Black Hole Information Paradox and Wormholes

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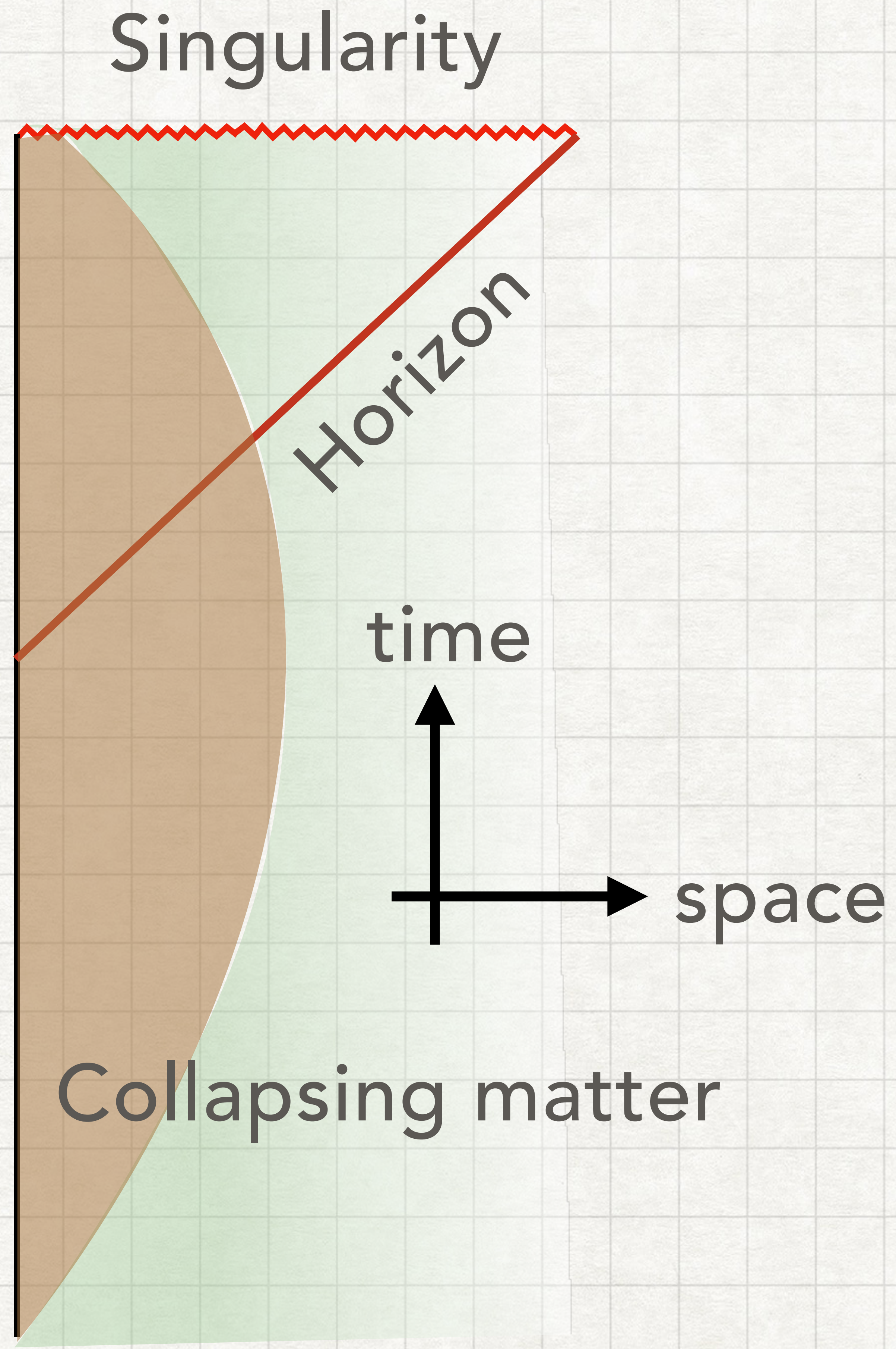
RIKEN iTHEMS

