

$$N = N_b + N_f$$

$$N_i = \langle \psi_i^\dagger(\mathbf{r}) \psi_i(\mathbf{r}) \rangle$$

Average coupling

$$U = \frac{U_{bf} + U_{bb}}{2}$$

Average kinetic energy

$$\langle \mathcal{E} \rangle = \frac{1}{N} \left\langle \psi_b^\dagger(\mathbf{r}) \frac{-\nabla^2}{2m_r} \psi_b(\mathbf{r}) + \psi_f^\dagger(\mathbf{r}) \frac{-\nabla^2}{2m_r} \psi_f(\mathbf{r}) \right\rangle$$

Average interaction energy

$$\langle V \rangle = \frac{1}{N} \left\langle \frac{U}{2} \psi_b^\dagger(\mathbf{r}) \psi_b^\dagger(\mathbf{r}) \psi_b(\mathbf{r}) \psi_b(\mathbf{r}) + U \psi_b^\dagger(\mathbf{r}) \psi_b(\mathbf{r}) \psi_f^\dagger(\mathbf{r}) \psi_f(\mathbf{r}) \right\rangle$$

# Gell-Mann-Oakes-Renner relation

Exact at up to the first order of explicit breakings  $X = \Delta\mu, \chi, \Delta U$

$$\begin{aligned}\omega_G &= -\Delta\mu + \chi\langle\mathcal{E}\rangle + \frac{2\Delta U}{U}\langle V\rangle + O(X^2) \\ &= -\Delta\mu + \chi\langle\mathcal{E}\rangle + \frac{1}{2\pi N} \left( \frac{a_{bf}}{2m_r} - \frac{a_{bb}}{m_b} \right) \left( \frac{\mathcal{C}_{bf}}{a_{bf}^2} + \frac{\mathcal{C}_{bb}}{a_{bb}^2} \right) \\ &\quad + O(X^2)\end{aligned}$$

Tan's contacts S. Tan, Ann. Phys. **323**, 2952, (2008).

$$\left( \frac{\partial E}{\partial a_{bf}^{-1}} \right)_{S,N_i} = \frac{\mathcal{C}_{bf}}{2\pi m_r} \quad \left( \frac{\partial E}{\partial a_{bb}^{-1}} \right)_{S,N_i} = \frac{\mathcal{C}_{bb}}{8\pi m_b}$$

$E$ : internal energy       $S$ : entropy

# Mean-field approximation

We evaluate the expectation values in the GOR relation above  $T_{\text{BEC}}$  in 3D

$$\langle \psi_b^\dagger(\mathbf{r}) \psi_b(\mathbf{r}) \psi_f^\dagger(\mathbf{r}) \psi_f(\mathbf{r}) \rangle \simeq N_b N_f \quad \langle \psi_b^\dagger(\mathbf{r}) \psi_b^\dagger(\mathbf{r}) \psi_b(\mathbf{r}) \psi_b(\mathbf{r}) \rangle \simeq 2N_b^2$$

► Kinetic energy

$$\langle \mathcal{E} \rangle_{HF} = \frac{1}{N} \int \frac{d^3 k}{(2\pi)^3} [n_f(\xi_k^f) + n_b(\xi_k^b)] \frac{k^2}{2m_r}$$

$$n_i: \text{distribution functions}$$

$$\xi_k^i = \frac{k^2}{2m_i} - \mu_i + \Sigma_i^H$$

► Number densities

$$N_f = T \int \frac{d^3 p}{(2\pi)^3} \sum_{i\omega_n} G_f(\mathbf{p}, i\omega_n) \quad N_b = -T \int \frac{d^3 q}{(2\pi)^3} \sum_{\mathbf{q}, i\nu_s} G_b(\mathbf{q}, i\nu_s)$$

► Single-particle Green's function

$$G_f(\mathbf{p}, i\omega_n) = \frac{1}{i\omega_n - \frac{p^2}{2m_f} + \mu_f - \Sigma_f^H}$$

$$G_b(\mathbf{q}, i\nu_s) = \frac{1}{i\nu_s - \frac{q^2}{2m_b} + \mu_b - \Sigma_b^H}$$

► Hartree shift

$$\Sigma_f^H = U_{bf} N_f$$

$$\Sigma_b^H = U_{bf} N_f + 2U_{bb} N_b$$

# Random phase approximation for Goldstino propagator

GOR relation is exact up to the first order of explicit breakings  
→ We check the validity of GOR by comparing it with RPA

$$\Gamma = \Pi + U_{bf} + \dots$$

$$\Gamma^R(\mathbf{p}, \omega) = \frac{\Pi(\mathbf{p}, \omega)}{1 + U_{bf}\Pi(\mathbf{p}, \omega)}$$

Lowest-order bubble for fermion-boson exchange

$$\Pi(\mathbf{p}, \omega) = - \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{n_f(\xi_{\mathbf{k}}^f) + n_b(\xi_{\mathbf{k}-\mathbf{p}}^b)}{\omega + i\delta + \xi_{\mathbf{k}-\mathbf{p}}^b - \xi_{\mathbf{k}}^f}$$

