Toward simulating Superstring/M-theory on a Quantum Computer

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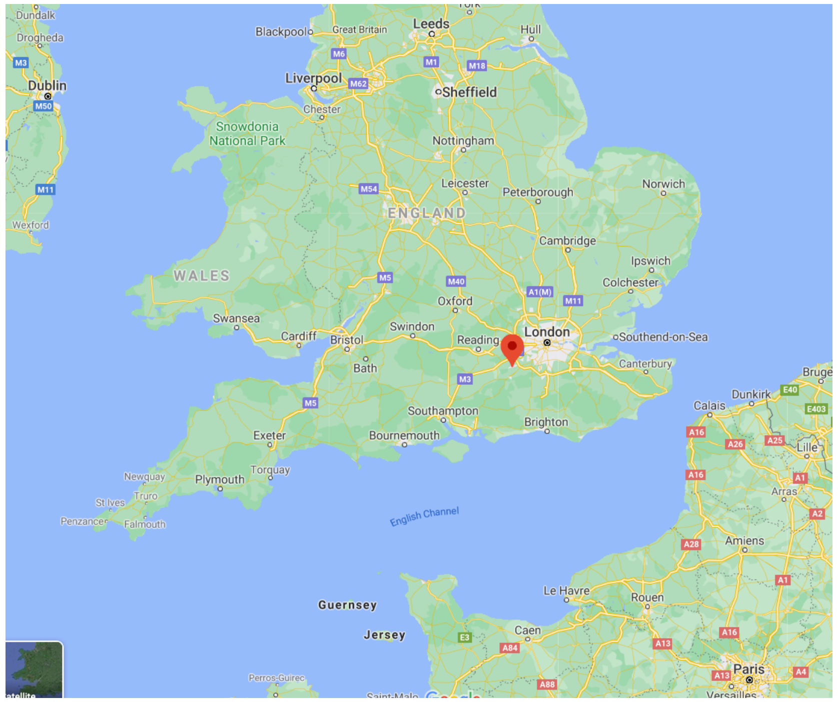

with Alex Buser, Hrant Gharibyan, Junyu Liu (Caltech), Masazumi Honda (Kyoto)

Gharibyan-MH-Honda-Liu, to appear (1st part)
Buser-Gharibyan-MH-Honda-Liu, to appear (2nd part)

Oct. 23, 2020, (virtually) @RIKEN

- Introduction
- BMN matrix model
- QFT from BMN matrix model
- Quantum simulation of BMN matrix model and QFT
- Orbifold lattice construction

QFT can be defined (or regularized) by using lattice.

$\boxed{\text{Holographic Principle}}$

Quantum Gravity can be defined (or regularized) by using QFT.

$\boxed{\text{Quantum Gravity can be defined (or regularized) by using lattice QFT.}}$

The Large N Limit of Superconformal field theories and supergravity

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By deriving various field theories from string theory and considering their large N limit we have shown that they contain in their Hilbert space excitations describing supergravity on various spacetimes. We further conjectured that the field theories are dual to the full quantum M/string theory on various spacetimes. In principle, we can use this duality to give a definition of M/string theory on flat spacetime as (a region of) the large N limit of the field theories. Notice that this is a non-perturbative proposal for defining such theories, since the corresponding field theories can, *in principle*, be defined non-perturbatively. We

Quantum Gravity can be defined (or regularized) by using lattice QFT.

What kind of QFT?

- (p+1)-d Super Yang-Mills, p=0,1,2,3
- 6d Super-Conformal Theory
- 3d Super-Conformal Theory (ABJM)

Lattice regularization is not easy!

(Known solution only for SYM with p=0,1,2)

Why difficult?

- Exact symmetries at regularized level are needed.
- Otherwise (usually) wrong continuum limit is obtained, due to the radiative corrections.
- Not easy to keep big enough supersymmetry.

Wilson's lattice gauge theory

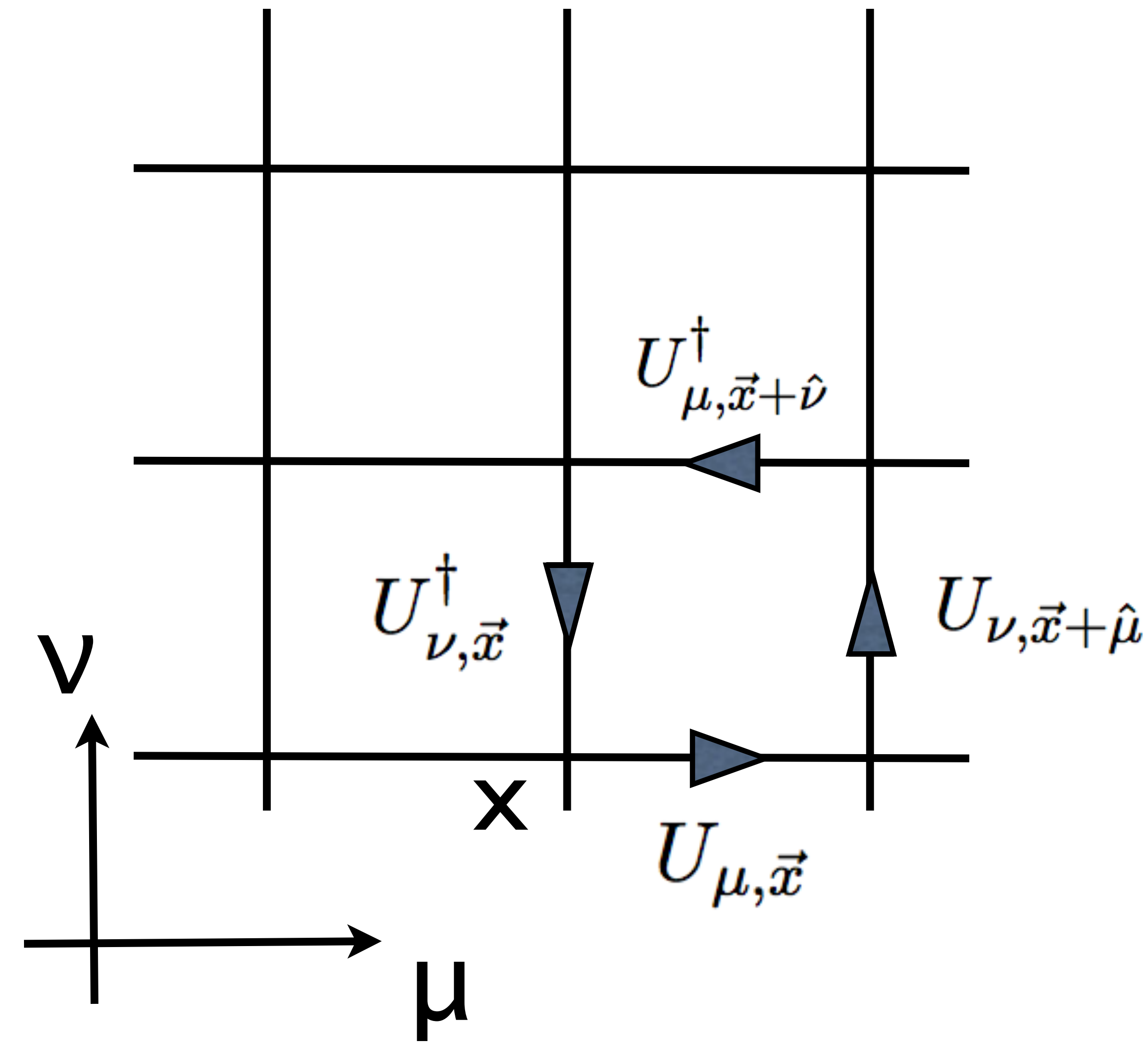
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr\left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}\right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^{\dagger}$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

These symmetries exist *at discretized level*.

‘No-Go’ for lattice SYM

- SUSY algebra contains infinitesimal translation.

$$\{Q, \bar{Q}\} \sim \partial$$

- Infinitesimal translation is broken on lattice by construction.
- Impossible to keep all SUSY on lattice. Radiative corrections spoil SUSY.
- Still it is possible to preserve a part of supercharges, though. (subalgebra which does not contain ∂)

Avoiding ‘No Go’

(Kaplan-Katz-Unsal 2002, Sugino 2003, Catterall 2003, ...)

- Keep a few supercharges exact on lattice.
- Use it (and other discrete symmetries) to forbid SUSY breaking radiative corrections.
- Only “extended” SUSY can be realized for a technical reason.
- Works for (0+1)-, (1+1)- and (2+1)-d SYM.
- <u>Euclidean simulations</u> are successful so far.

Quantum Gravity on a quantum device?

- Real-time features.
  Formation and evaporation of black hole?
  Graviton scattering?
- Direct access to quantum states.
  Emergent geometry?
- No sign problem.

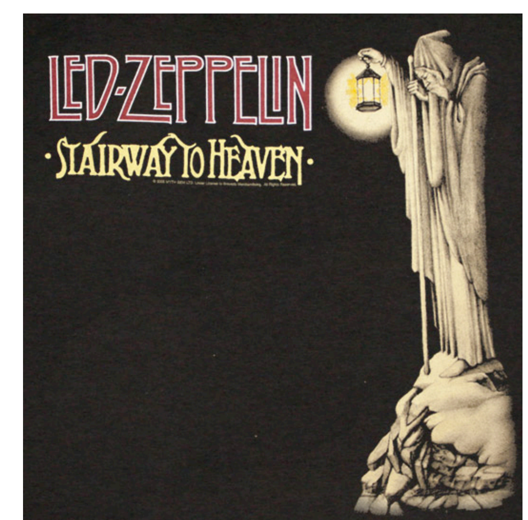
But it is (at least) as hard as Euclidean lattice.

Simulation on Quantum Computer

In the ideal world:

- Direct access to big Hilbert space (qubits).
- Any unitary time evolution can be programmed.

In the real world:



- How can we program the theory?
- How big resources?
- Fine tuning?