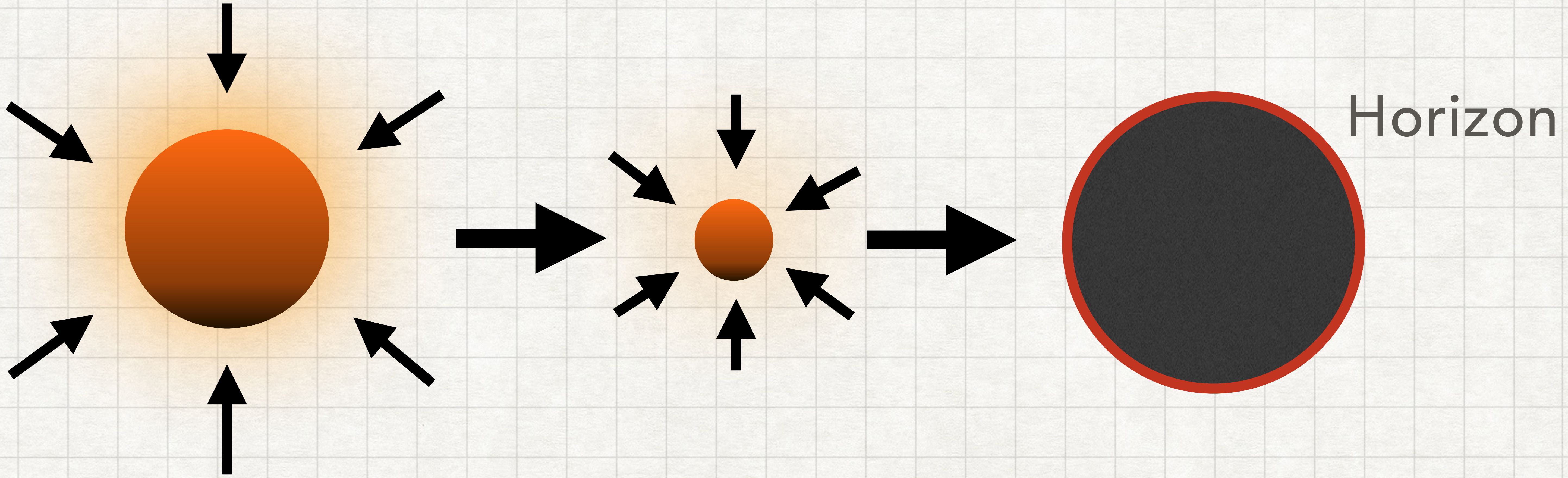
Based on work with T.Hartman and A. Tajdini JHEP04(2021)289 arXiv:2011.09043