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# Thermodynamics at $\Delta\mu = \Delta U = 0$

It is necessary to obtain the gaps

$N_f < N_b$  at weak coupling

$N_f \simeq N_b$  at strong coupling

► Hugenholtz-Pinez condition

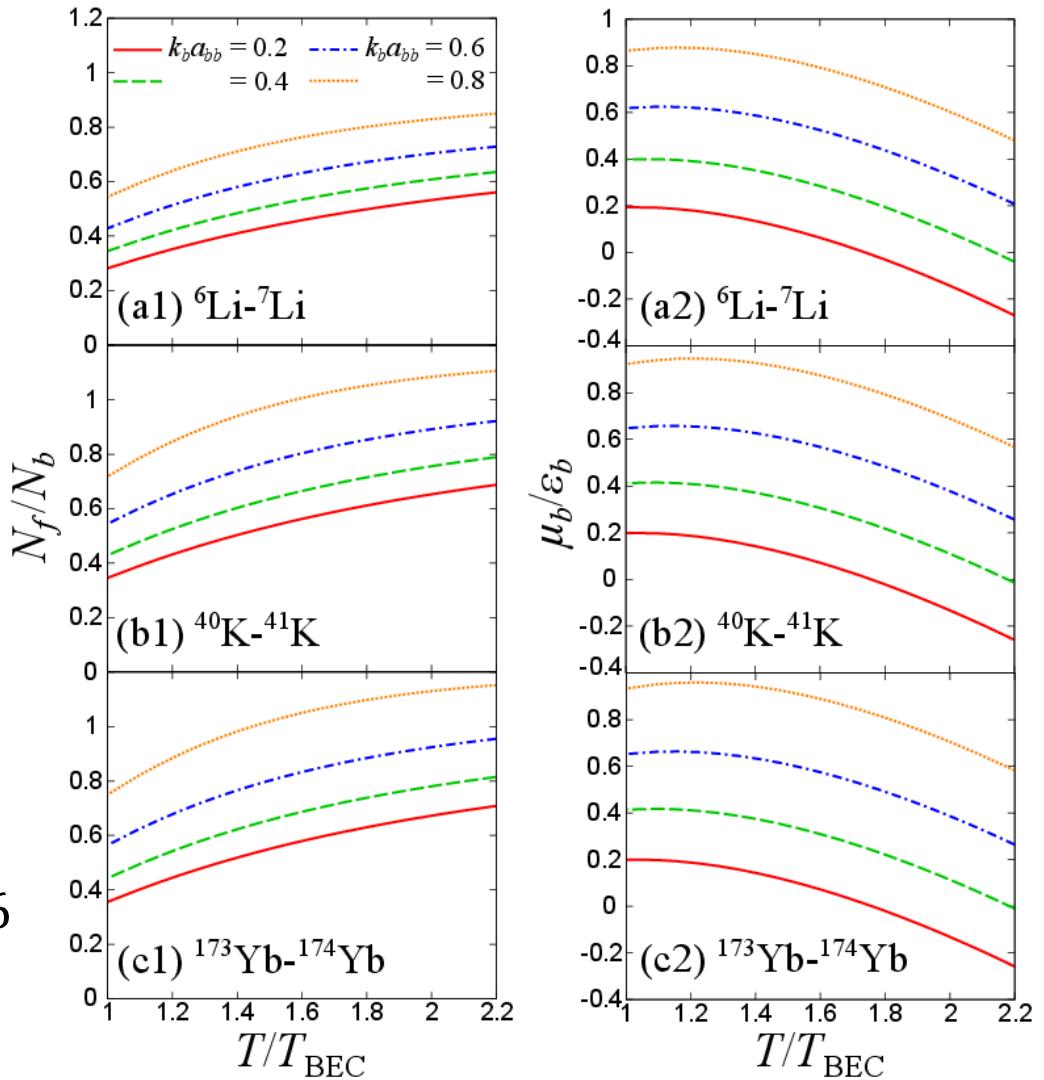
$$\mu_b - \Sigma_b^H = 0$$

$$\mu_b(T_{\text{BEC}}) = U_{bf}N_f + 2U_{bb}N_b$$

► Stability condition (at  $T = 0$ )

$$k_b a_{bb} \leq \pi \frac{m_b}{m_f + m_b} \left( \frac{N_b}{N_f} \right)^{\frac{1}{3}} \sim 1.6$$

L. Viverit, *et al.*, PRA **61**, 053605 (2000).



# Mass-balanced case ( $m_f = m_b$ )

The Goldstino gap  $\omega_G$  within RPA can analytically be obtained from

$$\Gamma^R(\mathbf{p} = \mathbf{0}, \omega_G^{\text{RPA}}) = 0 \rightarrow 1 + U_{bf}\Pi(\mathbf{p} = \mathbf{0}, \omega_G^{\text{RPA}}) = 0$$