QFT on quantum computer

(assuming we have an actual quantum computer)

- Construct lattice Hamiltonian $\hat{H}$.

$$\phi(\vec{x}) \to \hat{\phi}_{\vec{n}}$$

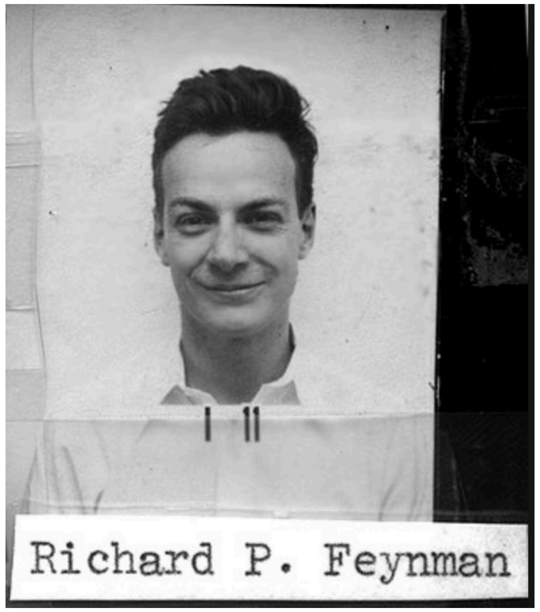
- Truncate the Hilbert space to finite dimension.

$$\hat{\phi}_{\vec{n}}|\phi_{\vec{n}}\rangle = \phi_{\vec{n}}|\phi_{\vec{n}}\rangle \to \hat{\phi}_{\vec{n}}|\phi_{\vec{n}}^{(i)}\rangle = \phi_{\vec{n}}^{(i)}|\phi_{\vec{n}}^{(i)}\rangle \quad (i = 1, \cdots, \Lambda)$$

- Hilbert space cutoff $\Lambda \to \infty$
  then lattice spacing $a \to 0$

Lattice may work but surely complicated.

Any alternative?

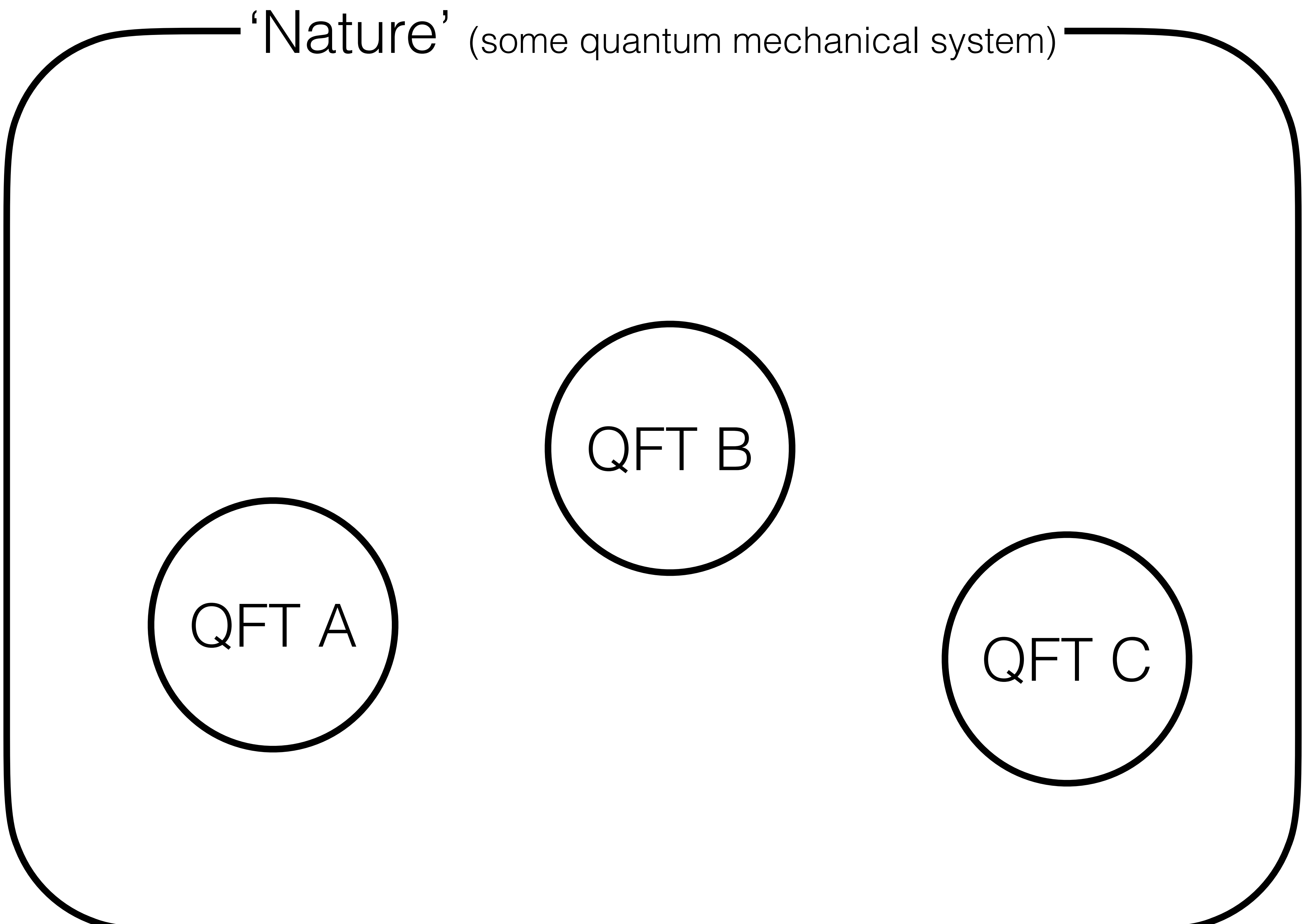


If you want to make a simulation of nature, you'd better make it quantum mechanical.

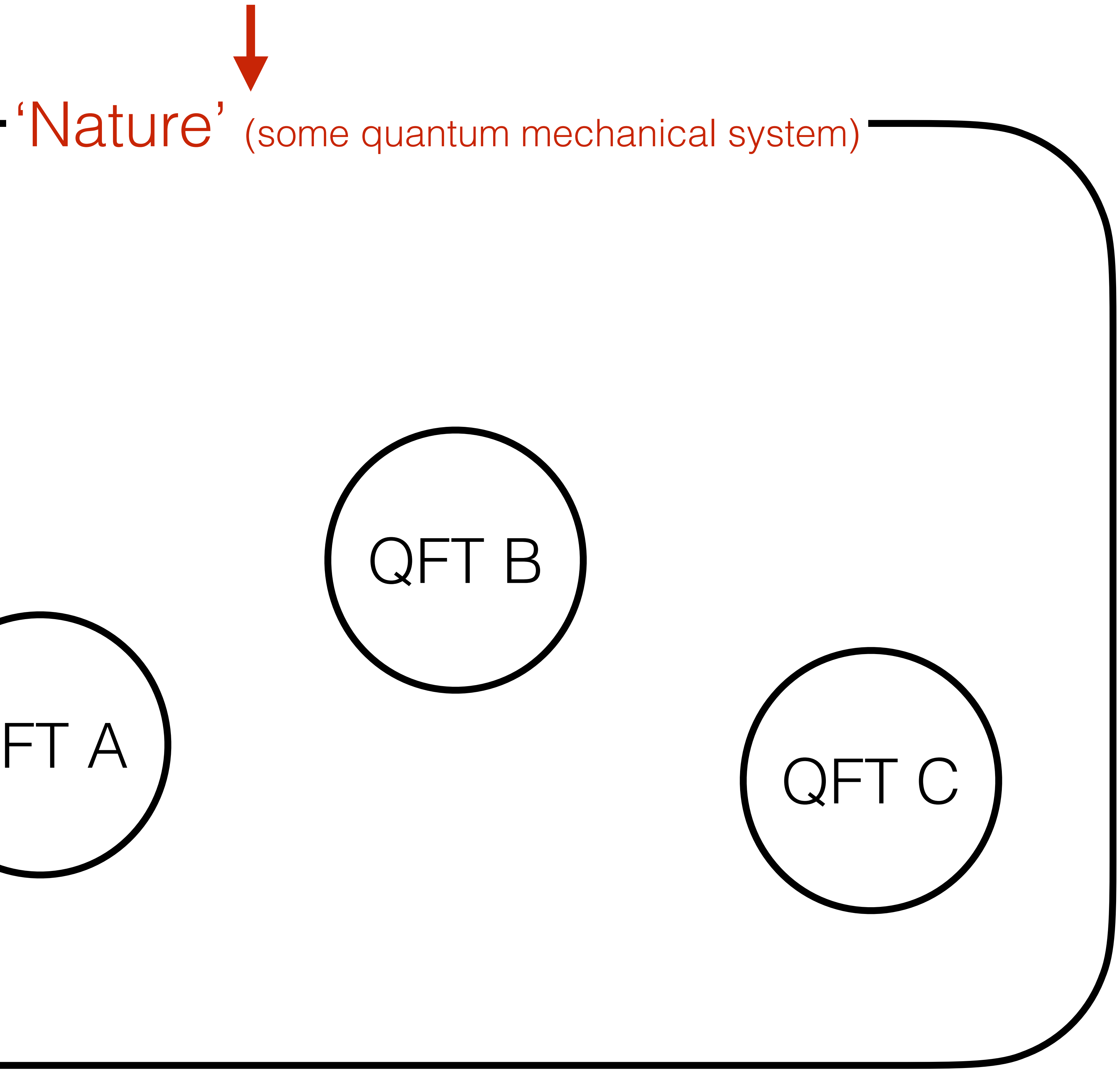
Nature is a quantum computer.

If you realize a QFT as a part of nature, nature takes care of the simulation.

