

Quantum mechanical description of energy dissipation and application to heavy-ion fusion reactions

Masaaki Tokieda

Department of Physics , Tohoku University

Kouichi Hagino

Department of Physics , Kyoto University



Quantum mechanical description of energy dissipation and application to heavy-ion fusion reactions

► **Introduction to heavy-ion fusion reactions**

- Langevin equation
- Coupled-channels method
- A unified description

► **Methodology**

- Caldeira-Leggett model
- Why good ?

► **Application to a fusion problem**

- Results
- Effects of energy dissipation
- Remark on a unified description

► **Summary**

Quantum mechanical description of energy dissipation and application to heavy-ion fusion reactions

▶ **Introduction to heavy-ion fusion reactions**

- Langevin equation
- Coupled-channels method
- A unified description

▶ **Methodology**

- Caldeira-Leggett model
- Why good ?

▶ **Application to a fusion problem**

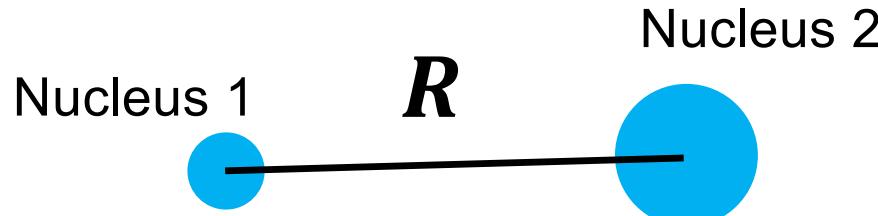
- Results
- Effects of energy dissipation
- Remark on a unified description

▶ **Summary**

Fusion: transmission of the coulomb barrier

4

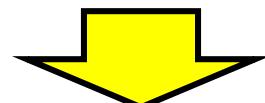
Nuclear Collision



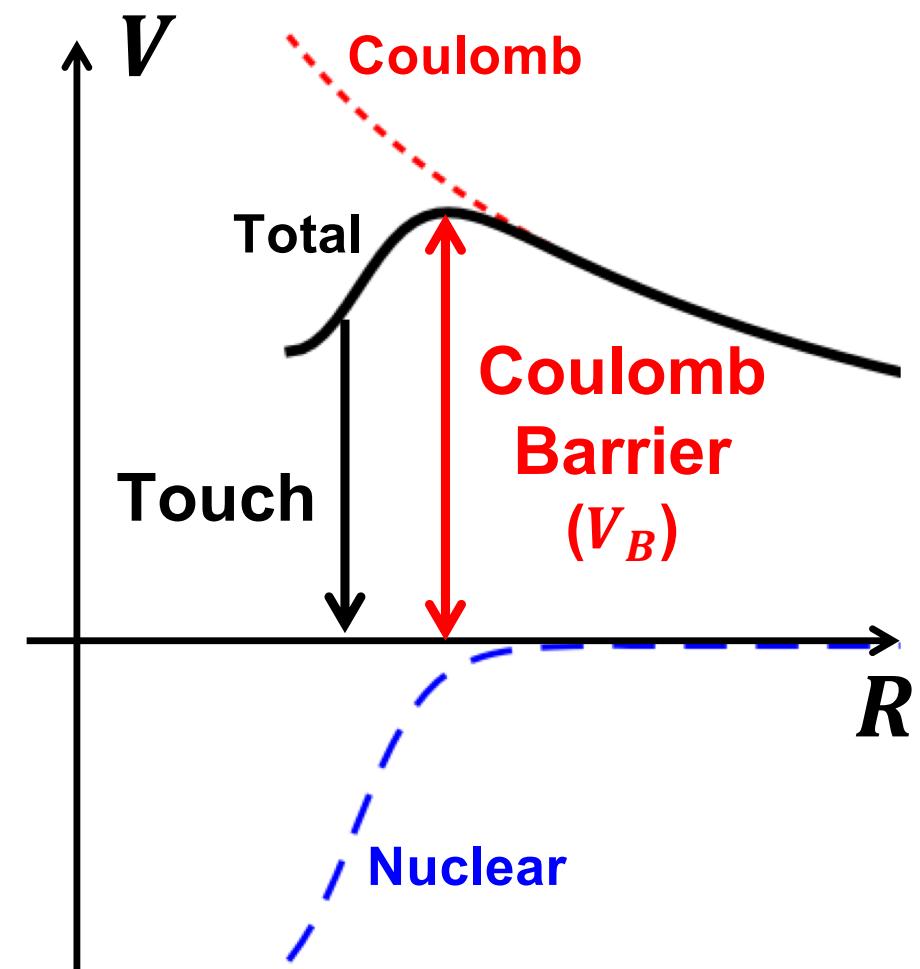
Potential = Coulomb + Nuclear

Repulsive	Attractive
Long range	Short range

Touch and fusion

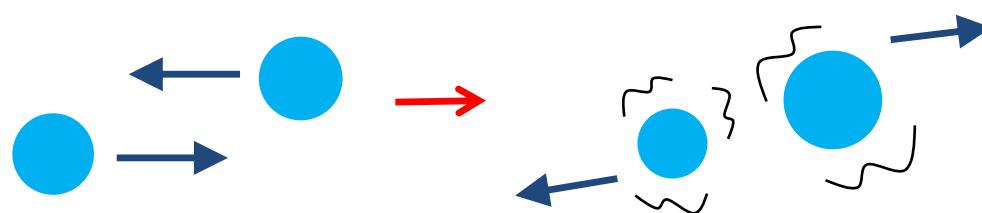


Barrier transmission problem
(Excitations of internal structure)



$E > V_B$: Deep inelastic scattering and Langevin equation 5

Deep inelastic scattering



Energy dissipation and Fluctuation
(Complex internal excitations)

D.A.Bromley, Treatise on Heavy-Ion Science, Vol2, (Plenum Press New York, 1984)

P.Frobrich and J.Marten, Z.Phys.A339(1991)175

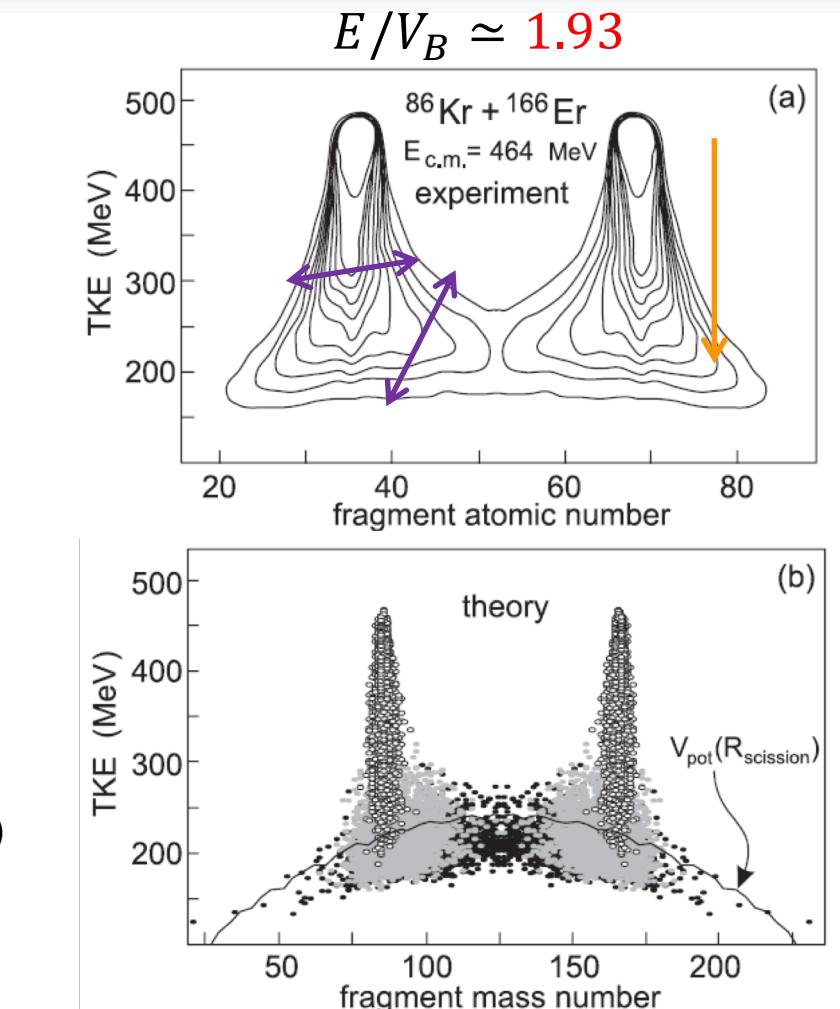
(Classical) Langevin equation

$$\begin{cases} \dot{Q} = P/M \\ \dot{P} = -V'(Q) - \gamma P + \xi(t) \end{cases}$$