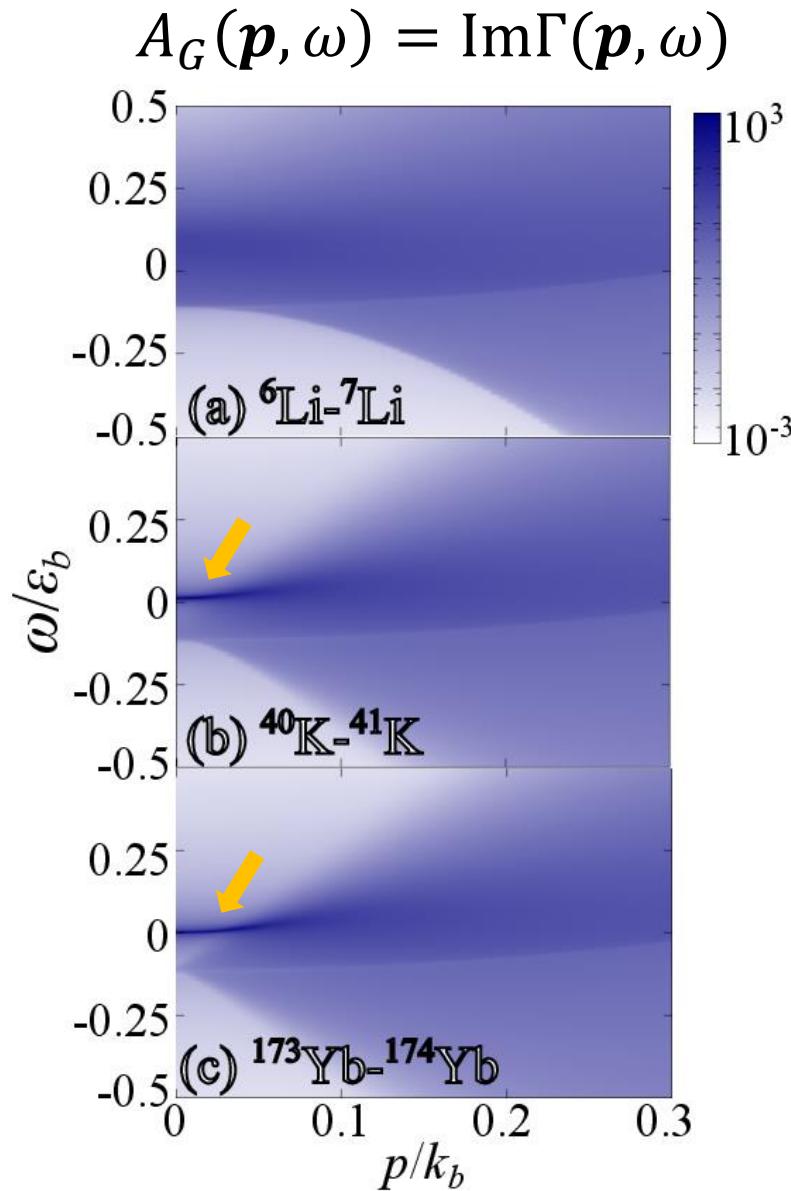


$$\omega_G^{\text{RPA}} = -\Delta\mu + 2N_b\Delta U$$

Consistent with the GOR relation!

The Goldstino gap can be estimated from the observables  $\mu_f$ ,  $\mu_b$ ,  $N_b$ ,  $a_{bf}$ , and  $a_{bb}$

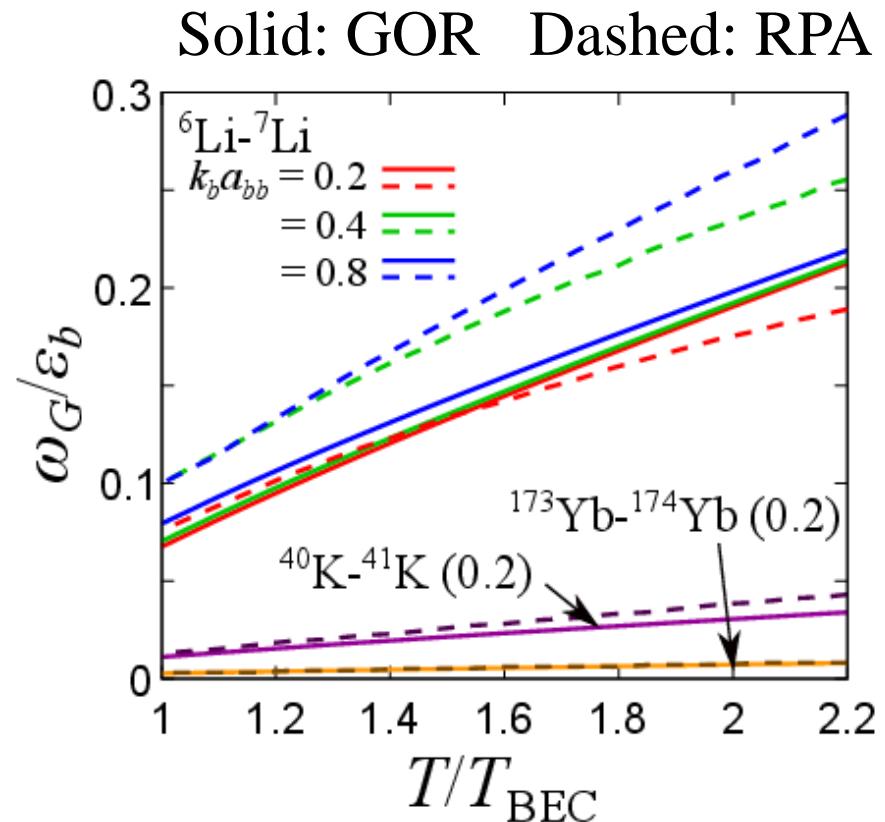
# Mass-imbalanced case ( $m_f \neq m_b$ )