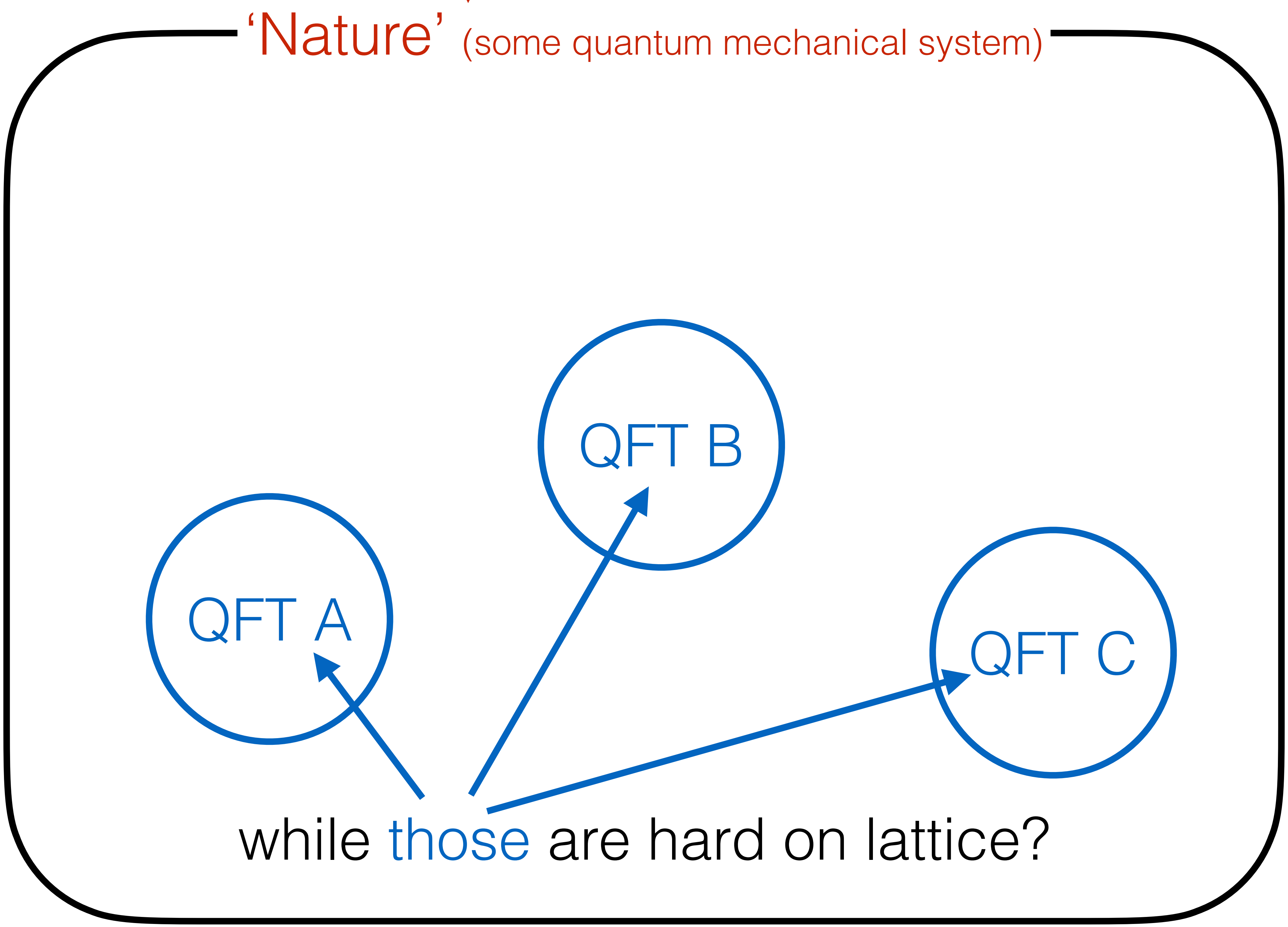
(~Hamiltonian engineering)


‘Nature’ (some quantum mechanical system)
QFT B
QFT A
QFT C

What if this can easily be realized

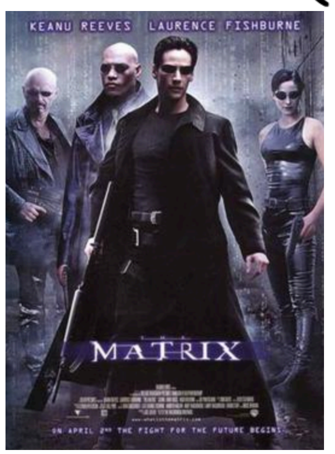


What if this can easily be realized



Build the nature (e.g. supersymmetric matrix model) first.

$$\hat{H} = \mathrm{Tr}\left\{ \frac{1}{2}(\hat{P}_I)^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2 + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k \right.$$



$$+g\hat{\psi}^{\dagger Ip}\sigma_p^{i\,q}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip}g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}]$$

$$\left. +\frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip} \right\}$$

BMN matrix model

$$\boxed{[\hat{X}, \hat{P}] = i\hbar}$$

(Berenstein, Maldacena, Nastase, 2002)

Then prepare appropriate states.

Some supersymmetric backgrounds of plane-wave matrix model

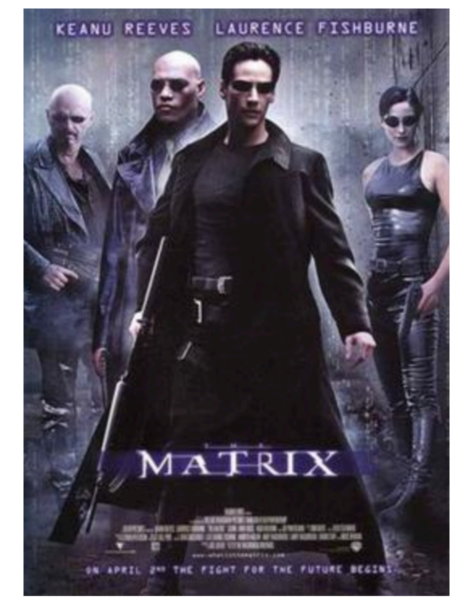
⟶ 3d SYM (any N), 4d SYM (N=∞),
6d superconformal theory (any N)

(Maldacena, Sheikh-Jabbari, Van Raamsdonk 2002,
Asano, Ishiki, Shimasaki, Terashima 2017,
Ishii, Ishiki, Shimasaki, Tsuchiya 2008, ….)

Other matrix models
→ SUSY QCD, QCD on noncommutative space, …

$$\begin{aligned}\hat{H} = \mathrm{Tr}\Bigg\{ & \frac{1}{2}(\hat{P}_I)^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2 + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k \\ & + g\hat{\psi}^{\dagger Ip}\sigma_p^{i\,q}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip} g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] \\ & + \frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip} \Bigg\}\end{aligned}$$

$$\boxed{[\hat{X}, \hat{P}] = i\hbar}$$



- Hamiltonian = harmonic oscillators + some interactions
- Standard Fock basis truncation is good enough
- Truncated Hamiltonian = Σ (product of Pauli matrices)

  → efficient quantum algorithms can be used.
- Gauss law is imposed when the states are prepared.
- Or perhaps the singlet constraint is not important. (Non-singlets are heavy.)

Maldacena-Milekhin 2018; Berkowitz-MH-Rinaldi-Vranas 2018

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4d 'minimal' super Yang-Mills

gauge field gaugino

$$A_{\mu=0,1,2,3},\ \psi_{\alpha=1,2,3,4}$$

$$\mathcal{L} = \mathrm{Tr}(F_{\mu\nu}^2 + \bar{\psi}\gamma^\mu D_\mu \psi)$$

$$D_\mu \psi = \partial_\mu \psi - i[A_\mu, \psi]$$

10d 'minimal' & 'maximal' super Yang-Mills

$$\mathcal{L} = \mathrm{Tr}(F_{\mu\nu}^2 + \bar{\psi}\gamma^\mu D_\mu \psi)$$

'maximal' because
More SUSY → spin > 1

$$A_{\mu=0,1,2,\cdots,9},\ \psi_{\alpha=1,2,\cdots,16}$$

dimensional reduction ↓

4d 'maximal' super Yang-Mills

$$A_{\mu=0,1,2,3}(x_0,\cdots,x_9) \to A_{\mu=0,1,2,3}(x_0,x_1,x_2,x_3)$$

$$A_{\mu=4,5,\cdots,9}(x_0,\cdots,x_9) \to X_{I=1,2,\cdots,6}(x_0,x_1,x_2,x_3)$$

$$\psi_{\alpha=1,2,\cdots,16}(x_0,\cdots,x_9) \to \psi_{\alpha=1,2,\cdots,16}(x_0,\cdots,x_3)$$

dimensional reduction ↓

1d 'maximal' super Yang-Mills = BFSS matrix model

$$A_0(t),\ X_{I=1,2,\cdots,9}(t),\ \psi_{\alpha=1,2,\cdots,16}(t)$$

10d 'minimal' & 'maximal' super Yang-Mills

dimensional reduction

$$A_{\mu=0,1,2,\cdots,9},\ \psi_{\alpha=1,2,\cdots,16}$$

4d 'maximal' super Yang-Mills

1d 'maximal' super Yang-Mills = BFSS matrix model

$$A_0(t), X_{I=1,2,\cdots,9}(t), \psi_{\alpha=1,2,\cdots,16}(t)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$$

$$\rightarrow \partial_0 X_I - \partial_I A_0 - i[A_0, X_I] = \partial_0 X_I - i[A_0, X_I] = D_0 X_I$$