Black Hole Evaporation



Penrose diagram

Black holes are created by gravitational collapse



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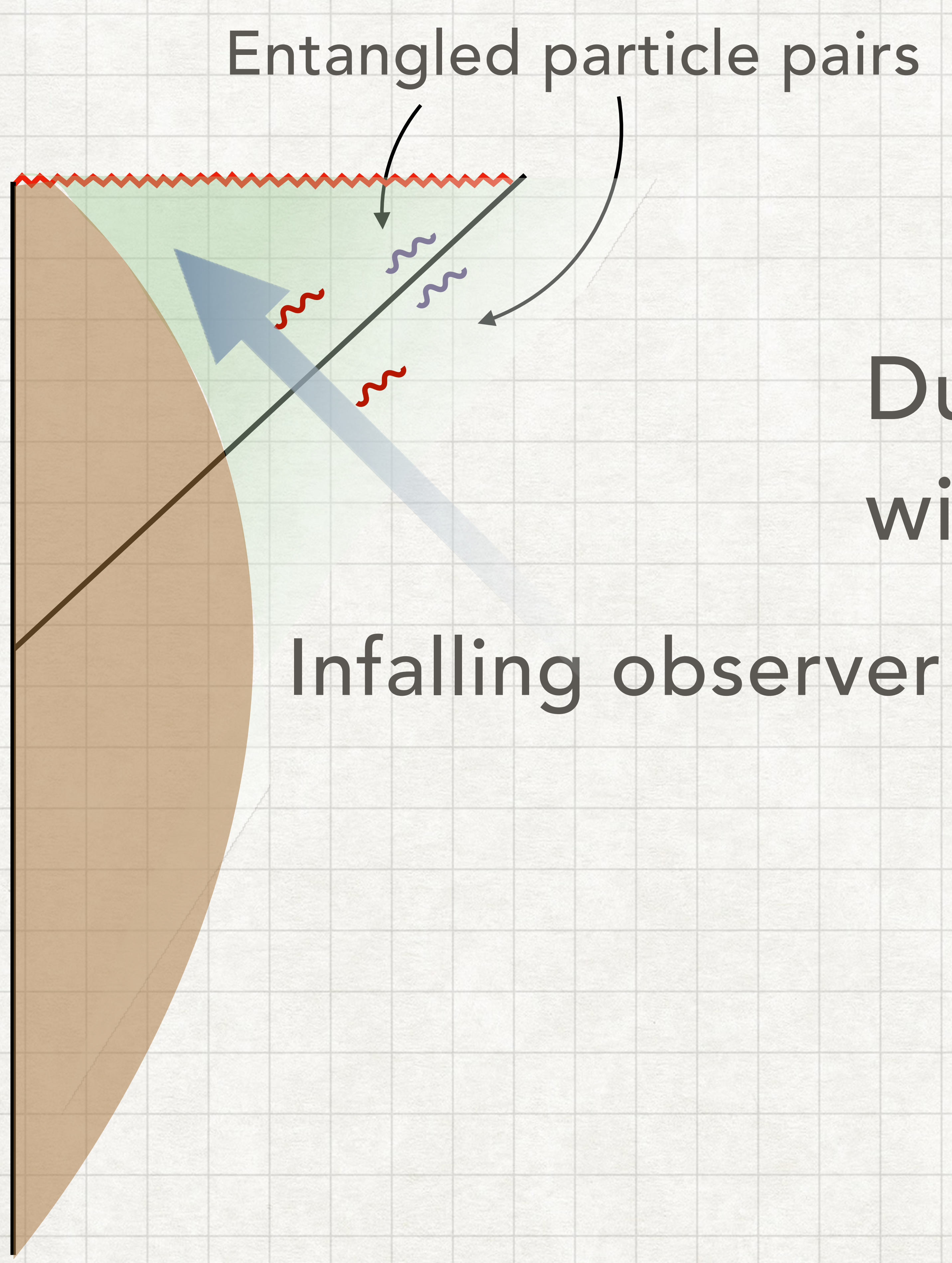
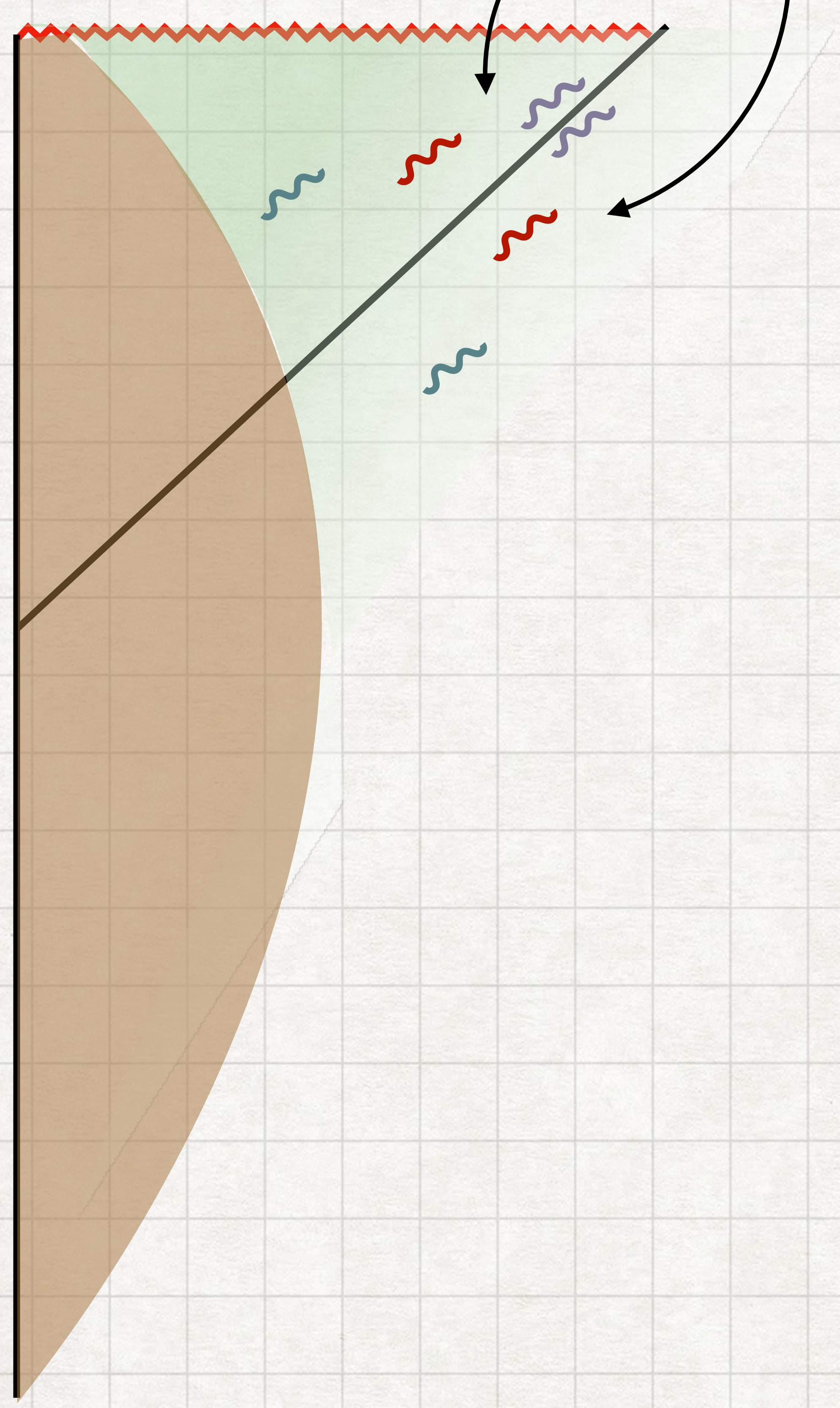


Due to the equivalence principle, an infalling observer will see the smooth spacetime near the horizon