$$*\Delta\mu = \Delta U = 0$$

$$\omega_G^{\text{RPA}} \neq \omega_G^{\text{GOR}}$$

Comparison of the Goldstino gaps  $\omega_G$



# Why $\omega_G^{RPA} \neq \omega_G^{GOR}$ when $m_f \neq m_b$ ?

A. The expansion with respect to the mass-imbalance  $\chi$  is not valid when the Goldstino pole is close to the branch point  $\omega_{BP}$

$$\begin{aligned}\omega_{BP} &= \xi_0^f - \xi_0^b = -\Delta\mu + 2N_b\Delta U - U_{bf}N \\ &= \omega_G^{GOR} - \chi\langle\mathcal{E}\rangle_{HF} - U_{bf}N\end{aligned}$$

$$1 + U_{bf}\Pi(\mathbf{0}, \omega) = \frac{1}{U_{bf}N} [\omega - \omega_G^{GOR} - \tilde{\Phi}(\omega)]$$

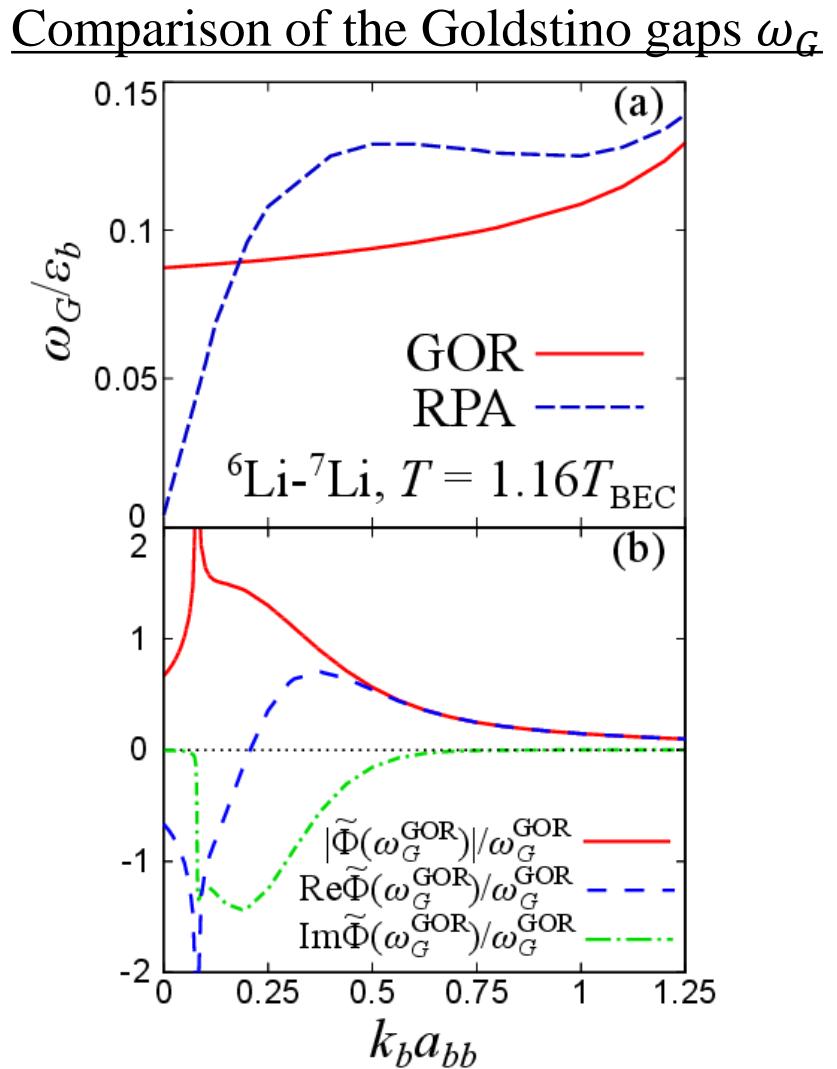
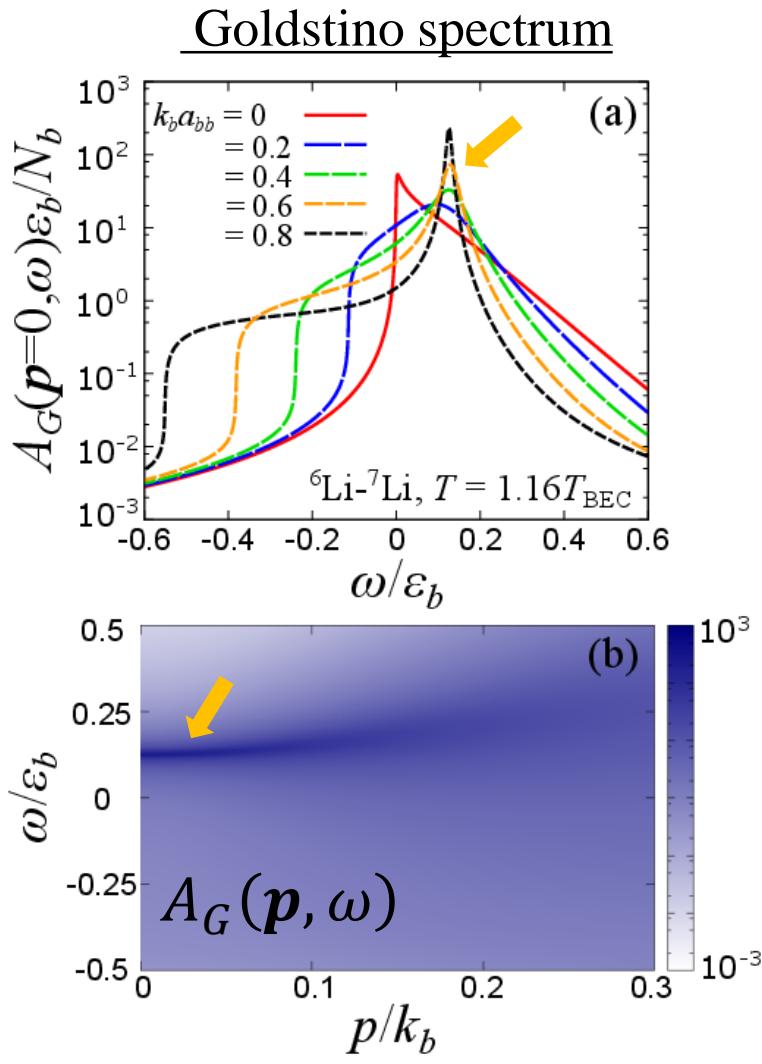
$$\tilde{\Phi}(\omega_G^{GOR}) = \frac{\chi^2}{N} \int \frac{d^3k}{(2\pi)^3} [n_f(\xi_k^f) + n_b(\xi_k^b)] \frac{[k^2/(2m_r) - \langle\mathcal{E}\rangle_{HF}]^2}{\omega_G^{GOR} - \omega_{BP} - \chi k^2/(2m_r) + i\delta}$$