$$\partial_I X_J - \partial_J X_I - i[X_I, X_J] = -i[X_I, X_J]$$

4d 'maximal' super Yang-Mills

on $R^{1,3}$

|

dimensional reduction

↓

BFSS matrix model

on $R\times S^3$ $\quad \text{radius} \sim \frac{1}{\mu}$

|

dimensional reduction

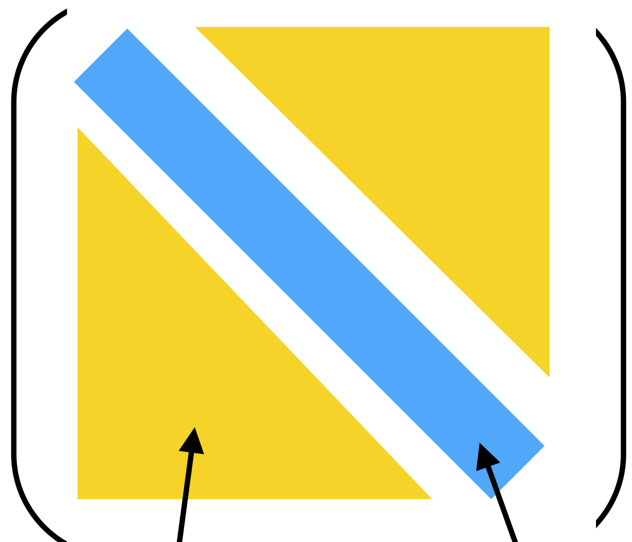
↓

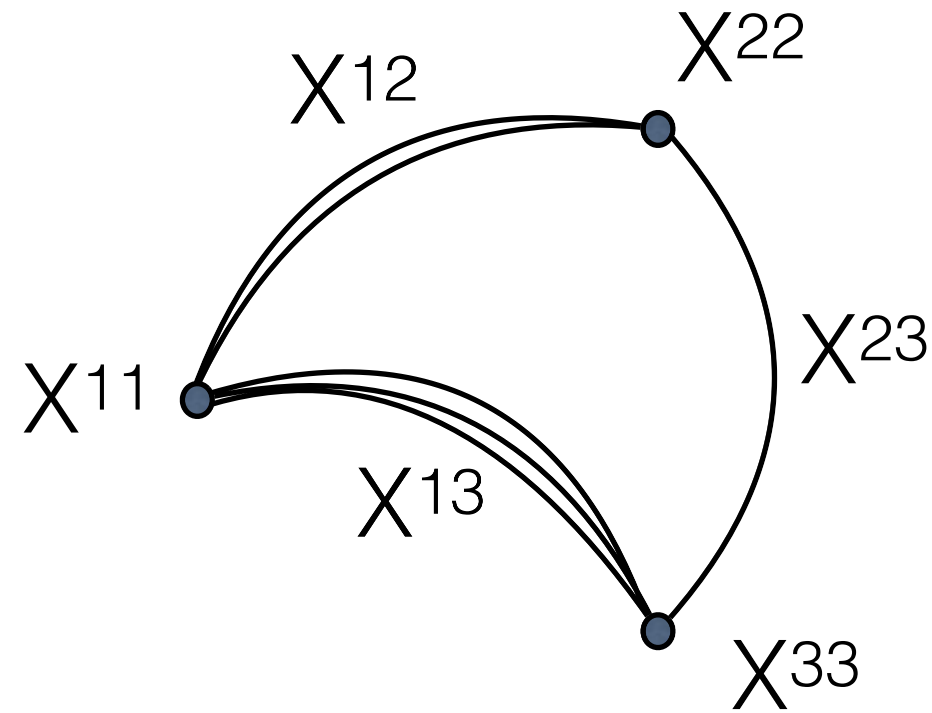
BMN matrix model

$$L = \mathrm{Tr}\left\{ \frac{1}{2}(D_t X_I)^2 + \frac{1}{2}\Psi^T D_t \Psi + \frac{g^2}{4}[X_I, X_J]^2 - \frac{ig}{2}\Psi^T \gamma_I [X_I, \Psi] \right.$$
$$\left. - \frac{\mu^2}{18} X_i^2 - \frac{\mu^2}{72} X_a^2 - \frac{\mu}{8}\Psi^T \gamma_{123}\Psi - \frac{i\mu g}{3}\epsilon^{ijk} X_i X_j X_k \right\}$$

I,J=1,...,9; i,j,k=1,2,3; a=4,...,9

$\boxed{\text{BFSS = BMN @ } \mu=0}$

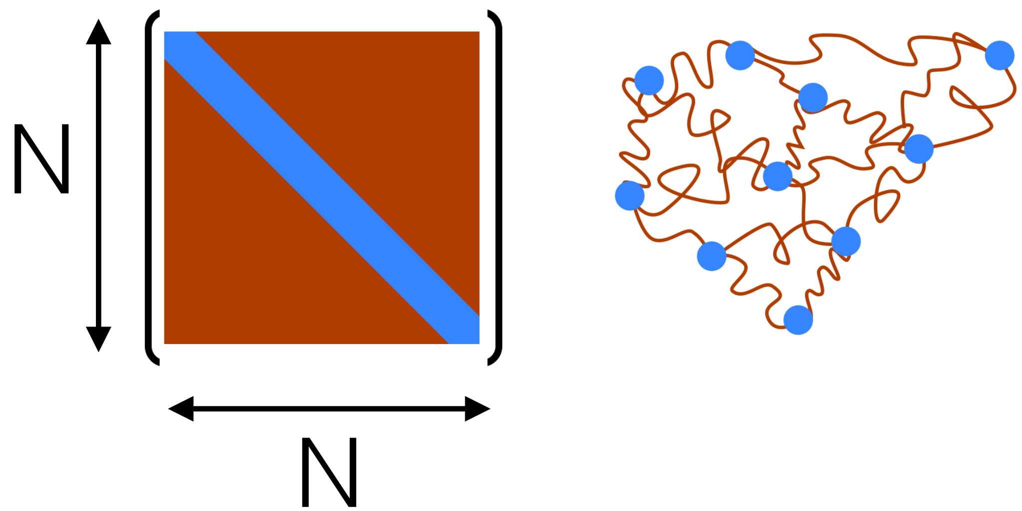
$X_M =$ 

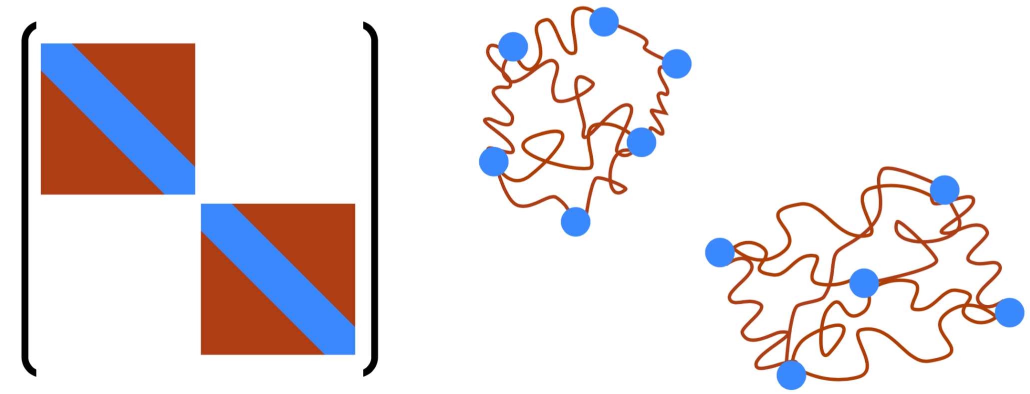


$(X_1^{ii}, X_2^{ii}, \ldots, X_6^{ii})$
location of i-th D-brane

X_M^{ij} : open strings connecting i-th and j-th D-branes.
large value → a lot of strings are excited

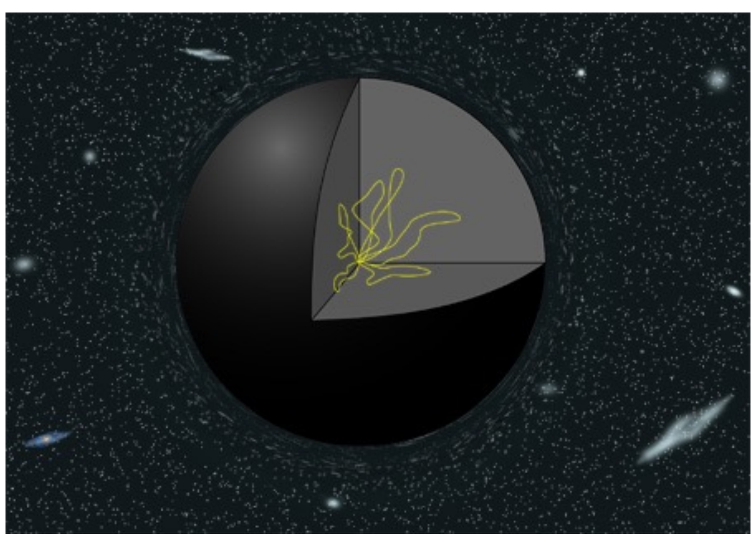
(Witten, 1994)





diagonal elements = particles (D-branes)
off-diagonal elements = open strings

(Witten, 1994)