Macroscopic coordinates
(Relative distance, Deformations, etc)

$$\langle \zeta(t) \rangle = 0, \langle \zeta(t)\zeta(s) \rangle = (2M\gamma/\beta)\delta(t-s)$$

Brownian motion



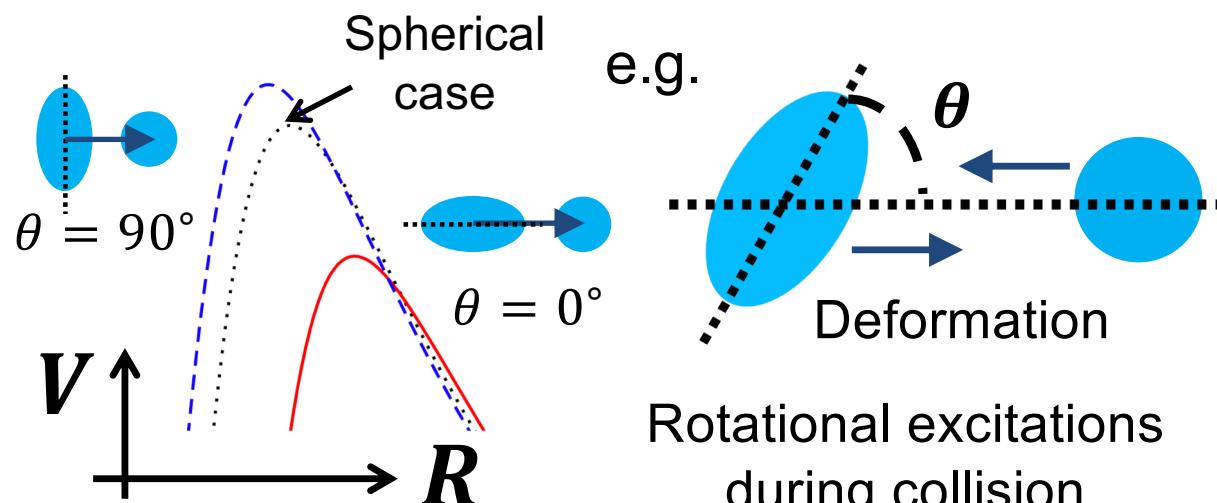
V.Zagrebaev and W.Greiner, J.Phys.G:Nucl.Part.Phys.31(2005)825

$E < V_B$: Quantum tunneling and channel coupling

6

(Incident energy E) < (Coulomb barrier V_B)