The asymptotic expansion is not always convergent!

$$\tilde{\Phi}(\omega_G^{GOR}) \sim \chi^2 \sum_{n=0}^{n=\infty} \left| \frac{\chi\langle\mathcal{E}\rangle_{HF}}{\omega_G^{GOR} - \omega_{BP}} \right|^n \quad \xrightarrow{\hspace{1cm}} \quad U_{bf} \gg \frac{\chi\langle\mathcal{E}\rangle_{HF}}{N}$$

# $U_{bf}$ dependence in ${}^6\text{Li}-{}^7\text{Li}$ mixtures

$\omega_G^{\text{RPA}} \simeq \omega_G^{\text{GOR}}$  at strong  $U_{bf}$



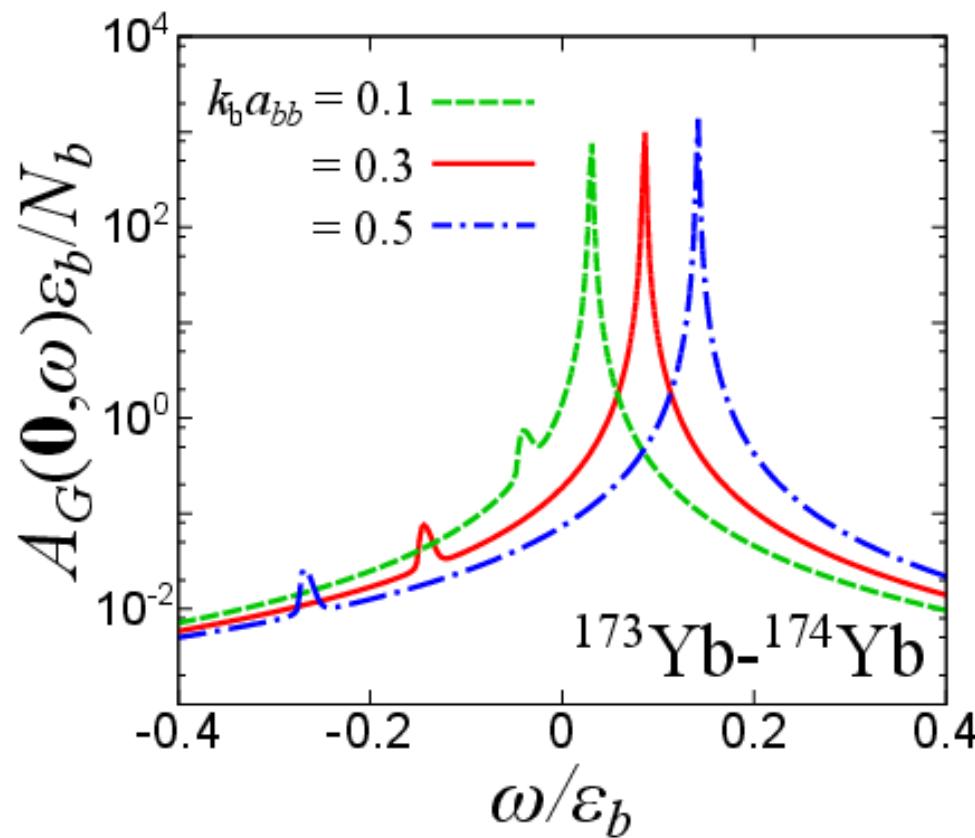
# $^{173}\text{Yb}$ - $^{174}\text{Yb}$ mixture at $T = T_{\text{BEC}}$

Observed scattering lengths

$$a_{bf} = 7.34 \text{ nm} \quad a_{bb} = 5.55 \text{ nm}$$

M. Kitazawa, *et al.*, PRA 77, 012719 (2008).

We can see a sharp peak of the Goldstino with realistic parameters!



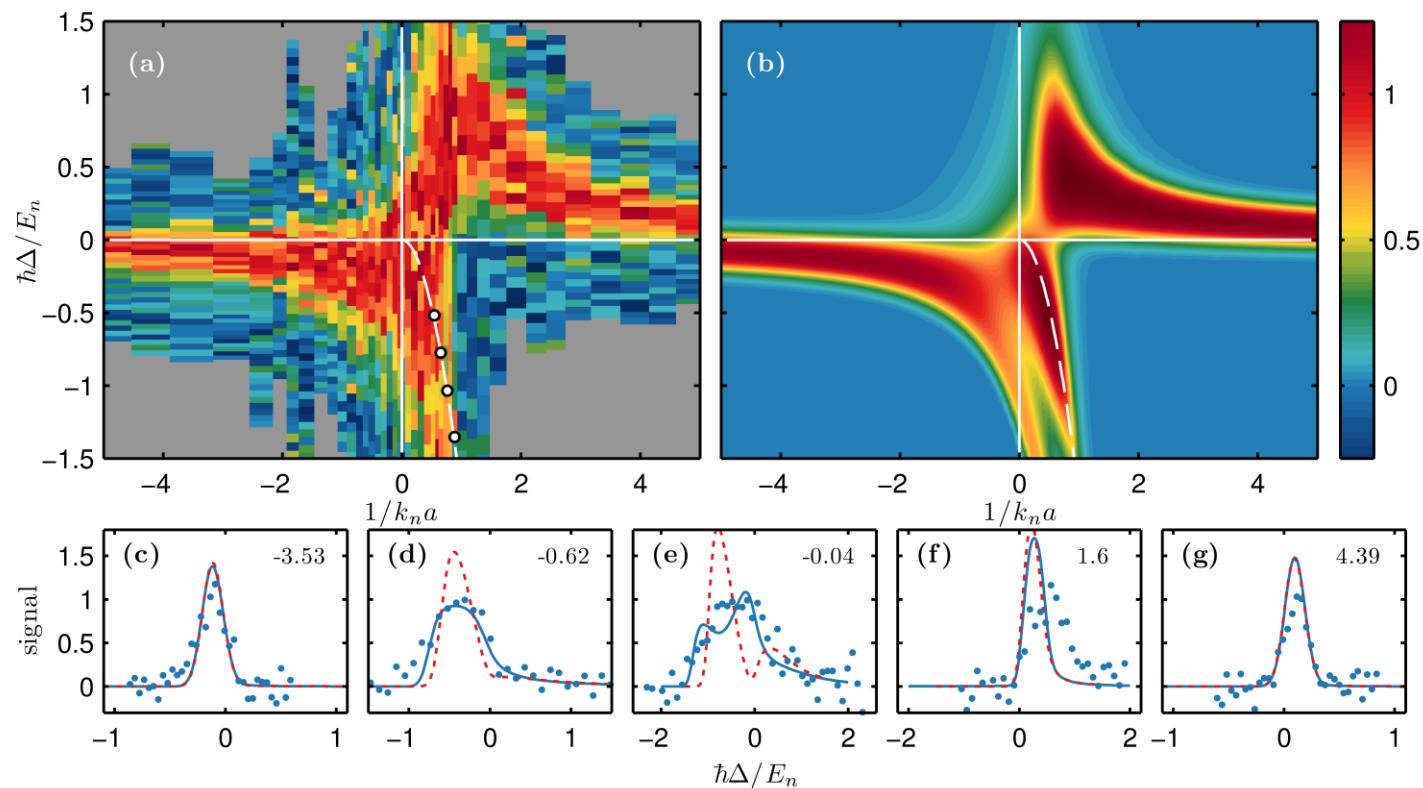
# How can we observe the Goldstino?