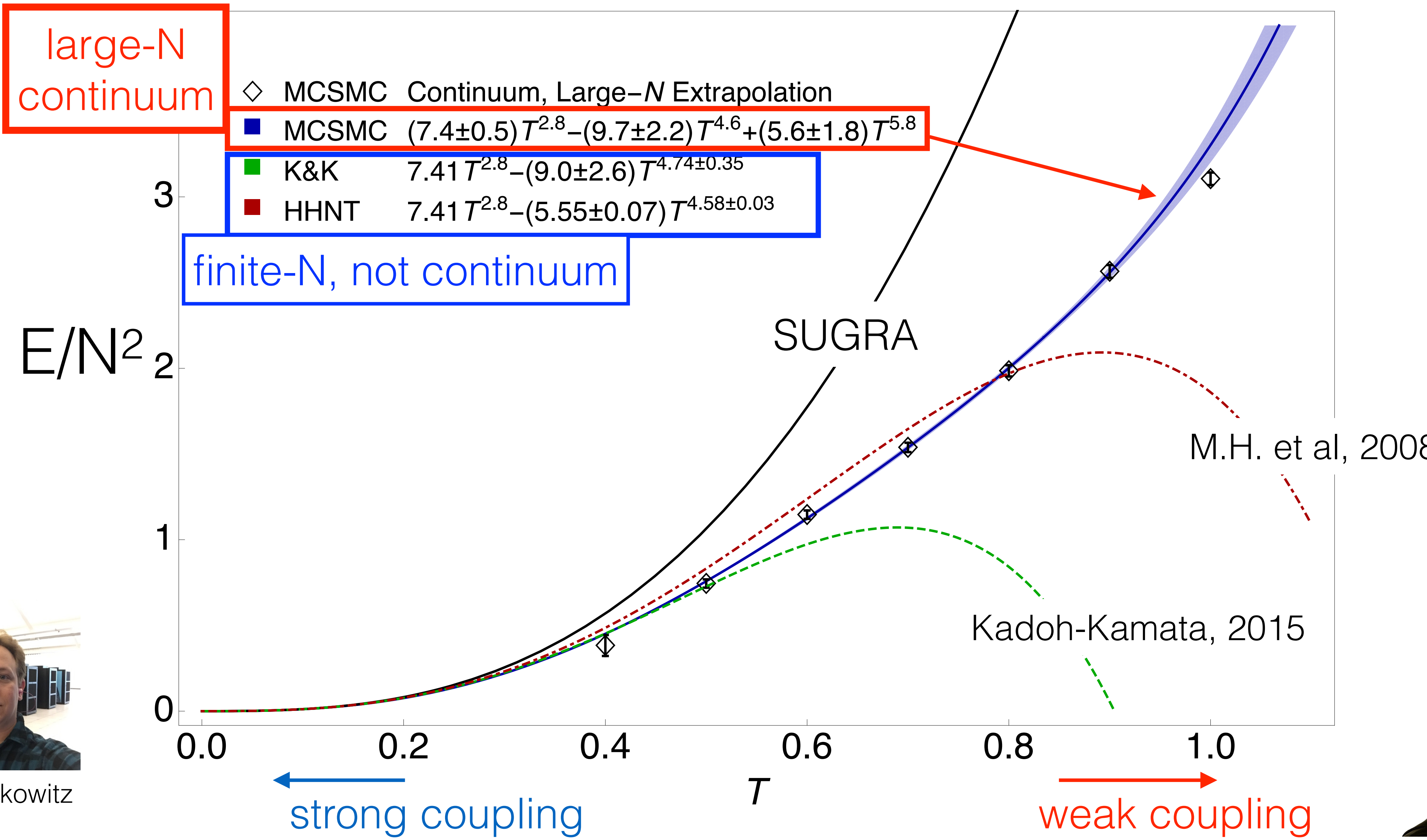


$\boxed{\text{black hole = bound state of D-branes and strings}}$

Energy (mass) of BH = energy in matrix model

Check it by lattice simulation.

Energy (mass) of BH = energy in matrix model ✔



Evan Berkowitz

Enrico Rinaldi
(pre-beard period)

Monte Carlo String/M-theory Collaboration, 2016

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- BMN has multiple supersymmetric vacua.
- M2/D2-brane, M5/NS5-brane, ...
- Various QFT's appear in appropriate large-N limits
- "Lattice" is embedded in matrices.

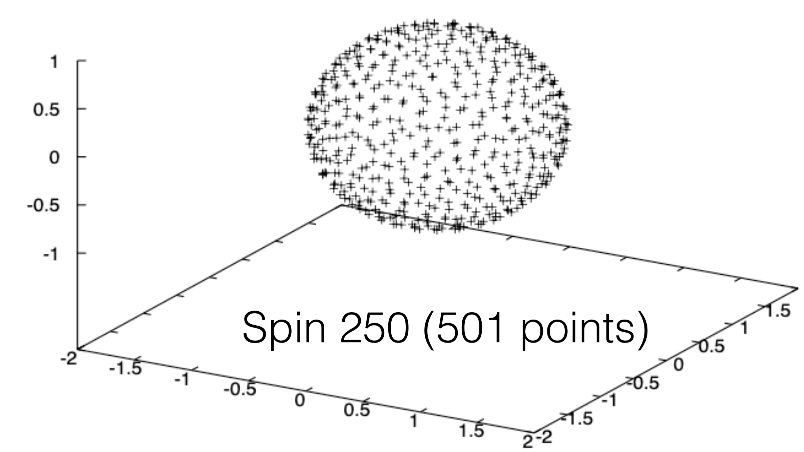
'Fuzzy sphere'

$$X_i = \frac{1}{\sqrt{j(j+1)}} J_i \quad (i = 1, 2, 3)$$

$$X_i = \frac{\mu}{3g} J_i, \qquad X_a = 0, \qquad A_t = 0, \qquad \Psi = 0$$

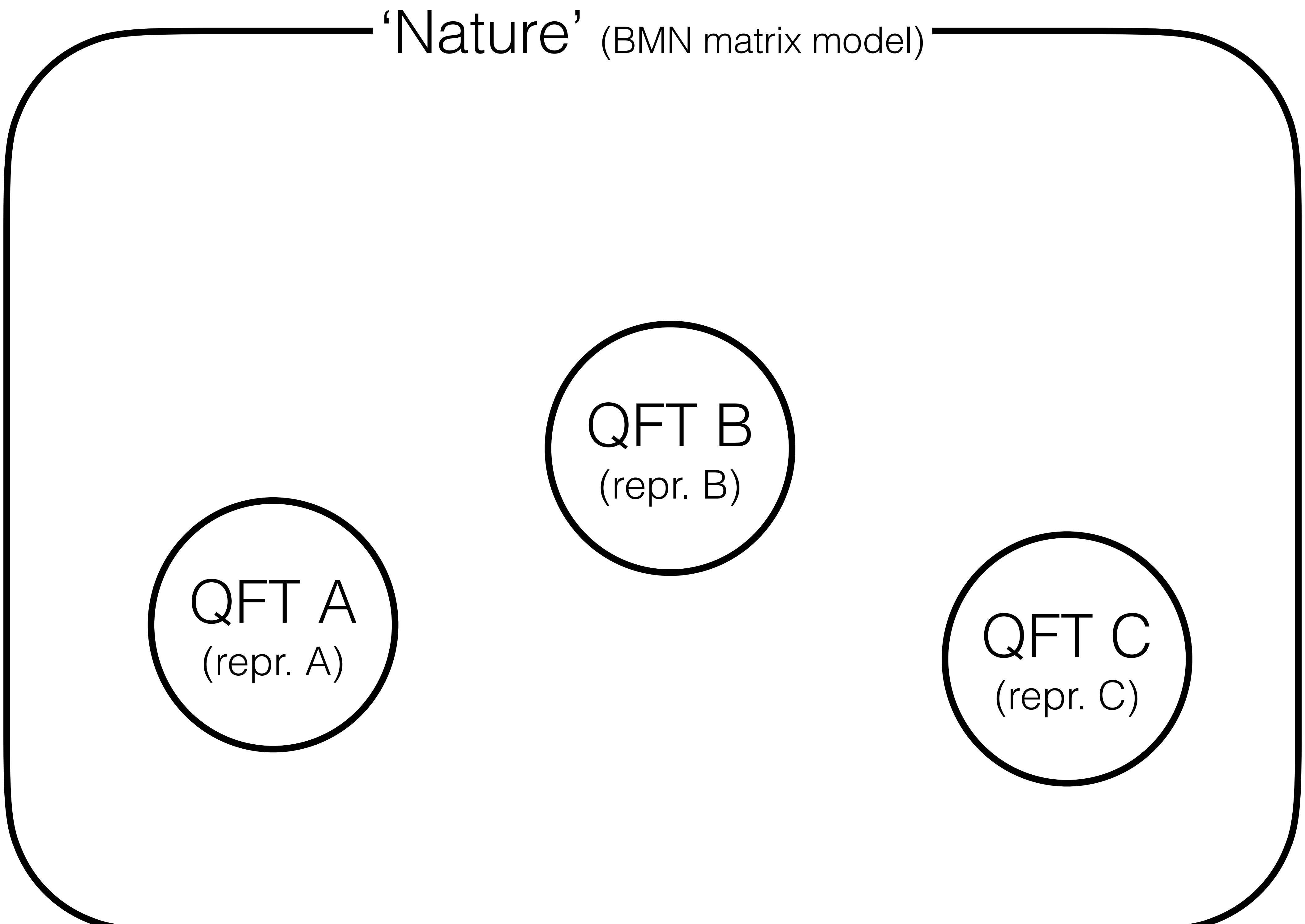
$$\sum_{\mu=1}^{3} \sum_{i=1}^{N} \left[\left(U X_\mu U^\dagger \right)_{ii} \right]^2$$ is maximized

$$[J_i, J_j] = i\epsilon^{ijk} J_k$$ (SU(2) algebra)



Different reducible representation
= different vacuum

(Azeyanagi-MH-Hirata-Shimada, 2009)



‘Nature’ (BMN matrix model)
QFT B
(repr. B)
QFT A
(repr. A)
QFT C
(repr. C)

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$$L = \mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{2}\Psi^T D_t \Psi + \frac{g^2}{4}[X_I, X_J]^2 - \frac{ig}{2}\Psi^T \gamma_I [X_I, \Psi]\right.$$
$$\left. -\frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{\mu}{8}\Psi^T\gamma_{123}\Psi - \frac{i\mu g}{3}\epsilon^{ijk}X_i X_j X_k\right\},$$

$$\Big\downarrow$$

(modulo some field redefinitions)

$$\hat{H} = \mathrm{Tr}\left\{\underline{\frac{1}{2}(\hat{P}_I)^2} - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \underline{\frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2} + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k\right.$$
$$\left. + g\hat{\psi}^{\dagger Ip}\sigma^{i\ q}_{p}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip}g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \underline{\frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip}}\right\}$$

Free part (bosonic/fermionic harmonic oscillators)

Gauge-singlet constraint

(A_0=0 gauge)

$$\hat{G}_\alpha|\mathrm{phys}\rangle = 0 \quad \text{with} \quad \hat{G}_\alpha \equiv \sum_{\beta,\gamma=1}^{N^2} f_{\alpha\beta\gamma}\left(\sum_{I=1}^{9}\hat{X}_I^\beta \hat{P}_I^\gamma + i\sum_{I,p}\hat{\psi}^{\dagger Ip\beta}\hat{\psi}^\gamma_{Ip}\right)$$

$$\hat{H} = \mathrm{Tr}\Bigg\{ \underline{\frac{1}{2}(\hat{P}_I)^2} - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \underline{\frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2} + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k$$

$$+ g\hat{\psi}^{\dagger Ip}\sigma_p^{i\,q}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip}g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \underline{\frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip}} \Bigg\}$$