→ **Quantum tunneling**



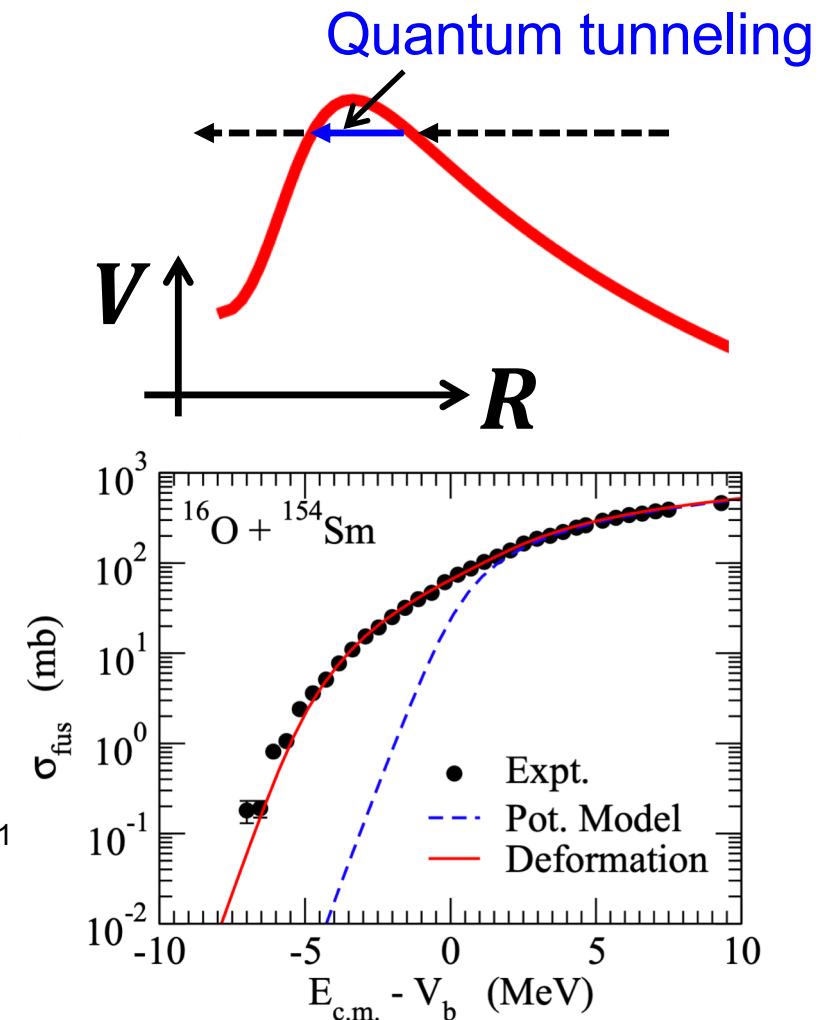
(Quantum) Coupled-channels method

K.Hagino and N.Takigawa, Prog.Theor.Phys.128(2012)1061

$$|\Psi\rangle = \sum_i \psi_i |i\rangle$$

Internal eigenstate

$$i\hbar \dot{\psi}_i = \sum_j H_{i,j} \psi_j$$

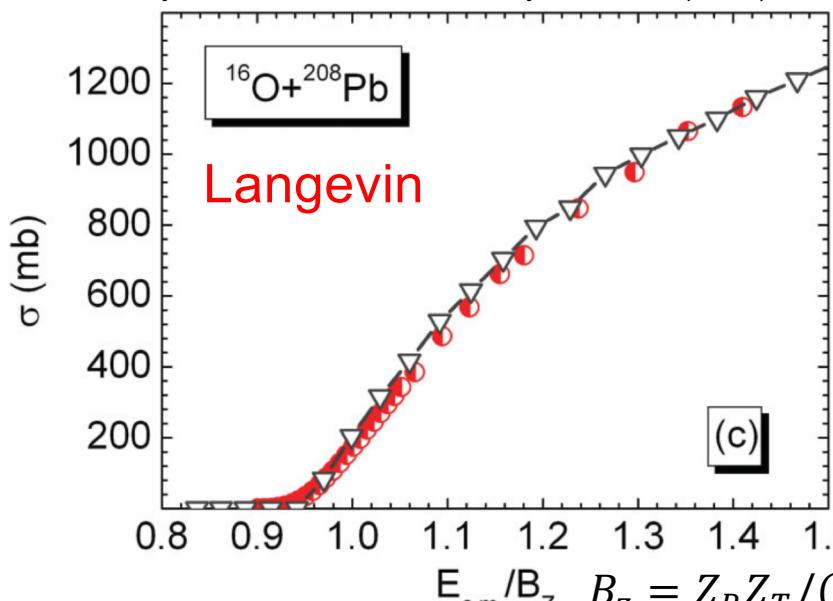


Unified description

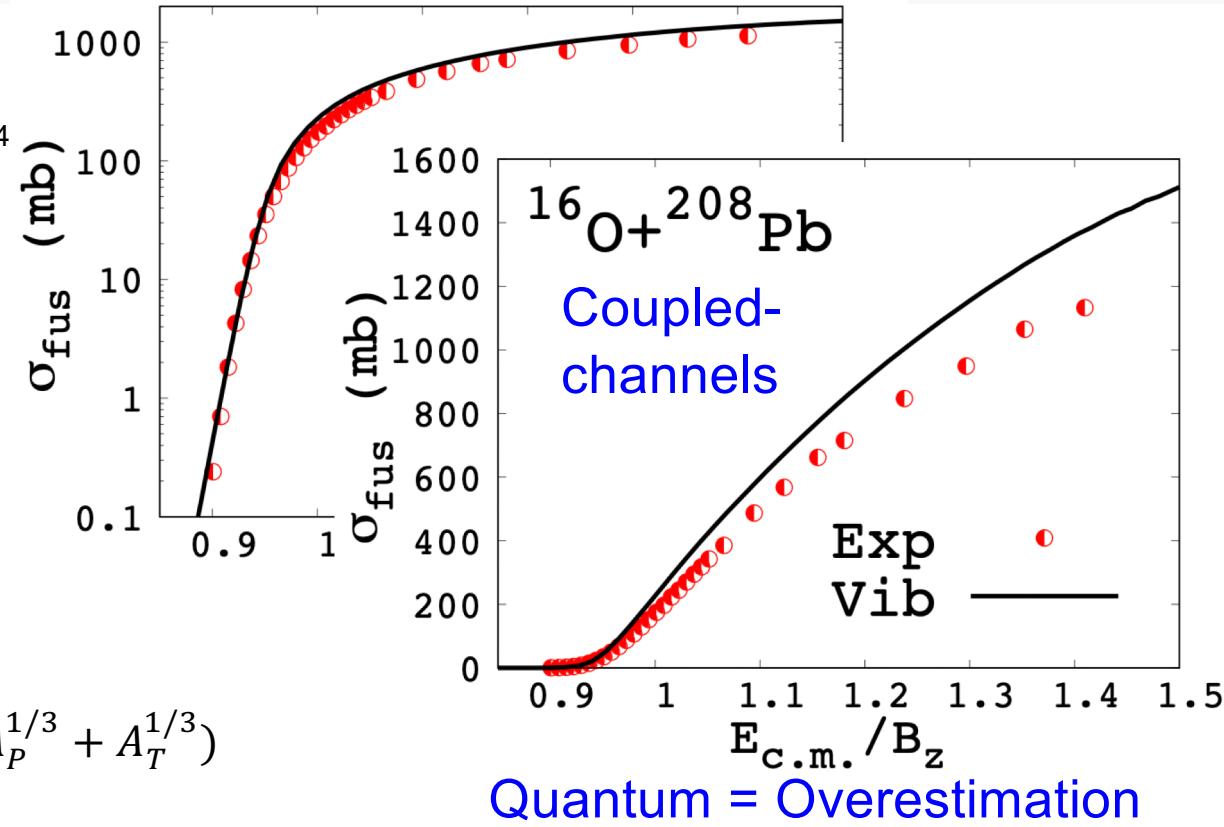
Experimental Data

C.R.Morton et al, Phys.Rev.C**60**(1999)044608

M.V.Chushnyakova and I.I.Gontchar, Phys.Rev.C**87**(2013)014614



Classical = No tunneling



J.O. Newton, et al, Phys.Rev.C**70**(2004)024605

To construct a unified model ...

Quantum mechanical description of dissipation and fluctuation

Quantum mechanical description of energy dissipation and application to heavy-ion fusion reactions

► Introduction to heavy-ion fusion reactions

- Langevin equation
- Coupled-channels method
- A unified description

► Methodology

- Caldeira-Leggett model
- Why good ?

► Application to a fusion problem

- Results
- Effects of energy dissipation
- Remark on a unified description

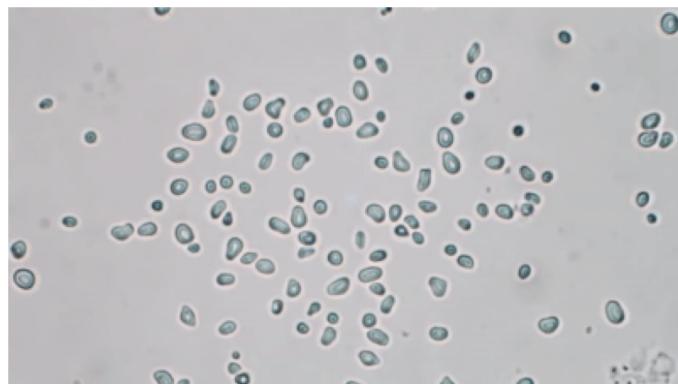
► Summary

Quantum dissipation model

9

Old model of dissipation ...

originally for **Brownian motion**

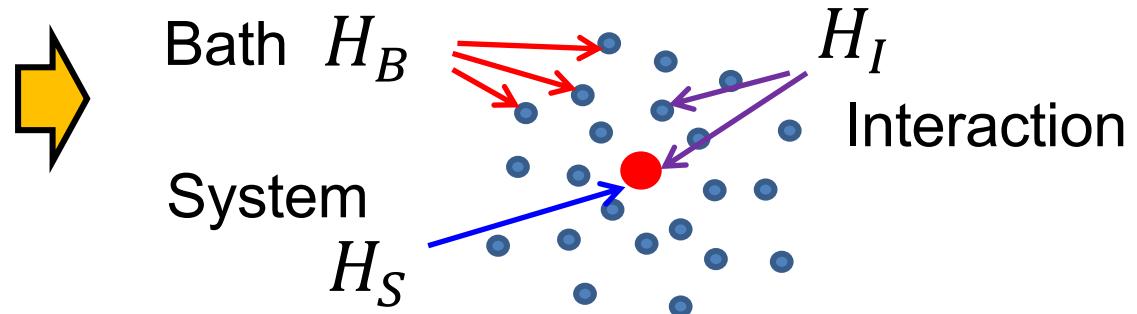


Pollen Grains in Water - Brownian Motion - YouTube

(Classical) Langevin equation

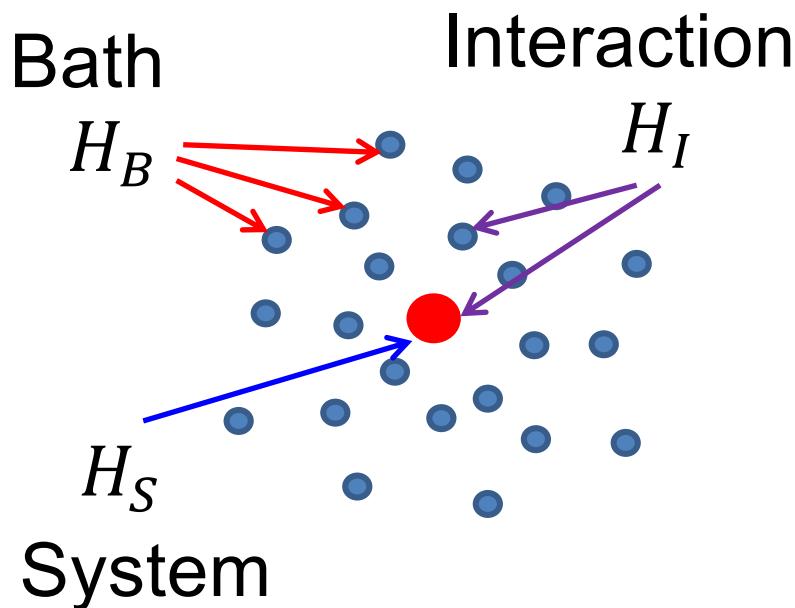
- ─ Based on classical e.o.m
- ─ Quantum mechanics: Hamiltonian

Model: Necessity of bath



$$H_{\text{tot}} = H_S + H_B + H_I$$

- ─ **System**(H_S): Relative motion
- ─ **Bath**(H_B): Internal motion



$$H_{\text{tot}} = H_S + H_B + H_I$$

A.O.Caldeira and A.J.Leggett, Physica, **121A**(1983)587
 A.O.Caldeira and A.J.Leggett, Ann. Phys. **149**(1983)374

System (Arbitrary)

$$H_S = \frac{\vec{P}^2}{2\mu} + U(\vec{R})$$

Bath (Oscillators)

$$H_B = \sum_i \hbar\omega_i a_i^\dagger a_i$$

$$[a_i, a_j] = 0, [a_i, a_j^\dagger] = \delta_{i,j}$$

Interaction (Linear)

$$H_I = \underbrace{h(\vec{R})}_{\text{Interaction form factor}} \sum_i d_i (a_i + a_i^\dagger)$$

$$H_{\text{tot}} = \frac{\vec{P}^2}{2\mu} + U(\vec{R}) + \sum_i \hbar\omega_i a_i^\dagger a_i + h(\vec{R}) \sum_i d_i (a_i + a_i^\dagger) \quad (*)$$

Classical limit of (*):

U.Weiss, "Quantum Dissipative Systems" (2008)
 G.W.Ford, J.T.Lewis, R. F.O'Connell, Phys.Rev.A37(1988)4419

$$\dot{\vec{P}}(t) = -\underbrace{\vec{\nabla}V(\vec{R}(t))}_{\text{Potential}} - \underbrace{\int_0^t ds \vec{\gamma}(t,s) \vec{P}(s)}_{\text{Frictional force}} + \underbrace{\vec{\nabla}h(\vec{R}(t)) \xi(t)}_{\text{Random force}}$$

Potential

$$\Delta(t) = \sum_i (2d_i^2/\hbar\omega_i) \cos(\omega_i t)$$

$$V(\vec{R}) = U(\vec{R}) - h^2(\vec{R})\Delta(0)/2$$

Frictional force

$$\vec{\gamma}(t,s) = \Delta(t-s) \vec{\nabla}h(\vec{R}(t)) (\vec{\nabla}h(\vec{R}(s)))^T / \mu$$

Random force

$\xi(t)$: Gaussian stochastic process

$$\begin{cases} \langle \xi(t) \rangle = 0 \\ \langle \xi(t) \xi(s) \rangle = \sum_i d_i^2 \cos(\omega_i(t-s)) \end{cases}$$

Solving (*) quantum mechanically ...