## Single-particle spectral function

Example: RF spectra of impurity atoms immersed in BEC

$$H_{RF} = \Omega e^{-i\omega_{RF}t} \sum_{\mathbf{k}} a_{\mathbf{k},i}^\dagger a_{\mathbf{k},j} + \text{h. c.}$$

N. B. Jorgensen, *et al.*, PRL **117**, 055302 (2016).



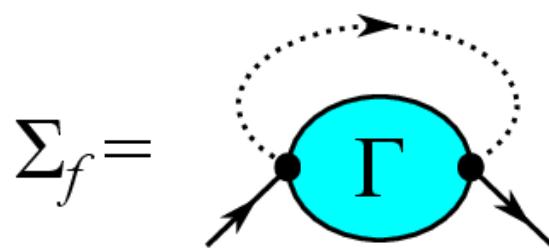
# Fermionic single-particle spectra

$$A_f(\mathbf{p} = \mathbf{0}, \omega) = -\frac{1}{\pi} \text{Im} \tilde{G}_f(\mathbf{p} = \mathbf{0}, i\omega_\ell \rightarrow \omega + i\delta)$$

- ▶ Dressed Green's function beyond the mean-field approach

$$\tilde{G}_f(\mathbf{p}, i\omega_\ell) = \frac{1}{i\omega_\ell - \frac{\mathbf{p}^2}{2m_f} + \mu_f - U_{bf}N_b - \Sigma_f(\mathbf{p}, i\omega_\ell)}$$

- ▶ Fermion self-energy with supersymmetric fluctuations

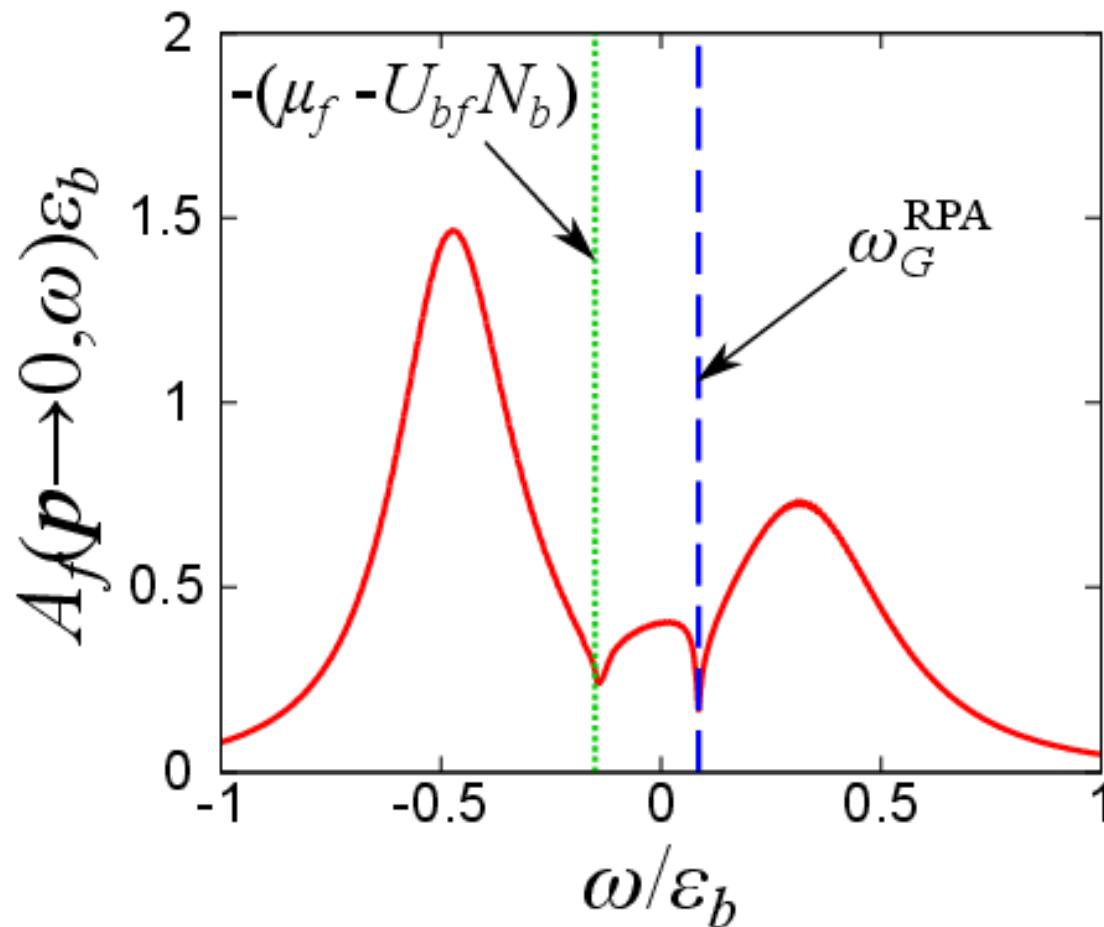


$$\Sigma_f(\mathbf{p}, i\omega_\ell) = -U_{bf}^2 T \sum_{i\omega_n} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \Gamma(\mathbf{k}, i\omega_n) G_b(\mathbf{p} - \mathbf{k}, i\omega_\ell - i\omega_n)$$