Free part (bosonic/fermionic harmonic oscillators)

Fock basis

$$X_I = \sum_{\alpha=1}^{N^2} X_I^\alpha \tau_\alpha, \quad \psi_{Ip} = \sum_{\alpha=1}^{N^2} \psi_{Ip}^\alpha \tau_\alpha \qquad [\tau_\alpha, \tau_\beta] = i f_{\alpha\beta\gamma}\tau_\gamma, \quad \mathrm{Tr}(\tau_\alpha\tau_\beta) = \delta_{\alpha\beta}$$

$$\hat{A}_{I\alpha} = \sqrt{\frac{\omega_I}{2}}\hat{X}_{I\alpha} + \frac{i\hat{P}_{I\alpha}}{\sqrt{2\omega_I}}, \qquad \hat{A}^\dagger_{I\alpha} = \sqrt{\frac{\omega_I}{2}}\hat{X}_{I\alpha} - \frac{i\hat{P}_{I\alpha}}{\sqrt{2\omega_I}}, \qquad \omega_I = \begin{cases} \frac{\mu}{3} & \text{for } I = 1,2,3 \\ \frac{\mu}{6} & \text{for } I = 4,5,\cdots,9 \end{cases}$$

$$|\{n_{I\alpha}\}\rangle \equiv \otimes_{I,\alpha}|n_{I\alpha}\rangle_{I\alpha} = \left(\prod_{I,\alpha}\frac{\hat{A}_{I\alpha}^{\dagger n_{I\alpha}}}{\sqrt{n_{I\alpha}!}}\right)|\mathrm{VAC}_{\mathrm{free}}\rangle, \qquad \hat{A}_{I\alpha}|\mathrm{VAC}_{\mathrm{free}}\rangle = 0.$$

Regularization: $0 \le n_{I\alpha} \le \Lambda - 1$

(No regularization needed for fermions)

$$\hat{a}^\dagger = \sum_{j=0}^{\Lambda-2} \sqrt{j+1}|j+1\rangle\langle j|$$

$$|j\rangle = |b_0\rangle\,|b_1\rangle \ldots |b_{K-1}\rangle$$

b, b’ = 0 or 1

$$|j+1\rangle = |b'_0\rangle\,|b'_1\rangle \ldots \left|b'_{K-1}\right\rangle$$

$$|j+1\rangle\langle j| = \otimes_{l=0}^{K-1}\left(|b'_l\rangle\langle b_l|\right)$$

$K = \log_2 \Lambda$

$$|0\rangle\langle 0| = \frac{\mathbf{1}_2 - \sigma_z}{2}, \qquad |1\rangle\langle 1| = \frac{\mathbf{1}_2 + \sigma_z}{2},$$

$$|0\rangle\langle 1| = \frac{\sigma_x + i\sigma_y}{2}, \qquad |1\rangle\langle 0| = \frac{\sigma_x - i\sigma_y}{2}.$$

H = Σ(Pauli strings)

$$\hat{a}^\dagger = \sum_{j=0}^{\Lambda-2} \sqrt{j+1}|j+1\rangle\langle j|$$

$\sim 2^K = \Lambda$ Pauli strings of length $K = \log_2 \Lambda$ for each j

➡ $\sim \Lambda^2$ Pauli strings of length $K = \log_2 \Lambda$

$$\sum_{I \neq J} \mathrm{Tr}[\hat{X}_I, \hat{X}_J]^2 = -\sum_{I \neq J} \sum_{\alpha,\beta,\gamma,\rho,\sigma=1}^{N^2} f_{\alpha\beta\sigma} f_{\gamma\rho\sigma} \hat{X}_I^\alpha \hat{X}_J^\beta \hat{X}_I^\gamma \hat{X}_J^\rho$$

$\sim N^4$ color combinations

$\sim \Lambda^8 N^4$ Pauli strings of length 4K

$$\dim\left(\mathcal{H}_{\mathrm{BMN}}\right)\big|_{\mathrm{regularized}} = \Lambda^{9N^2} \cdot 2^{8N^2}$$ (~N^4 nonzero components/row)

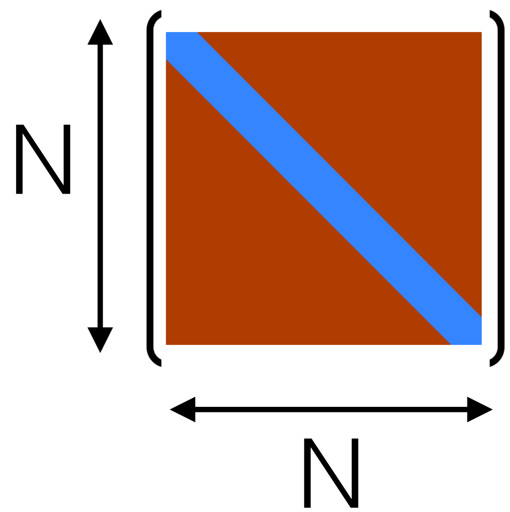
$$\hat{H} = \sum_{i=1}^{L} \alpha_i \hat{\Pi}_i, \qquad L \lesssim \Lambda^8 N^4$$

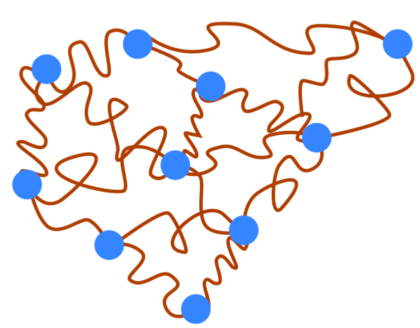
Pauli strings

How big Λ?

- Depend on the physics under consideration.
- Λ=2 can be already good for some interesting phenomena.

e.g., Deconfinement transition (black hole formation) at weak coupling





each matrix entry = harmonic oscillator

excitation level = # of strings

average excitation level < 1

Quantum Signal Processing(1)

Low-Chuang 2017; Babbush-Berry-Neven 2018

- Calculate time evolution efficiently using the Pauli-string form.
- Nice & cool math!

Bessel fnc. of 1st kind

Chebyshev polynomial of the first kind

$$T_n(x) = \cos(nt), \qquad x = \cos t$$

Jacobi-Anger expansion

$$e^{-i\hat{H}t} = J_0(-\lambda t) + 2\sum_{n=1}^{\infty} i^n J_n(-\lambda t) \times T_n\left(\frac{\hat{H}}{\lambda}\right)$$

$$\hat{H} = \sum_{i=1}^{L} \alpha_i \hat{\Pi}_i,$$

<u>Controlled-Pauli</u> $\hat{U}\,|i\rangle\,|\psi\rangle = |i\rangle\left(\hat{\Pi}_i\,|\psi\rangle\right)$

ancilla states

$$\frac{\hat{H}}{\lambda} = \left(\langle G| \otimes \hat{I}\right)\hat{U}\left(|G\rangle \otimes \hat{I}\right)$$

$$|G\rangle = \sum_{i=1}^{L} g_i\,|i\rangle\,, \qquad |g_i|^2 = \frac{\alpha_i}{\lambda}, \qquad \lambda = \sum_{i=1}^{L} \alpha_i$$

Quantum Signal Processing(2)

Low-Chuang 2017; Babbush-Berry-Neven 2018

$$\hat{H} = \sum_{i=1}^{L} \alpha_i \hat{\Pi}_i,$$

Bessel fnc. of 1st kind

Chebyshev polynomial of the first kind

$$T_n(x) = \cos(nt), \qquad x = \cos t$$

$$e^{-i\hat{H}t} = J_0(-\lambda t) + 2\sum_{n=1}^{\infty} i^n J_n(-\lambda t) \times T_n\left(\frac{\hat{H}}{\lambda}\right)$$

$$\hat{U}\left|i\right\rangle\left|\psi\right\rangle = \left|i\right\rangle\left(\hat{\Pi}_i\left|\psi\right\rangle\right)$$

$$\hat{R} = 2\left|G\right\rangle\left\langle G\right| - \hat{I}$$

Unitary & Hermitian

$$\hat{W} = \hat{R}\hat{U}$$

Unitary

$$\left\langle G\right|\hat{W}^n\left|G\right\rangle = T_n\left(\frac{\hat{H}}{\lambda}\right)$$

$$\boxed{T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)}$$

Quantum Signal Processing(3)

Low-Chuang 2017; Babbush-Berry-Neven 2018

$$\boxed{T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)}$$

$\hat{W} = \hat{R}\hat{U}$

$$\begin{aligned}
\langle G| \hat{W}^{n+1} |G\rangle &= \langle G| \hat{R}\hat{U}\,\hat{R}\hat{U}\hat{W}^{n-1} |G\rangle \\
&= \langle G| \hat{R}\hat{U} \left(2 |G\rangle \langle G| - \hat{I}\right) \hat{U}\hat{W}^{n-1} |G\rangle \\
&= 2 \langle G| \hat{R}\hat{U} |G\rangle \langle G| \hat{U}\hat{W}^{n-1} |G\rangle - \langle G| \hat{R}\hat{U}^2\hat{W}^{n-1} |G\rangle \\
&= 2 \langle G| \hat{R}\hat{U} |G\rangle \langle G| \hat{R}\hat{U}\hat{W}^{n-1} |G\rangle - \langle G| \hat{U}^2\hat{W}^{n-1} |G\rangle \\
&= 2 \langle G| \hat{W} |G\rangle \langle G| \hat{W}\hat{W}^{n-1} |G\rangle - \langle G| \hat{W}^{n-1} |G\rangle \\
&= 2\frac{\hat{H}}{\lambda}T_n\left(\frac{\hat{H}}{\lambda}\right) - T_{n-1}\left(\frac{\hat{H}}{\lambda}\right) \\
&= T_{n+1}\left(\frac{\hat{H}}{\lambda}\right).
\end{aligned}$$

$\hat{R} = 2 |G\rangle \langle G| - \hat{I}$

$\langle G|\hat{R} = \langle G|$

$\hat{U}^2 = \sum_i |i\rangle \langle i|,$

$\langle G| \hat{U}^2 = \langle G|$

assumption for mathematical induction

$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$

Quantum Signal Processing(4)

Low-Chuang 2017; Babbush-Berry-Neven 2018

$$e^{-i\hat{H}t} = \langle G| \left(J_0(-\lambda t) + 2\sum_{n=1}^{\infty} i^n J_n(-\lambda t)\hat{W}^n \right) |G\rangle \equiv \langle G| f(\hat{W}) |G\rangle$$

We want to construct this unitary operator efficiently.