↓

entangled particle pairs across the horizon

Black Hole Evaporation



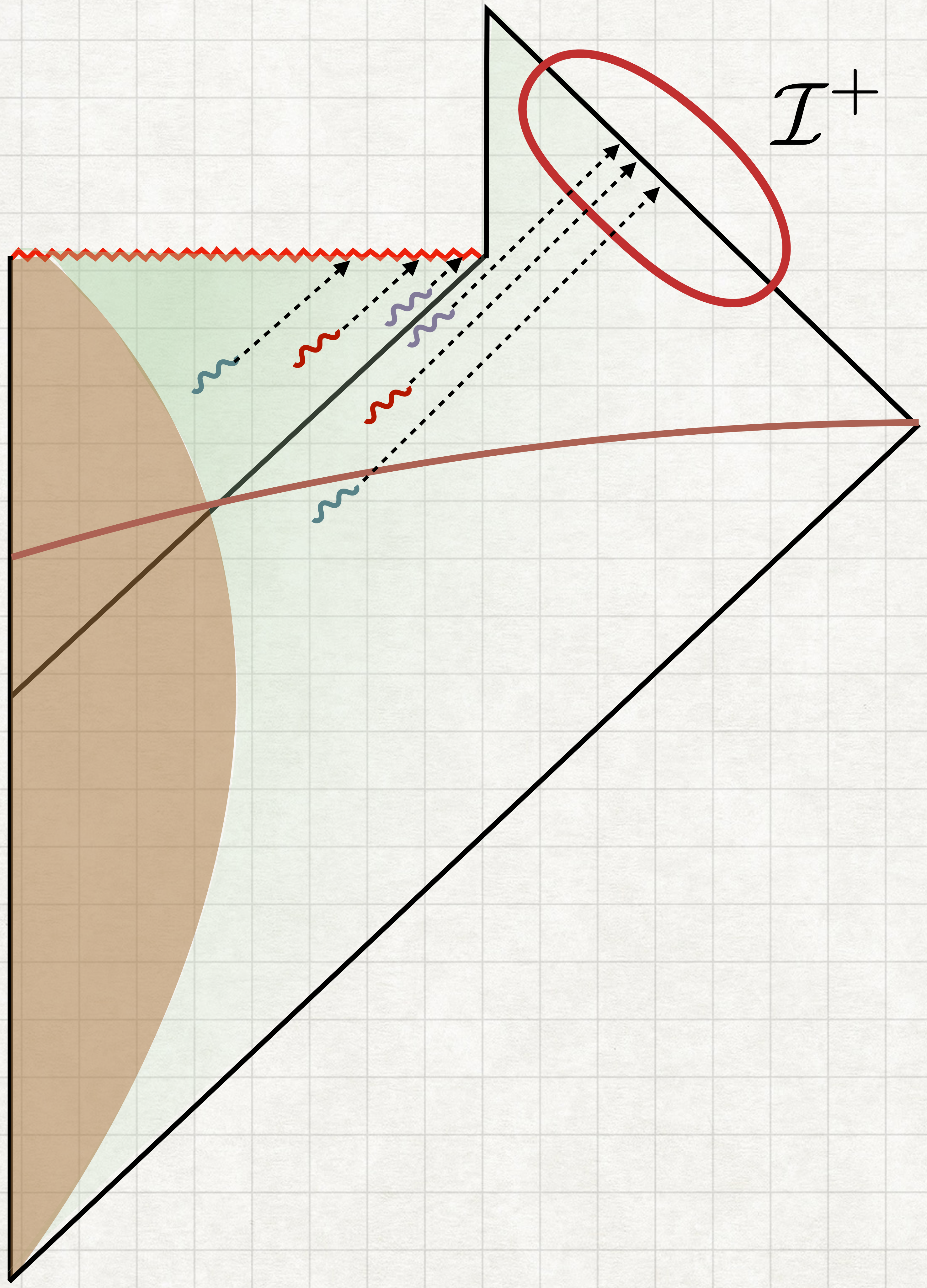
From the outside:

Only a partner of the entangled pair can be seen.

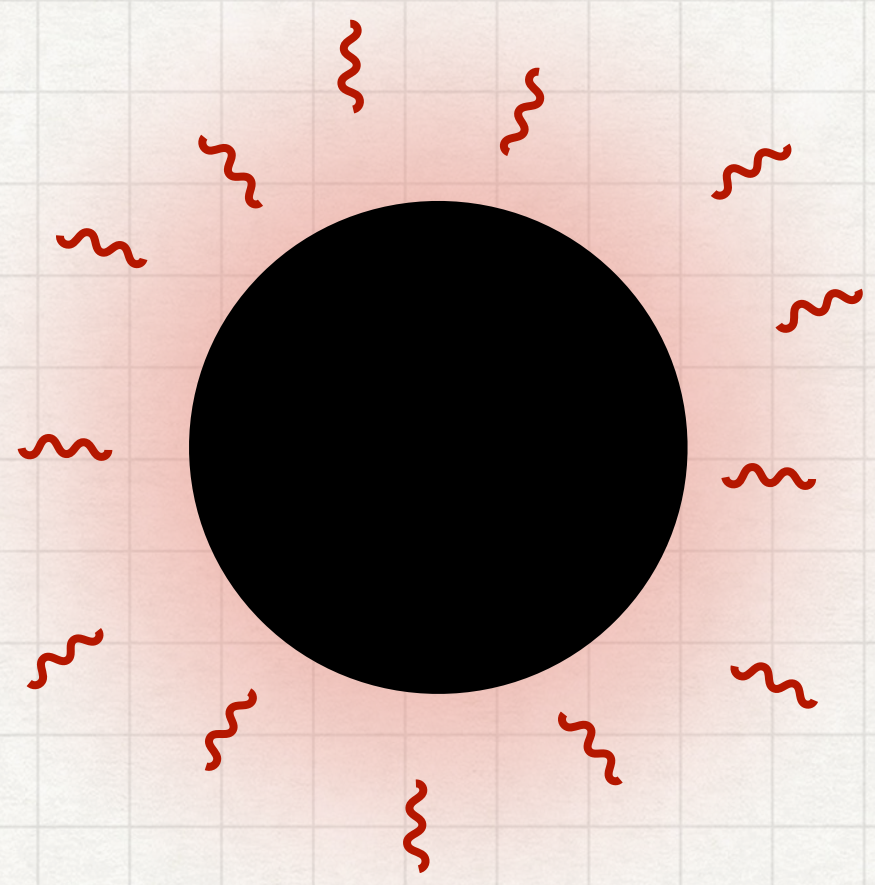
→Thermal radiation emitted from the horizon