Quantum mechanical extension of the Langevin equation

$$H_{\text{tot}} = \frac{\vec{P}^2}{2\mu} + U(\vec{R}) + \sum_i \hbar\omega_i a_i^\dagger a_i + h(\vec{R}) \sum_i d_i(a_i + a_i^\dagger)$$

Naive basis

$$\langle \vec{R} | \Psi(t) \rangle = \sum_{n_1, n_2 \dots} \Psi_{n_1, n_2 \dots}(\vec{R}, t) |n_1, n_2 \dots\rangle \stackrel{?}{=} \prod_i \left(a_i^\dagger \right)^{n_i} / \sqrt{n_i} |0\rangle$$

Not suitable for a large number of HO modes

New basis

M.T and K. Hagino, Ann. Phys. **412** (2020) 168005

$$e^{-i\omega_i t} = \sum_{k=1}^K \chi_k(\omega_i) v_k(t)$$

$$s.t. \quad \begin{cases} b_k^\dagger = \sum_i d_i \chi_k(\omega_i) a_i^\dagger \\ (k = 1, \dots, K) \\ [b_k, b_q^\dagger] = \delta_{k,q} \end{cases}$$

Boson operators

$$\langle \vec{R} | \Psi(t) \rangle = \sum_{j_1, \dots, j_K} \Psi_{j_1, \dots, j_K}(\vec{R}, t) |j_1, \dots, j_K\rangle \stackrel{?}{=} \prod_{k=1}^K \left(b_k^\dagger \right)^{j_k} / \sqrt{j_k} |0\rangle$$

The number of b -modes = K

- ❖ Independent of the number of HO modes
- ❖ Dependent on $\max(\omega_i) \times (\text{Running time})$

Quantum mechanical description of energy dissipation and application to heavy-ion fusion reactions

► Introduction to heavy-ion fusion reactions

- Langevin equation
- Coupled-channels method
- A unified description

► Methodology

- Caldeira-Leggett model
- Why good ?

► Application to a fusion problem

- Results
- Effects of energy dissipation
- Remark on a unified description

► Summary

$$H_{\text{tot}} = \frac{\vec{P}^2}{2\mu} + U(\vec{R}) + \sum_i \hbar\omega_i a_i^\dagger a_i + h(\vec{R}) \sum_i d_i (a_i + a_i^\dagger)$$

◆ System $^{16}\text{O} + ^{208}\text{Pb}$

□ Potential

$$V(R) = V_C(R) + V_N(R) + iW(R)$$

O.Akyüz and A.Winther,
ISPEF, Course LXXVII

Coulomb barrier

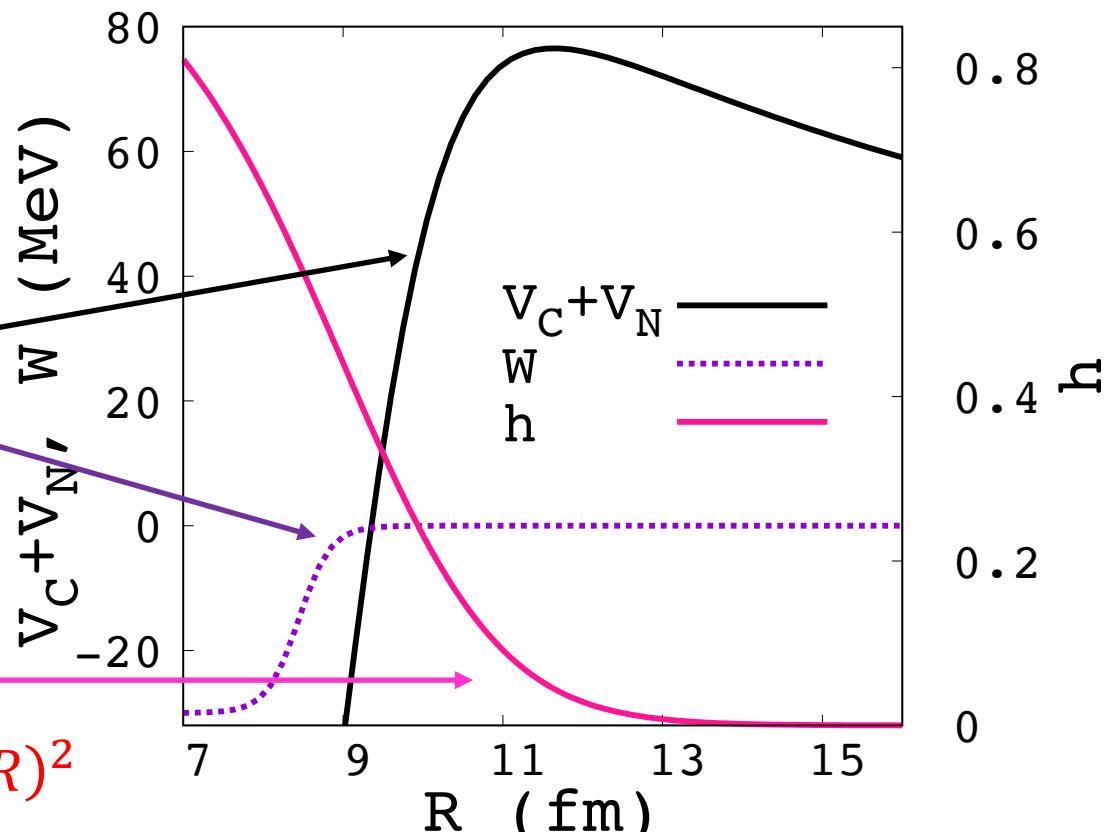
$$V_B = 76.52 \text{ MeV}$$

Absorption
(Fusion)

□ Interaction form factor

$$h(R) \sim V_N(R)$$

→ (Friction coefficient) $\sim (dV_N/dR)^2$



P.Frobrich and I.I.Gontchar, Phys.Rep.292(1998)131

$$H_{\text{tot}} = \frac{\vec{P}^2}{2\mu} + U(\vec{R}) + \sum_i \hbar\omega_i a_i^\dagger a_i + h(\vec{R}) \sum_i d_i (a_i + a_i^\dagger)$$

Rotational symmetry

$$U(\vec{R}) = U(|\vec{R}|), h(\vec{R}) = h(|\vec{R}|)$$

$$\rightarrow |\Psi(t)\rangle = \sum_{L,M} |Y_{L,M}\rangle \otimes |\Psi^L(t)\rangle$$

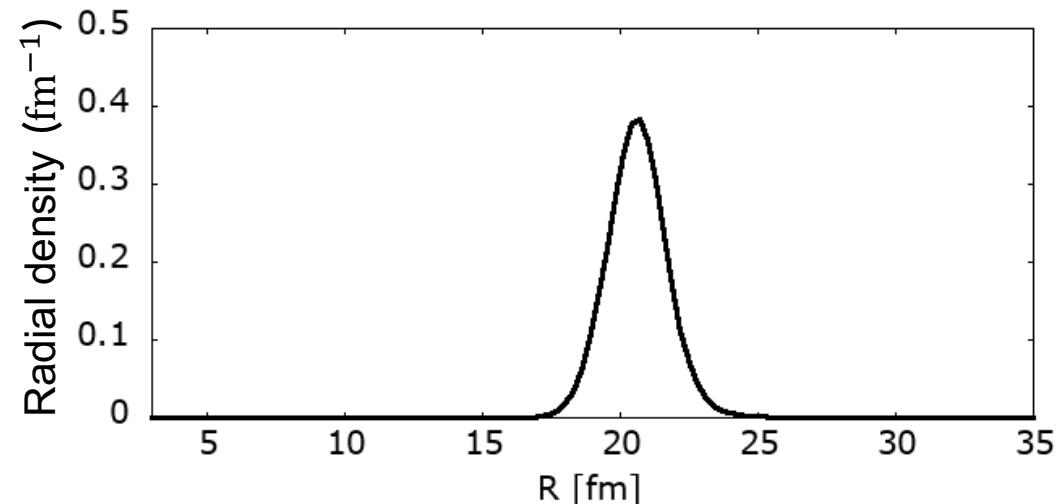
Time evolution of radial motion

Expansion with b -modes:

$$R\langle R|\Psi^L(t)\rangle = \sum_{j_1, \dots, j_K} u_{j_1, \dots, j_K}^L(R, t) |j_1, \dots, j_K\rangle$$

$$|j_1, \dots, j_K\rangle = \prod_{k=1}^K (b_k^\dagger)^{j_k} / \sqrt{j_k} |0\rangle$$

\rightarrow Schrödinger equation
 $(Bath_{t=0} = \text{Zero temperature})$



$|\Psi^L(t_f)\rangle$: Reflected wave packet

\longrightarrow Excitation spectrum

\longrightarrow Fusion cross sections

Result: Excitation spectrum (Kinetic energy loss dist.)