<u>Lemma</u>

2×2 special unitary matrix $\hat{V}(\theta) = A(\theta)\mathbf{1} + iB(\theta)\sigma_z + iC(\theta)\sigma_x + iD(\theta)\sigma_y$

with period 2π ($\hat{V}(\theta) = \hat{V}(\theta + 2\pi)$) can be approximated as

$$\hat{V}(\theta) \simeq \hat{R}_{\phi_n}(\theta)\hat{R}_{\phi_{n-1}}(\theta)\cdots\hat{R}_{\phi_1}(\theta)$$

where $\phi_1, \cdots, \phi_n \in \mathbb{R}$ & $\hat{R}_\phi(\theta) = e^{-i\frac{\phi}{2}\sigma_z}e^{-i\theta\sigma_x}e^{+i\frac{\phi}{2}\sigma_z}$

(use classical computer to find these coefficients)

Quantum Signal Processing(5)

Low-Chuang 2017; Babbush-Berry-Neven 2018

2×2 special unitary matrix $\hat{V}(\theta) = A(\theta)\mathbf{1} + iB(\theta)\sigma_z + iC(\theta)\sigma_x + iD(\theta)\sigma_y$
$\simeq \hat{R}_{\phi_n}(\theta)\hat{R}_{\phi_{n-1}}(\theta)\cdots\hat{R}_{\phi_1}(\theta)$

---

unitary

eigenvalue

eigenstate

$$\hat{W}\,|w\rangle = w\,|w\rangle = e^{i\theta}\,|w\rangle$$

<u>Controlled-W gate</u>

$$\widehat{\mathrm{CW}} : |0\rangle \otimes |w\rangle \mapsto w^{-1}\,|0\rangle\,|w\rangle\,, \qquad |1\rangle \otimes |w\rangle \mapsto w\,|1\rangle \otimes |w\rangle$$

$$\widehat{\mathrm{CW}} : |0\rangle \otimes |\psi\rangle \mapsto |0\rangle \otimes (\hat{W}^{-1}\,|\psi\rangle), \qquad |1\rangle \otimes |\psi\rangle \mapsto |1\rangle \otimes (\hat{W}\,|\psi\rangle)$$

$$\boxed{\hat{R}_\phi \equiv e^{-i\frac{\phi}{2}\sigma_z} \cdot \widehat{\mathrm{Had}} \cdot \widehat{\mathrm{CW}} \cdot \widehat{\mathrm{Had}} \cdot e^{+i\frac{\phi}{2}\sigma_z}}$$

Hadamard gate

$$\widehat{\mathrm{Had}} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\hat{R}_\phi(|b\rangle \otimes \left|w = e^{i\theta}\right\rangle) = (\hat{R}_\phi(\theta)\,|b\rangle) \otimes \left|w = e^{i\theta}\right\rangle$$

Quantum Signal Processing(6)

Low-Chuang 2017; Babbush-Berry-Neven 2018

2×2 special unitary matrix $\hat{V}(\theta) = A(\theta)\mathbf{1} + iB(\theta)\sigma_z + iC(\theta)\sigma_x + iD(\theta)\sigma_y$

$$\simeq \hat{R}_{\phi_n}(\theta)\hat{R}_{\phi_{n-1}}(\theta)\cdots\hat{R}_{\phi_1}(\theta)$$

$$\boxed{A(\theta) + iB(\theta) \simeq f(w = e^{i\theta})}$$

$$\boxed{\hat{R}_\phi \equiv e^{-i\frac{\phi}{2}\sigma_z} \cdot \widehat{\mathrm{Had}} \cdot \widehat{\mathrm{CW}} \cdot \widehat{\mathrm{Had}} \cdot e^{+i\frac{\phi}{2}\sigma_z}}$$

$$\hat{R}_\phi(|b\rangle \otimes |w = e^{i\theta}\rangle) = (\hat{R}_\phi(\theta)|b\rangle) \otimes |w = e^{i\theta}\rangle$$

$$\hat{V} \equiv \langle b = 0| \hat{R}_{\phi_n}\hat{R}_{\phi_{n-1}}\cdots\hat{R}_{\phi_1}|b = 0\rangle$$

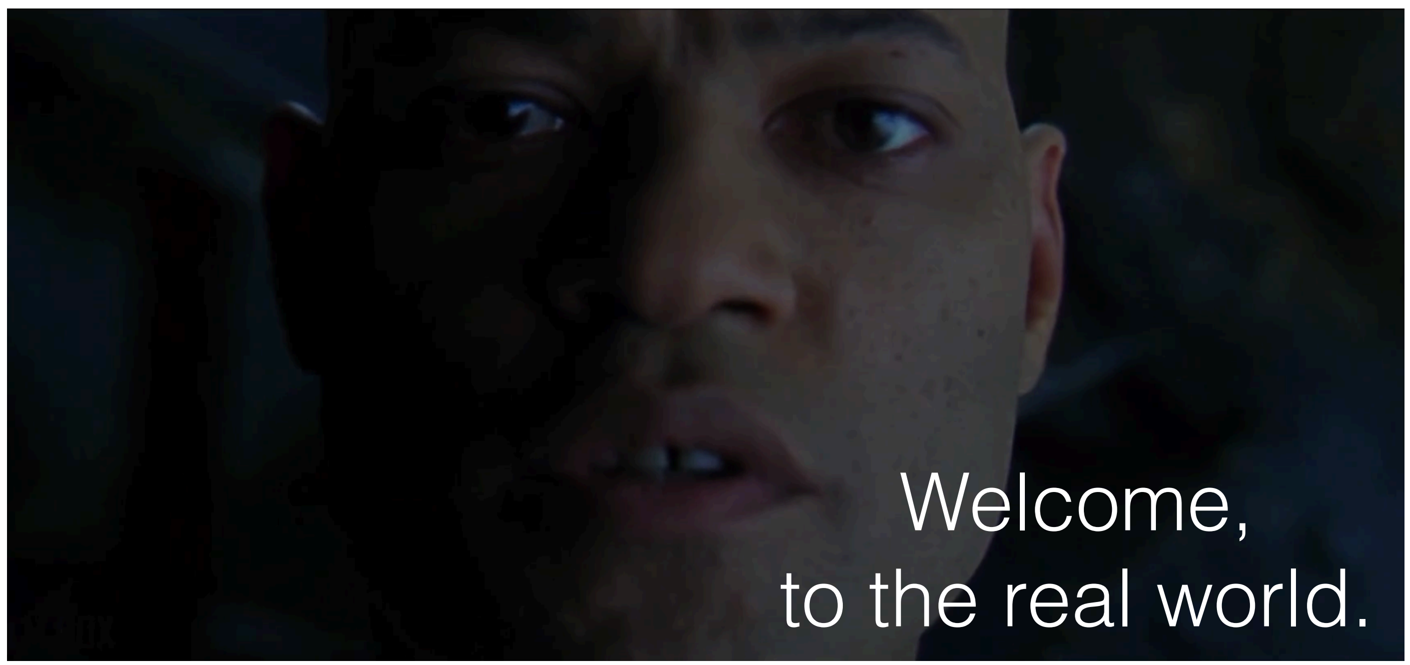
$$\boxed{\begin{aligned} &\hat{V} : |w\rangle \mapsto f(w)|w\rangle \\ &\hat{V} = f(\hat{W}) \end{aligned}}$$

- The same Hamiltonian + fuzzy sphere state → QFT
- State preparation is bit complicated but doable.
  (Please see the paper.)

- Introduction
- BMN matrix model
- QFT from BMN matrix model
- Quantum simulation of BMN matrix model and QFT
- Orbifold lattice construction

Some of you may not like string theory so much.

How about (3+1)-d U(k) YM?



- Kogut-Susskind Hamiltonian Formulation is commonly used.

- We will provide another option.

Kogut-Susskind Formulation

$$\hat{H} = \hat{H}_{\mathrm{E}} + \hat{H}_{\mathrm{B}}$$

$$\hat{H}_{\mathrm{E}} = \frac{a^3}{2} \sum_{\vec{n}} \sum_{\mu=1}^{3} \sum_{\alpha=1}^{k^2} \left(\hat{E}^{\alpha}_{\mu,\vec{n}} \right)^2$$

unitary, compact variable

$$\hat{H}_{\mathrm{B}} = \frac{1}{2ag^2} \sum_{\vec{n}} \sum_{\mu \neq \nu} \left(k - \mathrm{Tr} \left(\hat{U}_{\mu,\vec{n}} \hat{U}_{\nu,\vec{n}+\hat{\mu}} \hat{U}^{\dagger}_{\mu,\vec{n}+\hat{\nu}} \hat{U}^{\dagger}_{\nu,\vec{n}} \right) \right)$$

$$\left[\hat{E}^{\alpha}_{\mu,\vec{n}}, \hat{U}_{\nu,\vec{n}'} \right] = ag\delta_{\mu\nu}\delta_{\vec{n}\vec{n}'}\tau_{\alpha}\hat{U}_{\nu,\vec{n}'}, \qquad \left[\hat{E}^{\alpha}_{\mu,\vec{n}}, \hat{U}^{\dagger}_{\nu,\vec{n}'} \right] = -ag\delta_{\mu\nu}\delta_{\vec{n}\vec{n}'}\hat{U}^{\dagger}_{\nu,\vec{n}'}\tau_{\alpha}$$

$$\mathcal{H} = \otimes_{\mu,\vec{n}} \mathcal{H}_{\mu,\vec{n}} = \otimes_{\mu,\vec{n}} \left(\oplus_R \oplus_{i,j=1}^{\mathrm{dim}R} |R, ij\rangle_{\mu,\vec{n}} \right)$$

Complicated group theory. Not straightforward on QC.