"Hawking radiation"

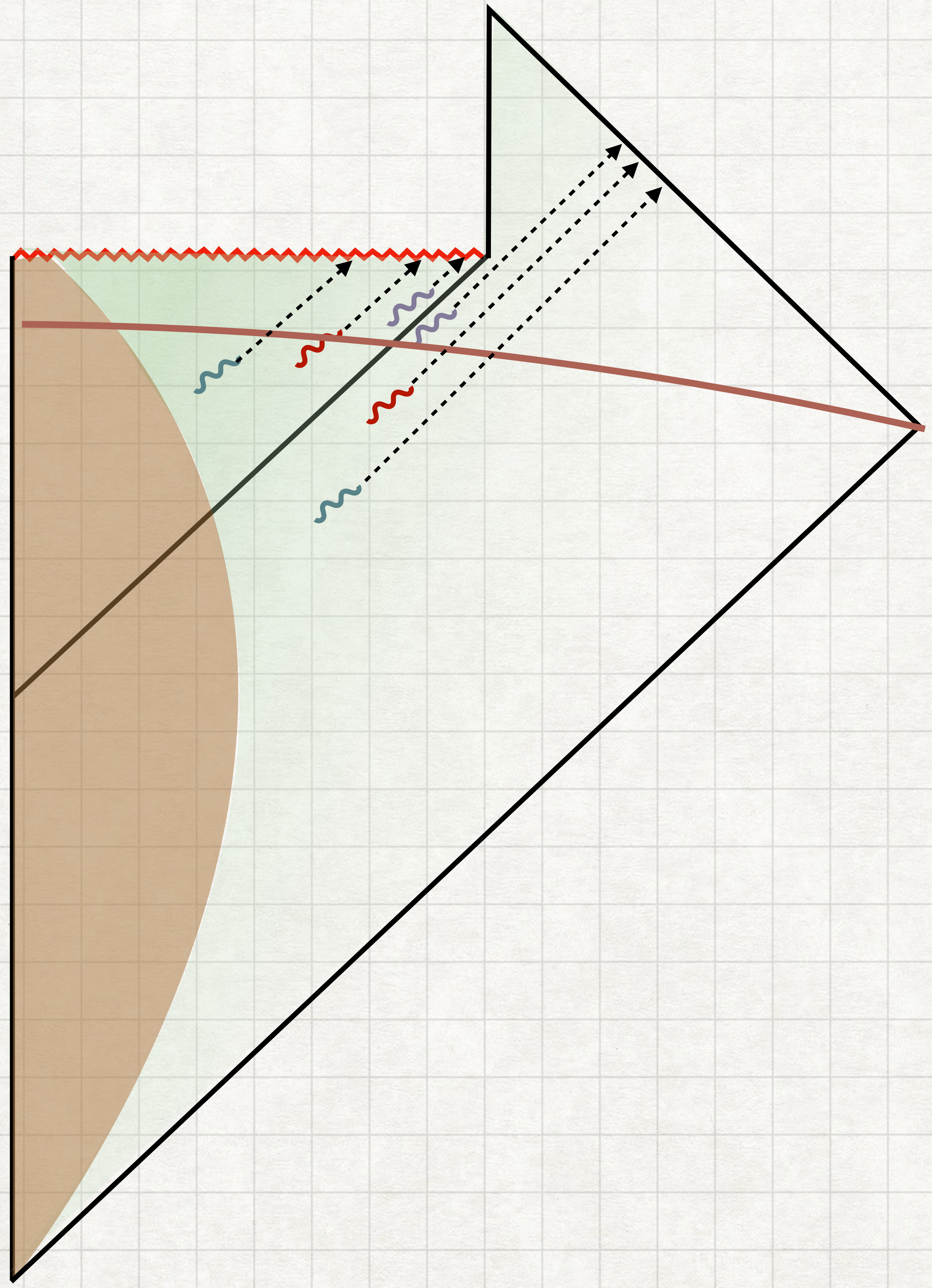
Black Hole Evaporation



Due to Hawking radiation BH loses its mass



Black Hole Evaporation