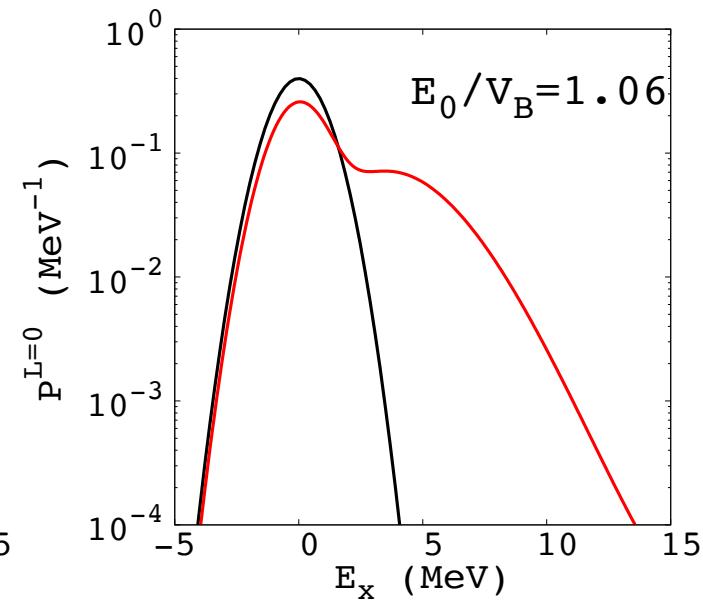
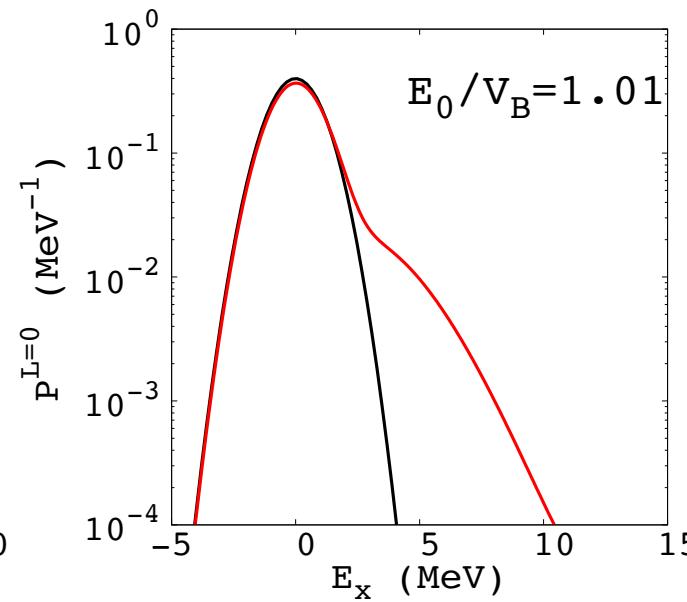
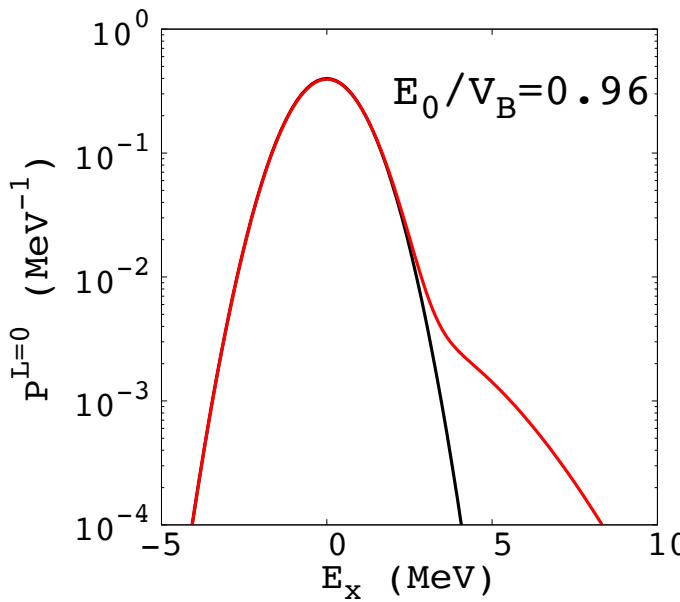
16

Excitation spectrum

Excitation energy
= Stored in the bath



$$P^L(E_x) = \frac{\langle \Psi^L(t_f) | f(H_B - E_x) | \Psi^L(t_f) \rangle}{\langle \Psi^L(t_f) | \Psi^L(t_f) \rangle}$$



— Free — Bath

— Increasing initial energies →

✓ The larger excitations with increasing incident energies

Result: Fusion cross sections

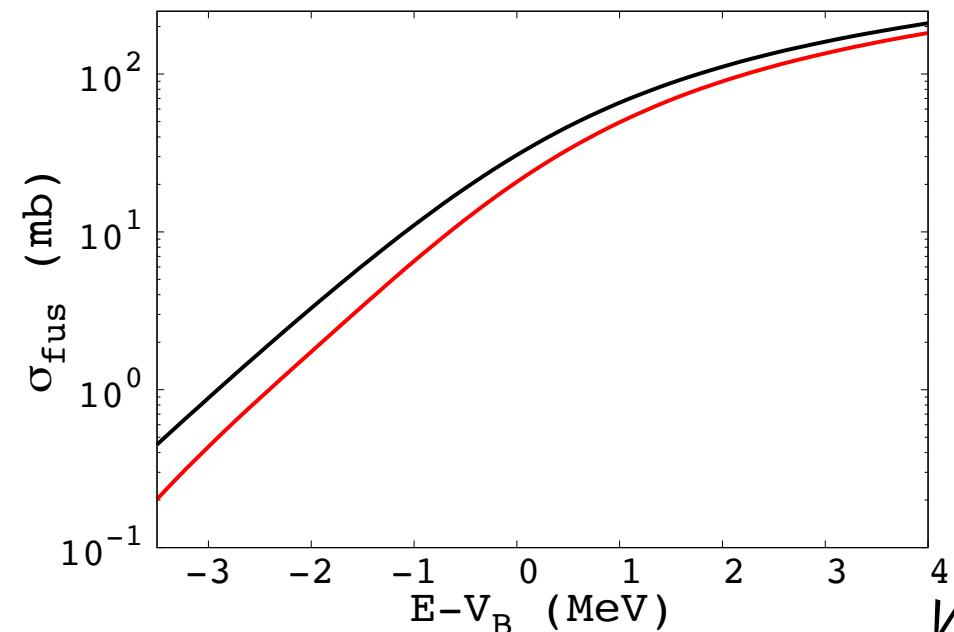
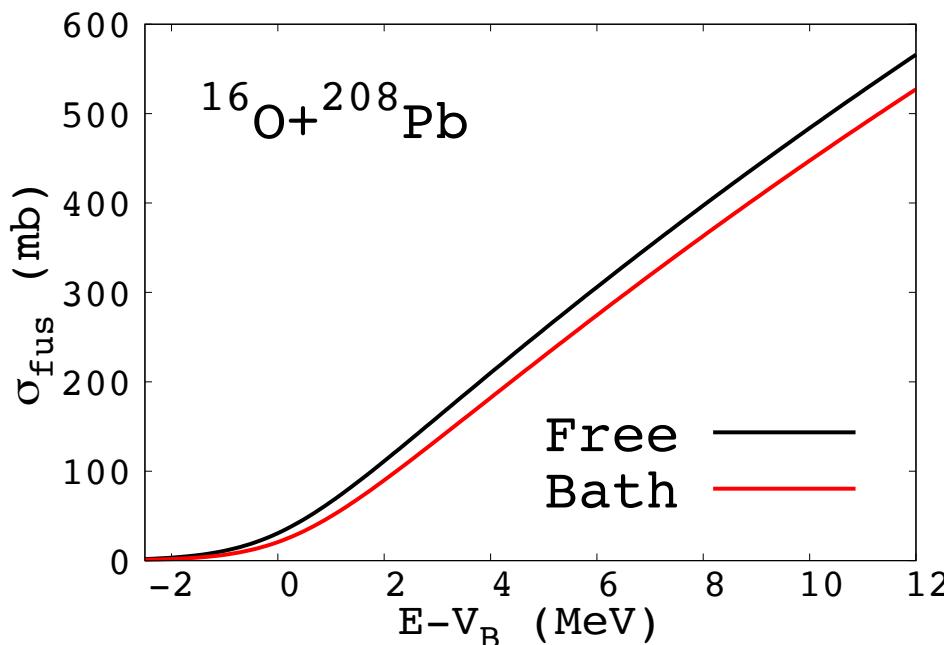
17

□ Fusion cross sections

$$R_L(E) = \frac{\langle \Psi^L(t_f) | \delta(H_S + H_B - E) | \Psi^L(t_f) \rangle}{\langle \Psi^L(0) | \delta(H_S + H_B - E) | \Psi^L(0) \rangle}$$