Orbifold lattice construction

- Matrix Model is easy partly because the variables are noncompact (~harmonic oscillators).
- Orbifold construction (Kaplan, Katz, Unsal 2002) gives lattice gauge theory with noncompact variables.
- Orbifold-projected matrix model (Kaplan, Katz, Unsal 2002) + dimensional deconstruction (Arkani-Hamed, Cohen, Georgi 2001)
- Original motivation was to build supersymmetric lattices, but it works without SUSY as well.

Example: (3+1)-d U(k) YM

(almost) Kaplan-Katz-Unsal 2002

$$
\begin{aligned}
L^{\text{lattice}} \quad = \quad & \sum_{\vec{n}} \mathrm{Tr}\Bigg(|D_t x_{\vec{n}}|^2 + |D_t y_{\vec{n}}|^2 + |D_t z_{\vec{n}}|^2 \\
& -\frac{g_{1\mathrm{d}}^2}{2} \left| x_{\vec{n}} \bar{x}_{\vec{n}} - \bar{x}_{\vec{n}-\hat{x}} x_{\vec{n}-\hat{x}} + y_{\vec{n}} \bar{y}_{\vec{n}} - \bar{y}_{\vec{n}-\hat{y}} y_{\vec{n}-\hat{y}} + z_{\vec{n}} \bar{z}_{\vec{n}} - \bar{z}_{\vec{n}-\hat{z}} z_{\vec{n}-\hat{z}} \right|^2 \\
& -2g_{1\mathrm{d}}^2 \left(\left| x_{\vec{n}} y_{\vec{n}+\hat{x}} - y_{\vec{n}} x_{\vec{n}+\hat{y}} \right|^2 + \left| y_{\vec{n}} z_{\vec{n}+\hat{y}} - z_{\vec{n}} y_{\vec{n}+\hat{z}} \right|^2 + \left| z_{\vec{n}} x_{\vec{n}+\hat{z}} - x_{\vec{n}} z_{\vec{n}+\hat{x}} \right|^2 \right) \Bigg)
\end{aligned}
$$

$$
\Delta L^{\text{lattice}} \equiv -\frac{m^2}{2a} \sum_{\vec{n}} \left(\left| x_{\vec{n}} \bar{x}_{\vec{n}} - \frac{1}{2a^2 g_{1\mathrm{d}}^2} \right|^2 + \left| y_{\vec{n}} \bar{y}_{\vec{n}} - \frac{1}{2a^2 g_{1\mathrm{d}}^2} \right|^2 + \left| z_{\vec{n}} \bar{z}_{\vec{n}} - \frac{1}{2a^2 g_{1\mathrm{d}}^2} \right|^2 \right)
$$

$$
\begin{aligned}
x &= \frac{1}{\sqrt{2} a g_{1d}} e^{a^{5/2} g_{1\mathrm{d}} s_1} e^{i a^{5/2} g_{1\mathrm{d}} A_1}, \\
y &= \frac{1}{\sqrt{2} a g_{1d}} e^{a^{5/2} g_{1\mathrm{d}} s_2} e^{i a^{5/2} g_{1\mathrm{d}} A_2}, \\
z &= \frac{1}{\sqrt{2} a g_{1d}} e^{a^{5/2} g_{1\mathrm{d}} s_3} e^{i a^{5/2} g_{1\mathrm{d}} A_3}.
\end{aligned}
$$

Example: (3+1)-d U(k) YM

(almost) Kaplan-Katz-Unsal 2002

$$L = \int d^3x \mathrm{Tr} \left(-\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_t s_I)^2 + \frac{g_{4\mathrm{d}}^2}{4} [s_I, s_J]^2 \right)$$

$$\Delta L = -\frac{m^2}{2} \int d^3x \mathrm{Tr} \left(s_1^2 + s_2^2 + s_3^2 \right)$$

$$\begin{aligned} x &= \frac{1}{\sqrt{2} a g_{1d}} e^{a^{5/2} g_{1\mathrm{d}} s_1} e^{i a^{5/2} g_{1\mathrm{d}} A_1}, \\ y &= \frac{1}{\sqrt{2} a g_{1d}} e^{a^{5/2} g_{1\mathrm{d}} s_2} e^{i a^{5/2} g_{1\mathrm{d}} A_2}, \\ z &= \frac{1}{\sqrt{2} a g_{1d}} e^{a^{5/2} g_{1\mathrm{d}} s_3} e^{i a^{5/2} g_{1\mathrm{d}} A_3}. \end{aligned}$$

Example: (3+1)-d U(k) YM

(almost) Kaplan-Katz-Unsal 2002

$$\begin{aligned}\hat{H} &= \sum_{\vec{n}} \mathrm{Tr}\Bigg(|\hat{p}_{x,\vec{n}}|^2 + |\hat{p}_{y,\vec{n}}|^2 + |\hat{p}_{z,\vec{n}}|^2 \\ &\quad + \frac{g_{1\mathrm{d}}^2}{2} \left| \hat{x}_{\vec{n}}\hat{\bar{x}}_{\vec{n}} - \hat{\bar{x}}_{\vec{n}-\hat{x}}\hat{x}_{\vec{n}-\hat{x}} + \hat{y}_{\vec{n}}\hat{\bar{y}}_{\vec{n}} - \hat{\bar{y}}_{\vec{n}-\hat{y}}\hat{y}_{\vec{n}-\hat{y}} + \hat{z}_{\vec{n}}\hat{\bar{z}}_{\vec{n}} - \hat{\bar{z}}_{\vec{n}-\hat{z}}\hat{z}_{\vec{n}-\hat{z}} \right|^2 \\ &\quad + 2g_{1\mathrm{d}}^2 \left(|\hat{x}_{\vec{n}}\hat{y}_{\vec{n}+\hat{x}} - \hat{y}_{\vec{n}}\hat{x}_{\vec{n}+\hat{y}}|^2 + |\hat{y}_{\vec{n}}\hat{z}_{\vec{n}+\hat{y}} - \hat{z}_{\vec{n}}\hat{y}_{\vec{n}+\hat{z}}|^2 + |\hat{z}_{\vec{n}}\hat{x}_{\vec{n}+\hat{z}} - \hat{x}_{\vec{n}}\hat{z}_{\vec{n}+\hat{x}}|^2 \right) \Bigg) + \Delta\hat{H}\end{aligned}$$

$$\Delta\hat{H} \equiv \frac{m^2}{2a} \sum_{\vec{n}} \mathrm{Tr}\left(\left| \hat{x}_{\vec{n}}\hat{\bar{x}}_{\vec{n}} - \frac{1}{2a^2 g_{1\mathrm{d}}^2} \right|^2 + \left| \hat{y}_{\vec{n}}\hat{\bar{y}}_{\vec{n}} - \frac{1}{2a^2 g_{1\mathrm{d}}^2} \right|^2 + \left| \hat{z}_{\vec{n}}\hat{\bar{z}}_{\vec{n}} - \frac{1}{2a^2 g_{1\mathrm{d}}^2} \right|^2 \right)$$

$$[\hat{x}_{\mu\vec{n},pq}, \hat{\bar{p}}_{\nu\vec{n}',rs}] = i\delta_{\mu\nu}\delta_{\vec{n}\vec{n}'}\delta_{ps}\delta_{qr}$$

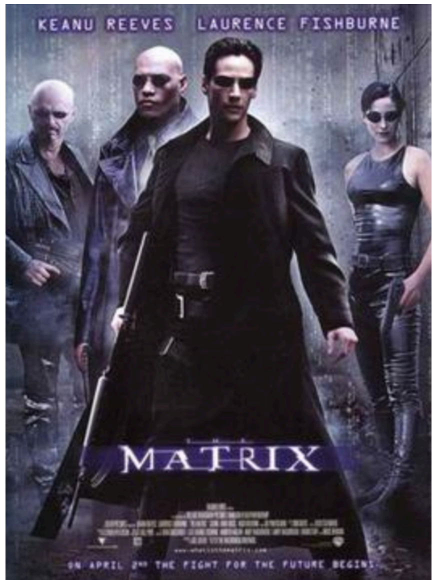
$$[\hat{x}, \hat{p}] = [\hat{\bar{x}}, \hat{\bar{p}}] = [\hat{x}, \hat{x}] = [\hat{\bar{x}}, \hat{\bar{x}}] = [\hat{p}, \hat{p}] = [\hat{\bar{p}}, \hat{\bar{p}}] = 0$$

- Hamiltonian = harmonic oscillators + some interactions
- Standard Fock basis truncation is good enough
- Truncated Hamiltonian = Σ (product of Pauli matrices)

  → efficient quantum algorithms can be used.

- Gauss law is imposed when the states are prepared.

Essentially the same as the matrix model.



Orbifold construction
vs
Kogut-Susskind formulation

- Orbifold lattice has simpler Hamiltonian made of Pauli matrices.

- Truncation to gauge-invariant sector — not easy but not impossible in both.

- State preparation — more or less the same level of hardness?

- Orbifold lattice is better when we want SUSY $\rightarrow$ potential application to quantum gravity via holography.

- We don’t know which is more economical in terms of the number of qubits.

- Probably we should study both, and choose a better approach depending on a concrete problem we want to solve.

Short Summary

- String/M-theory, Yang-Mills, maybe also QCD
  --- Simpler than expected.
- Standard quantum algorithms can be applied.
- Interesting to think about efficient simulation protocols.
- What would be the simplest model to simulate? (Various possibilities which I couldn't mention today)