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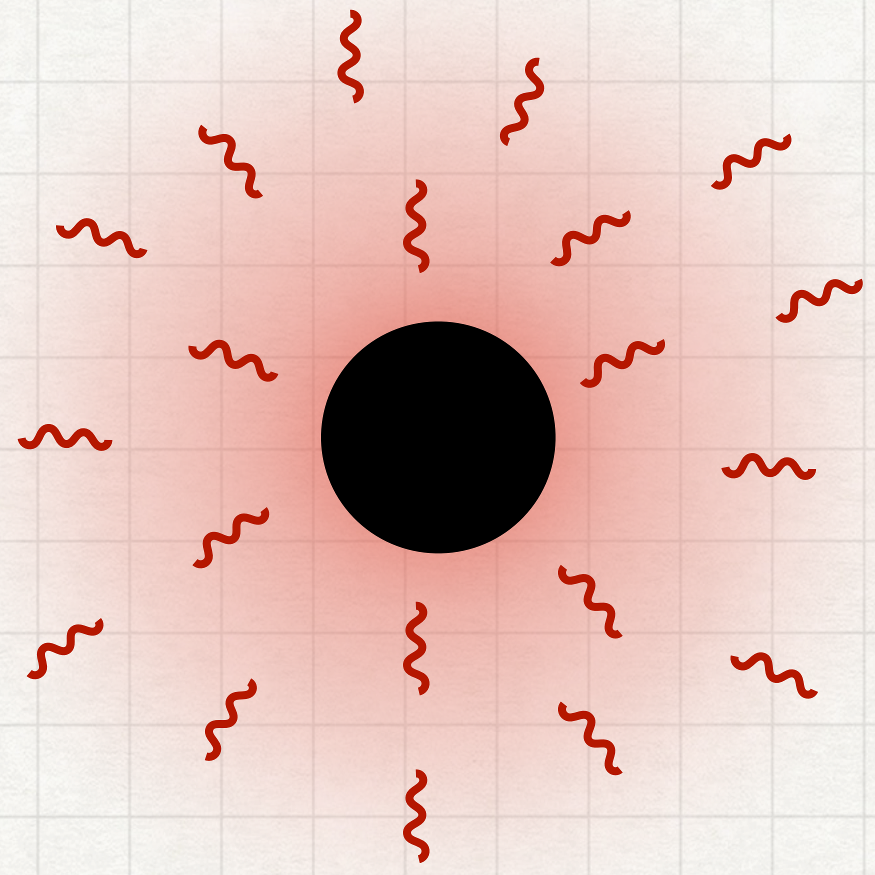

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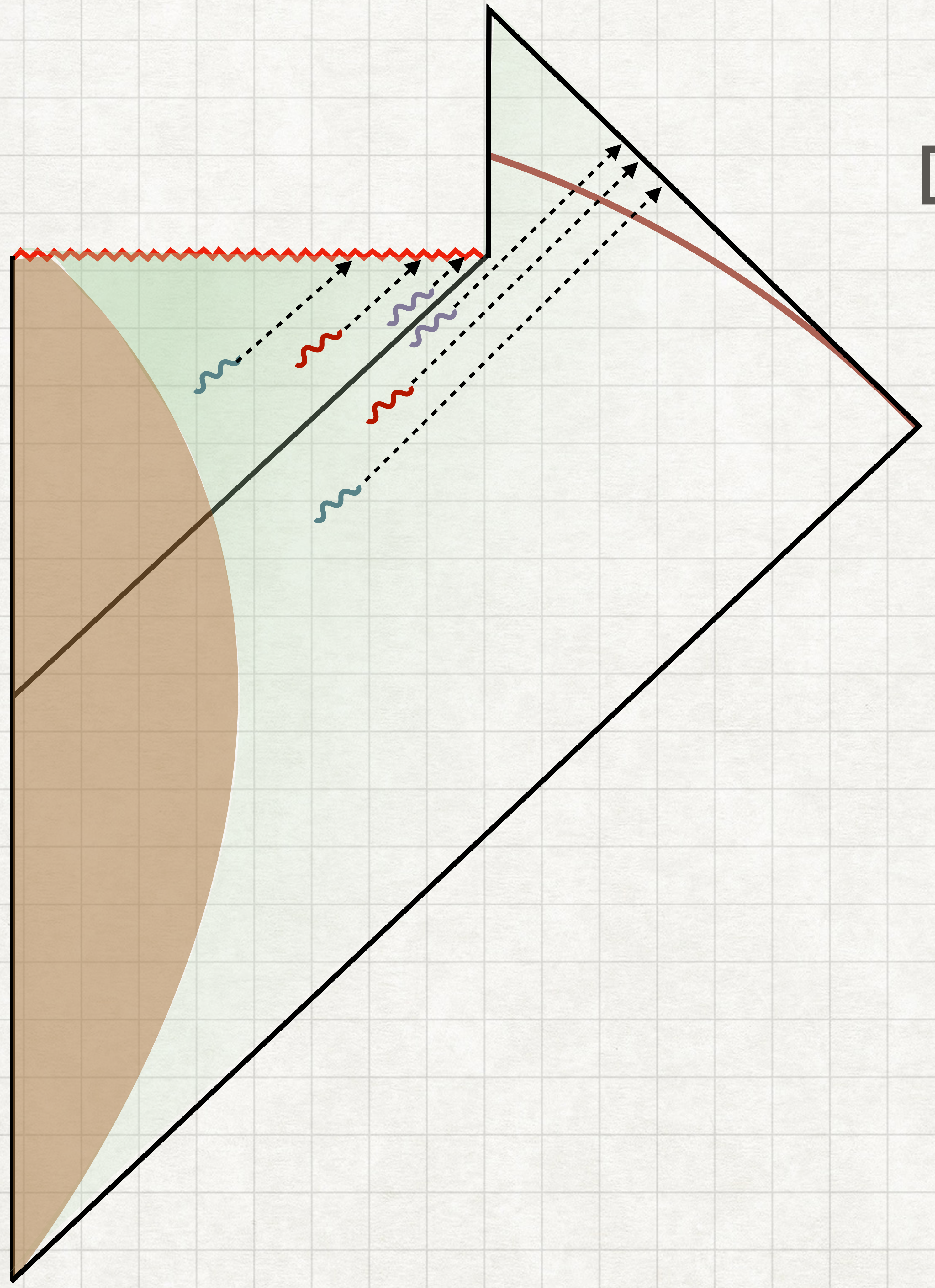