K. Yabana, Prog. Theor. Phys. 97, 437 (1997)

$$\sigma_{\text{fus}}(E) = \frac{\hbar^2 \pi}{2\mu E} \sum_L (2L+1) \frac{(1-R_L(E))}{\text{Penetrability}}$$



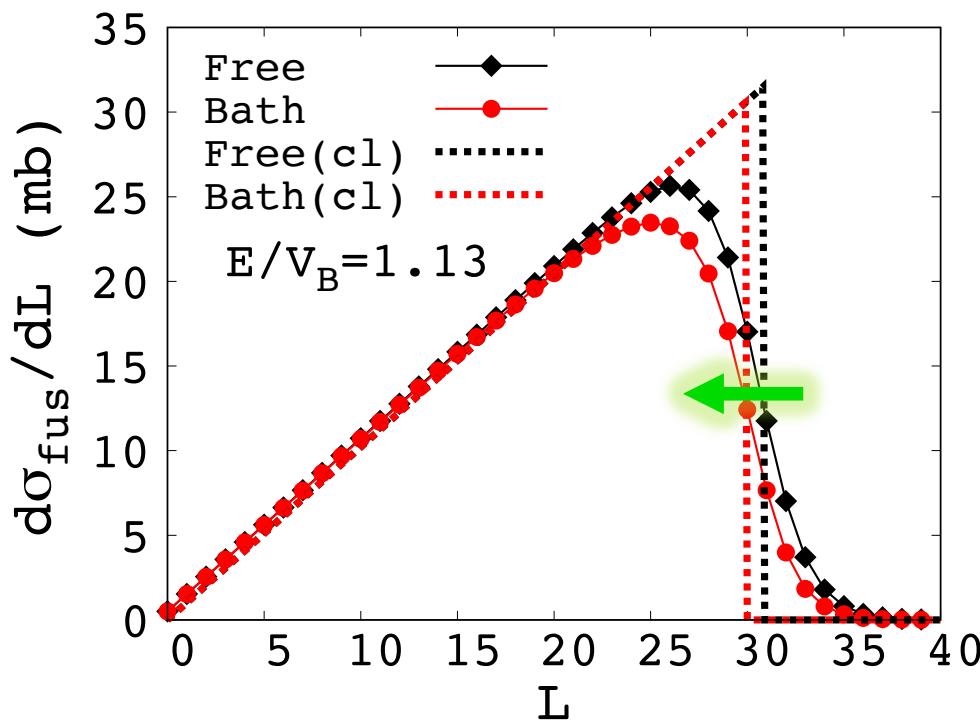
✓ Suppression at above and sub-barrier energies

Why ? ↗

$E > V_B$: Bath coupling before reaching the barrier

18

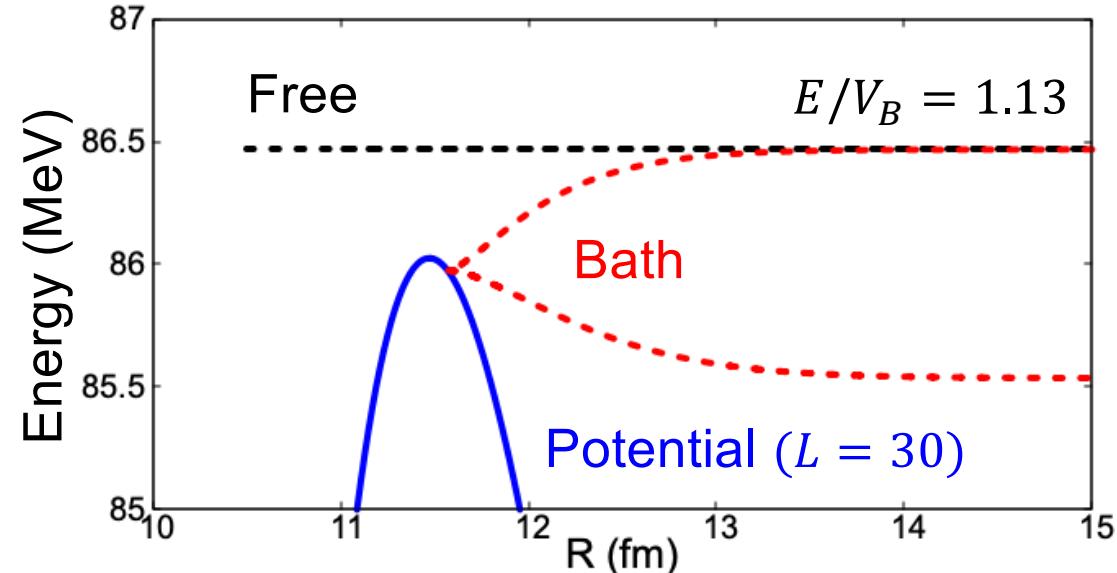
$$\begin{aligned}\sigma_{\text{fus}}(E) &= \frac{\hbar^2 \pi}{2\mu E} \sum_L (2L + 1)(1 - R_L(E)) \\ &\equiv \sum_L d\sigma_{\text{fus}}/dL(E)\end{aligned}$$



Classical limit

$T = 0 \rightarrow$ No random force

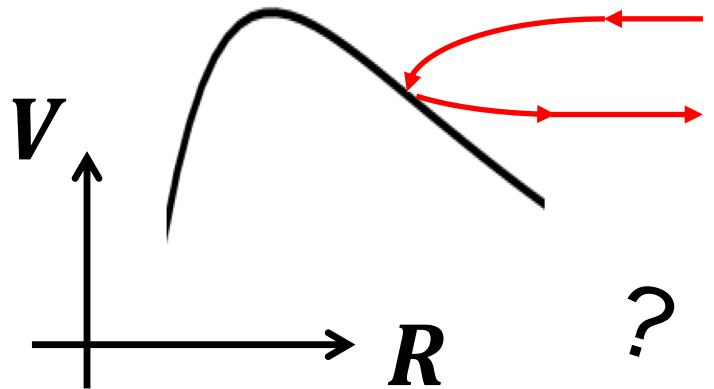
$$\dot{P}_R(t) = \frac{\hbar^2 L^2}{\mu R(t)^3} - V'(R(t)) - \int_0^t ds \gamma(t, s) P_R(s)$$



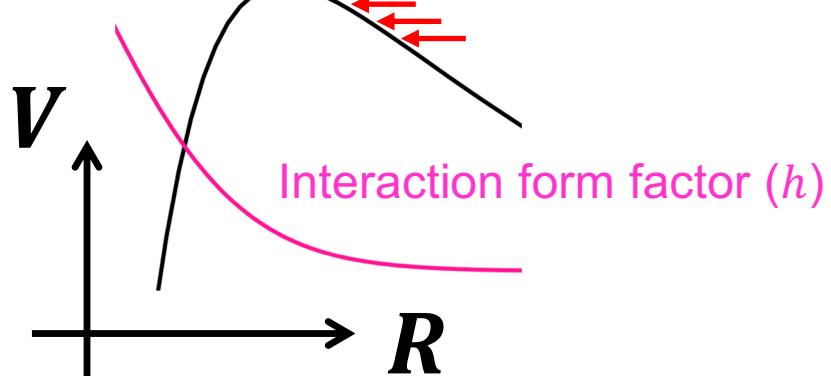
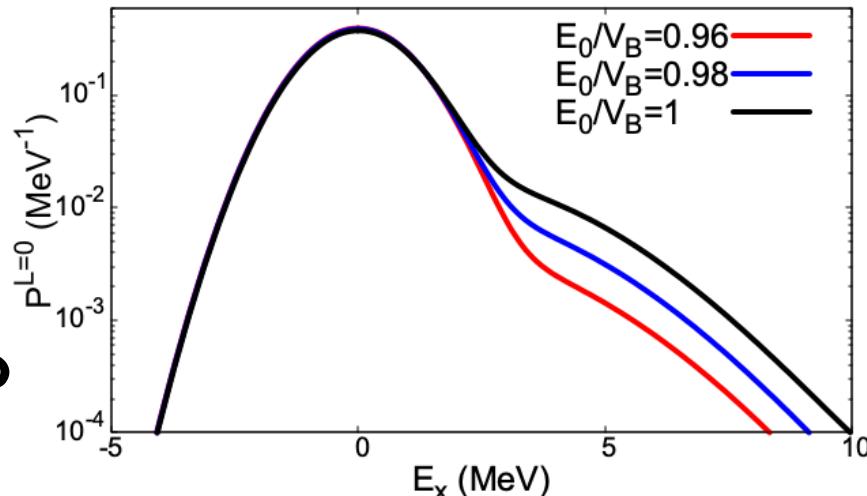
✓ Bath coupling before reaching the barrier

$E < V_B$: Bath coupling before reaching the barrier ?