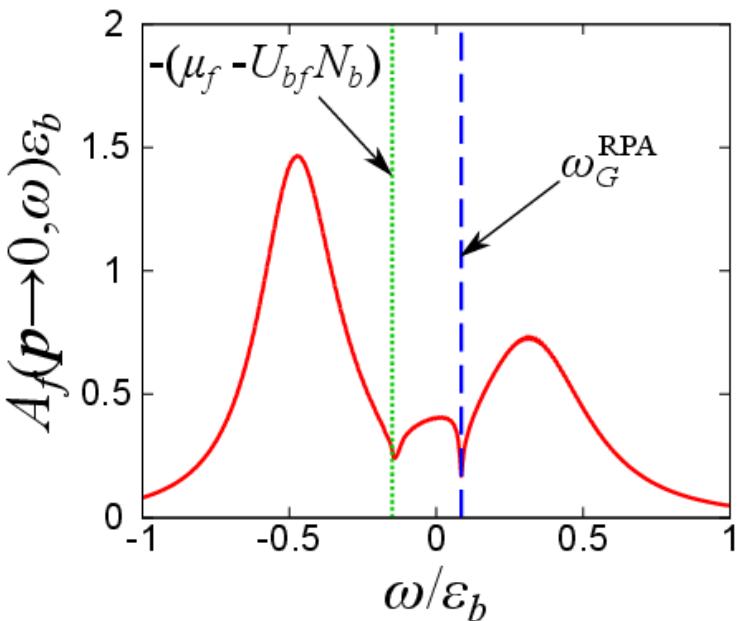
# Fermionic single-particle spectra

- Double-peak structure appears due to the Goldstino pole!

$^{173}\text{Yb}$ - $^{174}\text{Yb}$  mixture at  $T = T_{\text{BEC}}$  with  $k_b a_{bb} = 0.3$



# Origin of double-peak: level repulsion



The Goldstino pole contribution

$$\Gamma(\mathbf{k}, i\omega_n) \simeq \frac{Z_G}{i\omega_n - E_{\mathbf{k}}}$$

$$E_{\mathbf{k}} = \frac{k^2}{2m_G} + \omega_G$$

$Z_G$ : Goldstino residue

$E_{\mathbf{k}}$  : Goldstino dispersion

$$\Sigma_f(\mathbf{p}, i\omega_\ell) \simeq U_{bf}^2 Z_G \int \frac{d^3 k}{(2\pi)^3} \frac{1 - n_f(E_{\mathbf{p}-\mathbf{k}}) + n_b(\xi_{\mathbf{k}}^b)}{i\omega_\ell - E_{\mathbf{p}-\mathbf{k}} - \xi_{\mathbf{k}}^b}$$

$$\simeq \frac{U_{bf}^2 Z_G N_b}{i\omega_\ell - E_{\mathbf{p}}} \quad \text{Static approximation}$$

D. Kharga, et al., JPSJ **86**, 084301 (2017).

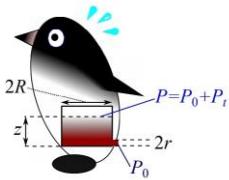
→  $A_f(0, \omega) \simeq \alpha_+ \delta(\omega - E_+) + \alpha_- \delta(\omega - E_-)$

$$E_{\pm} = \frac{\omega_G - \mu_f + U_{bf} N_b}{2} \pm \sqrt{\left( \frac{\omega_G + \mu_f - U_{bf} N_b}{2} \right)^2 + U_{bf}^2 Z_G N_b}$$

$$\alpha_{\pm} = \frac{1}{2} \left( 1 \mp \frac{\omega_G + \mu_f - U_{bf} N_b}{\sqrt{(\omega_G + \mu_f - U_{bf} N_b)^2 + 4U_{bf}^2 Z_G N_b}} \right)$$

# Outline

- Introduction
- Formalism
- Results
- Summary



# Summary

- We investigate how Goldstino acquires the energy gap in the presence of explicit supersymmetry breaking in ultracold Bose-Fermi mixture.
- We derived the Gell-Mann-Oakes-Renner relation for Goldstino and check its validity by comparing it with the random phase approximation.
- We showed that the Goldstino pole can be visible in the fermionic single-particle spectral function.

Future work: BEC phase, proposal for other observables, non-local interactions, etc...