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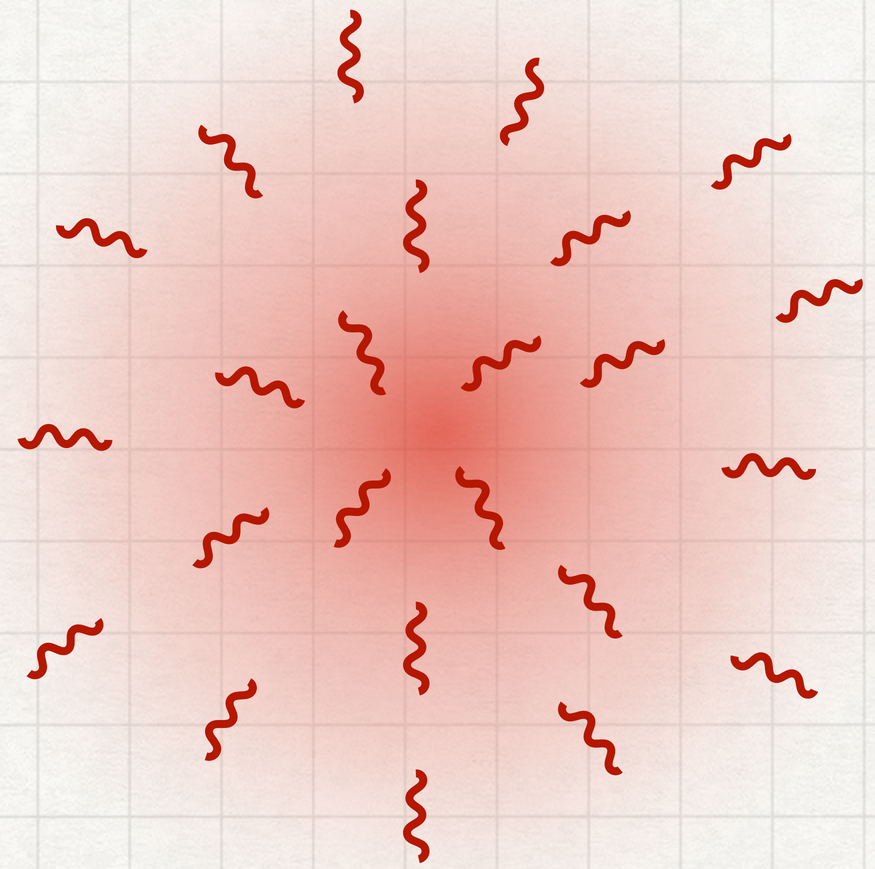
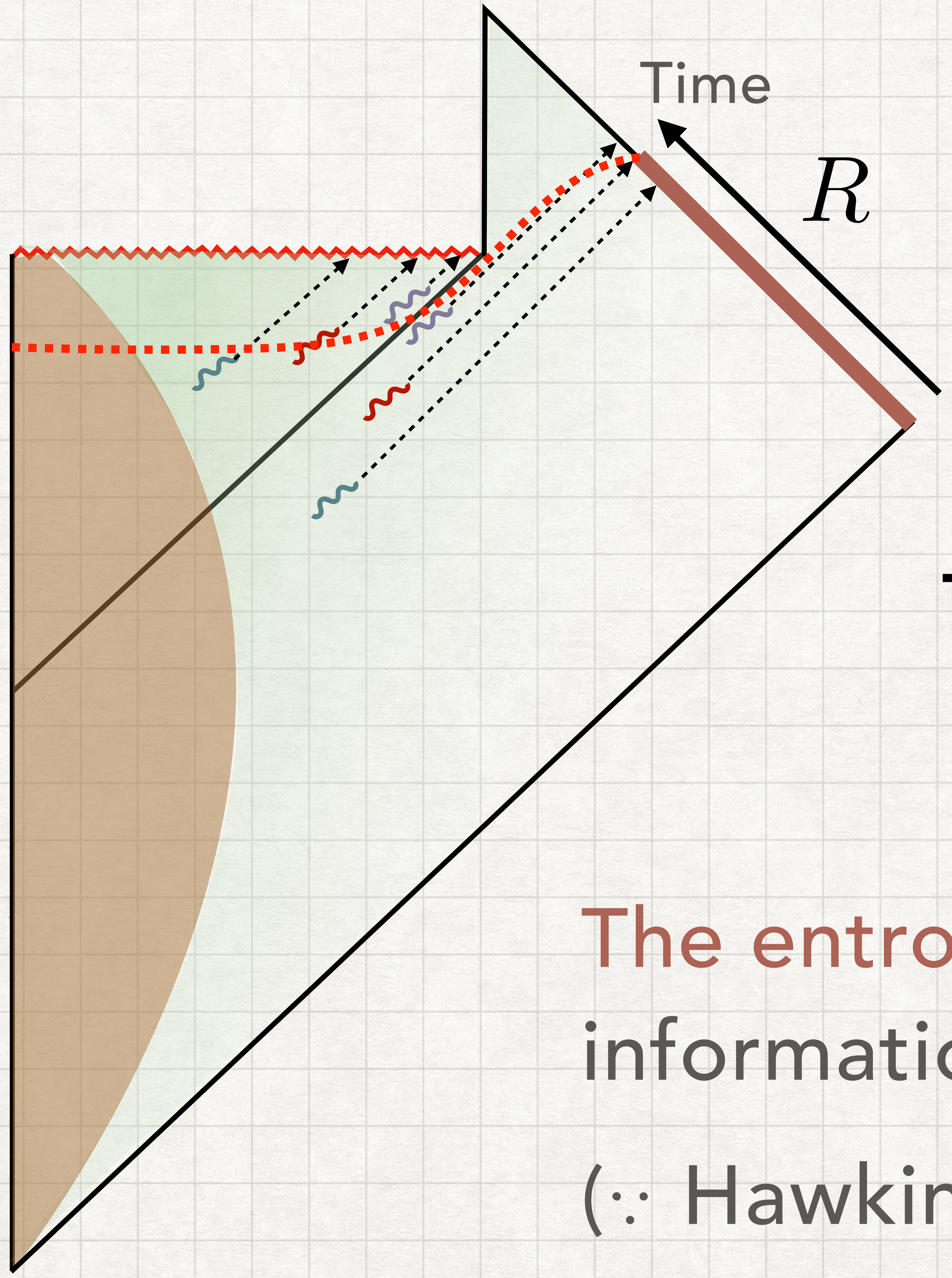