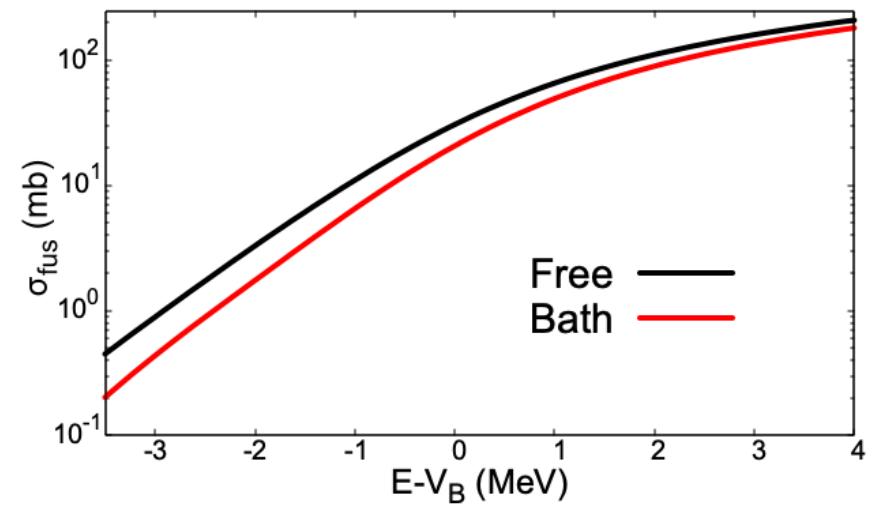
19



??



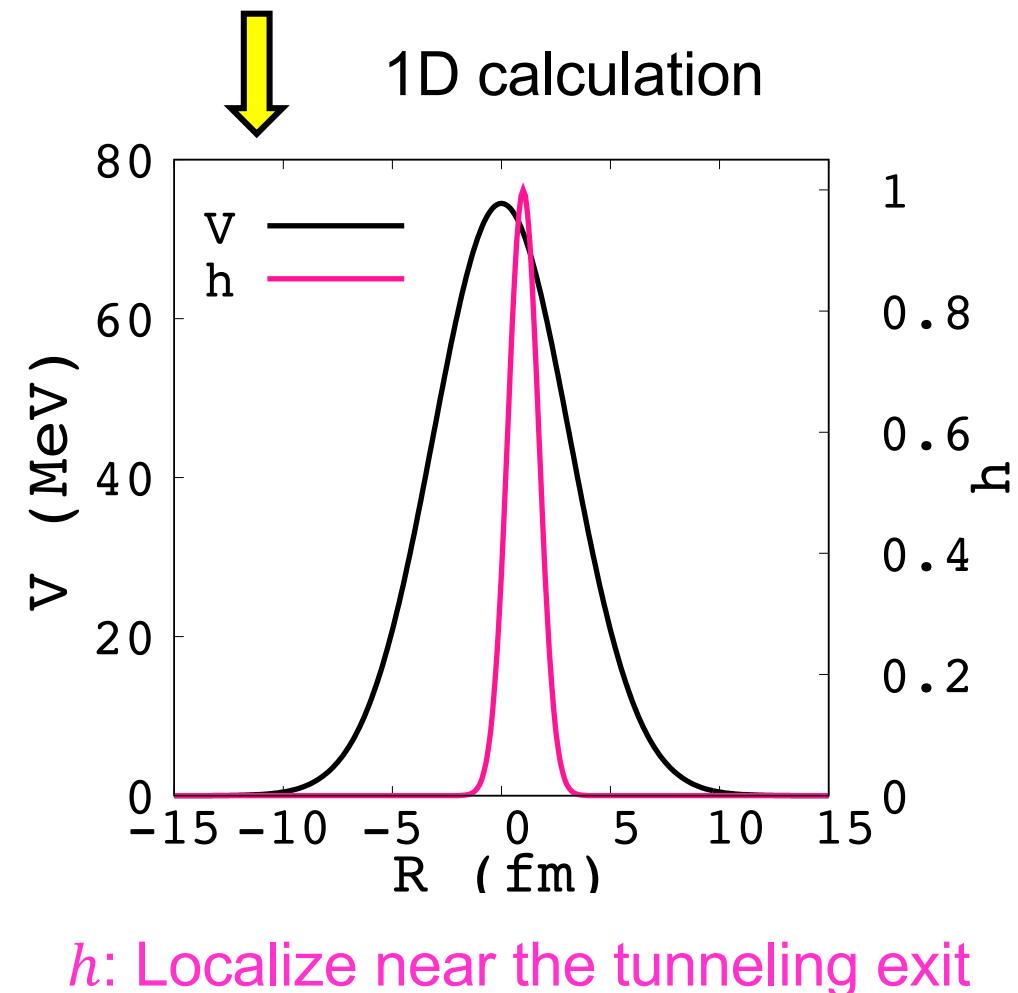
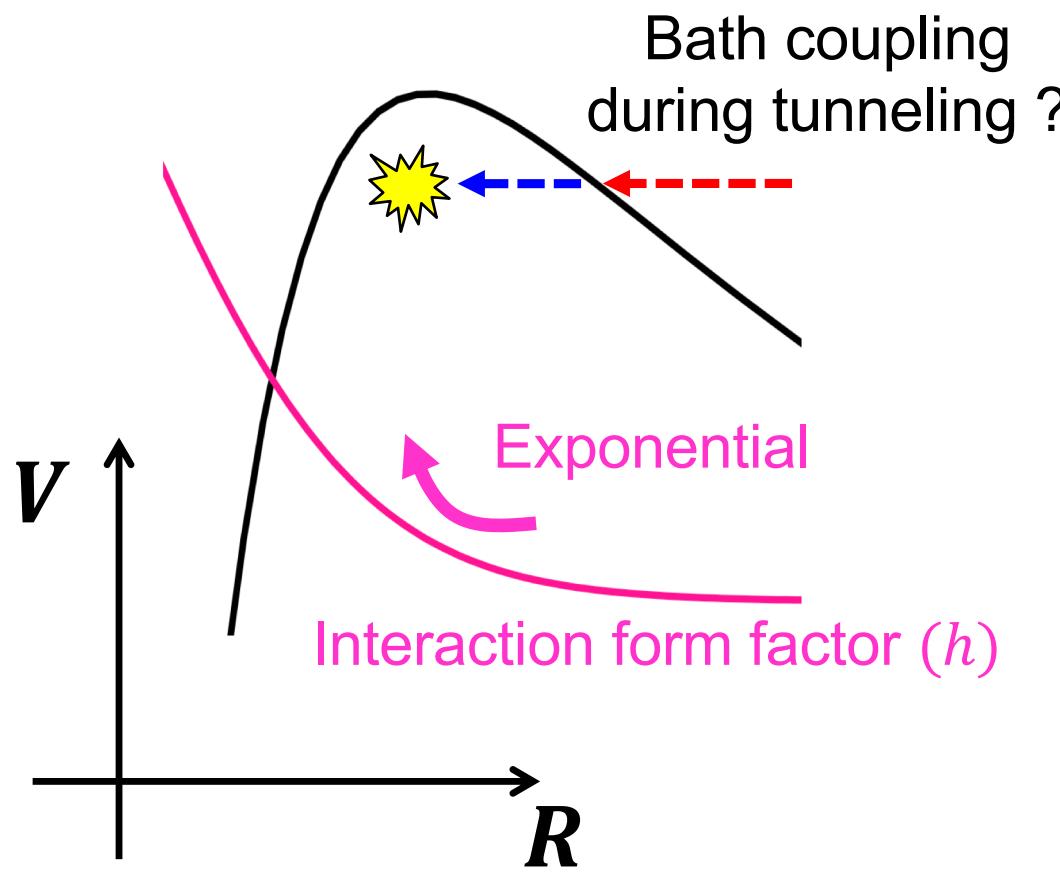
But ...



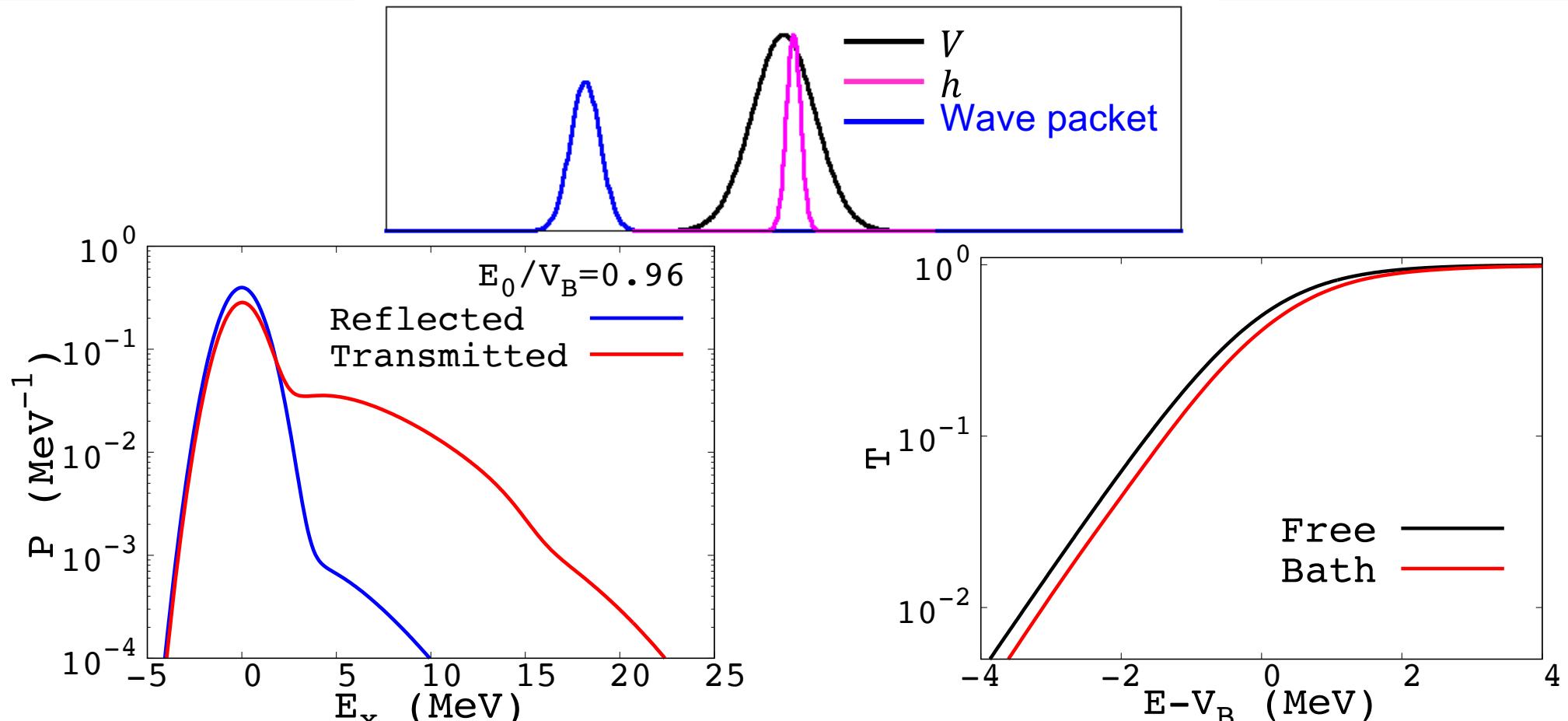
Overlap between h and Ψ decreases

Bath coupling during tunneling

20



h : Localize near the tunneling exit



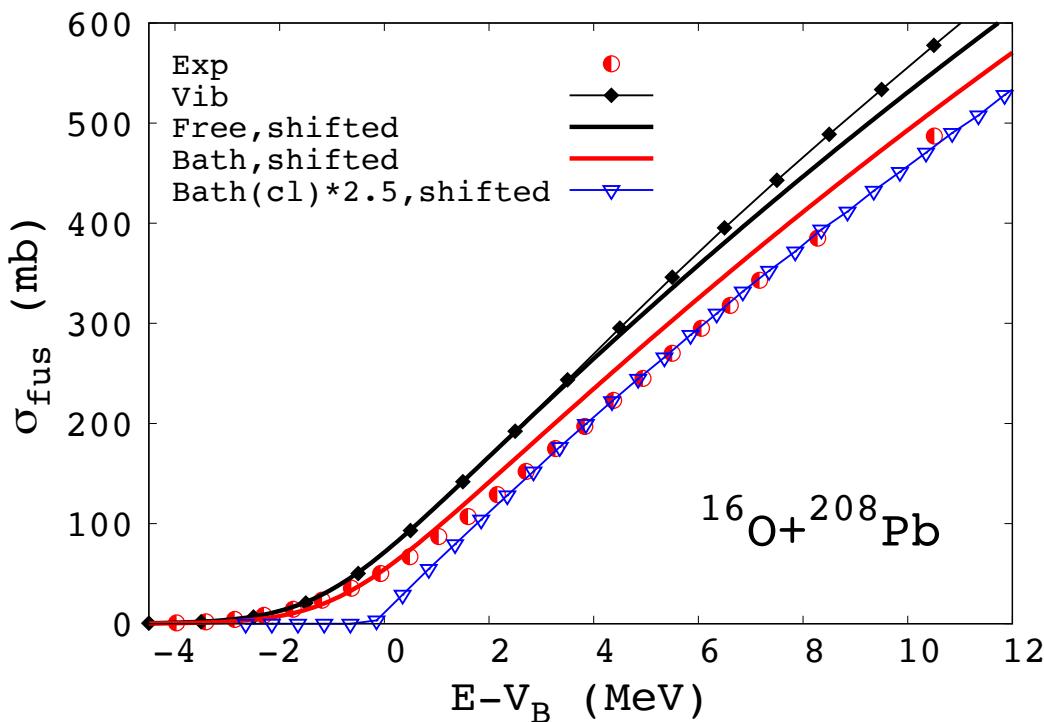
✓ Excitations during tunneling

✓ Bath coupling during tunneling

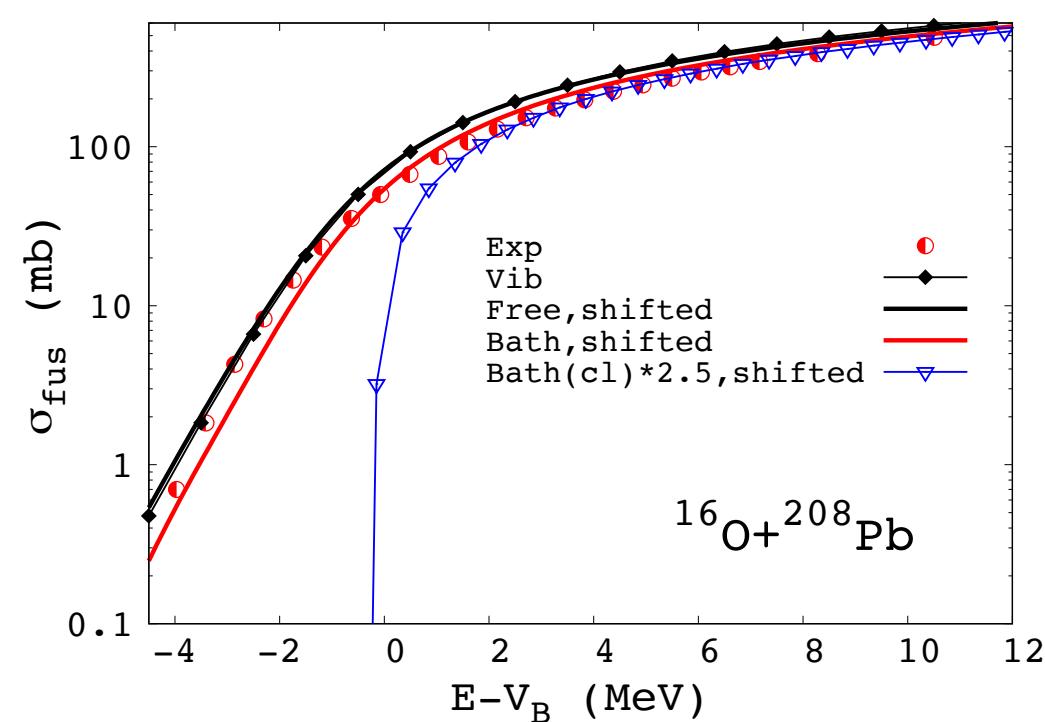
Unified description ?

22

Experimental data: C.R.Morton et al, Phys. Rev. C **60**(1999)044608



$V_B = 74.5 \text{ MeV}$



✓ Underestimation of sub-barrier fusion cross sections
Microscopic treatment of dissipation-fluctuation

K. Washiyama, D. Lacroix, and S. Ayik, Phys. Rev. C **79** (2009) 024609

Introduction to heavy-ion fusion reactions

- $E > V_B$: Classical Langevin equation (**Dissipation, Fluctuation**)
- $E < V_B$: Quantum coupled-channels method (**Tunneling, Low-lying excitations**)
- Quantum mechanical description of dissipation and fluctuation

Methodology

- Caldeira-Leggett model
- Classical limit = Langevin equation
- Numerical method based on non-trivial boson operators

$$H_{\text{tot}} = \vec{P}^2/2\mu + U(\vec{R}) + \sum_i \hbar\omega_i a_i^\dagger a_i + h(\vec{R}) \sum_i d_i (a_i + a_i^\dagger)$$

Application to a fusion problem

- Excitation spectrum and fusion cross sections
 - Suppression at above and sub-barrier energies
 - Unified description: Microscopic treatment of dissipation-fluctuation (Conjecture)
- $E > V_B$: **Before** the barrier
 - $E < V_B$: **During** tunneling