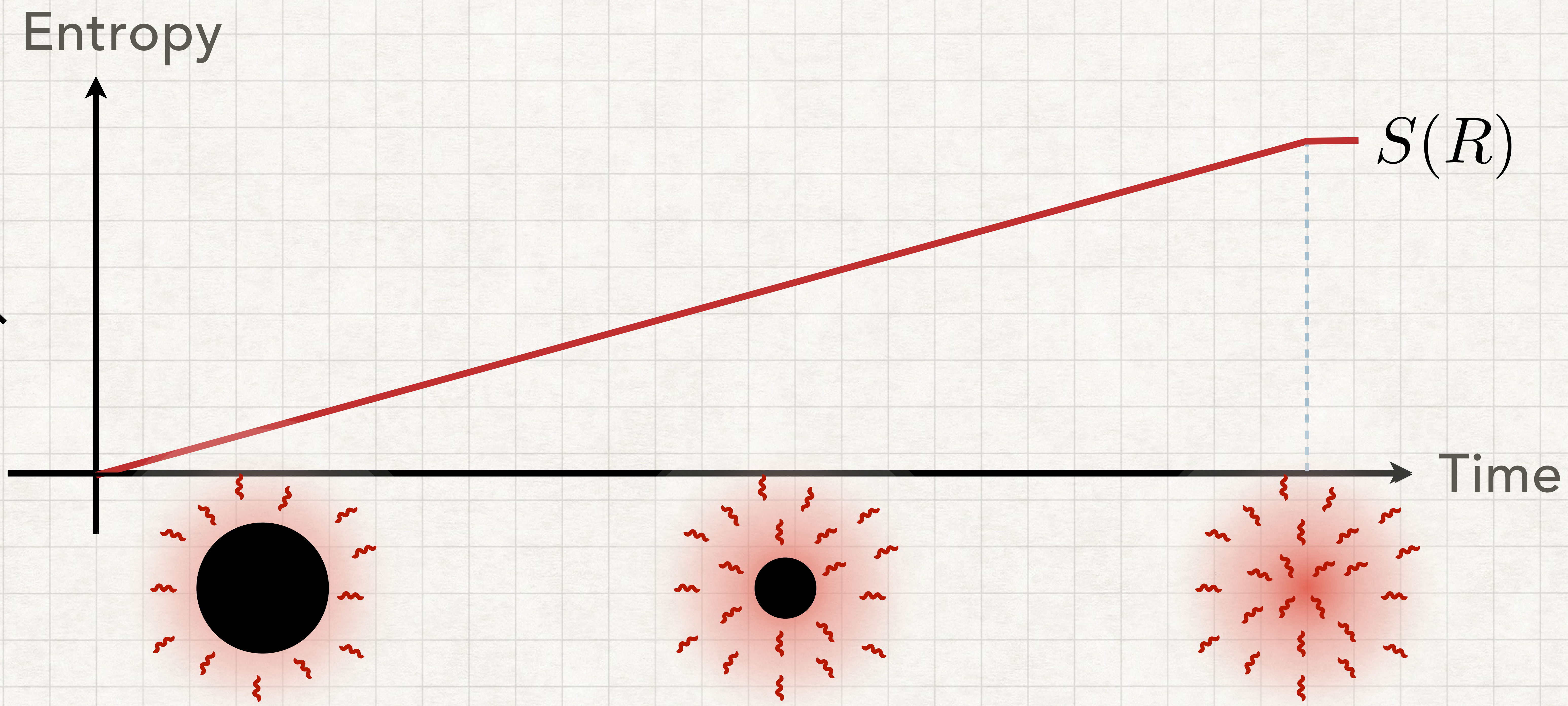


Eventually there only remains thermal radiation

Black Hole Information Paradox



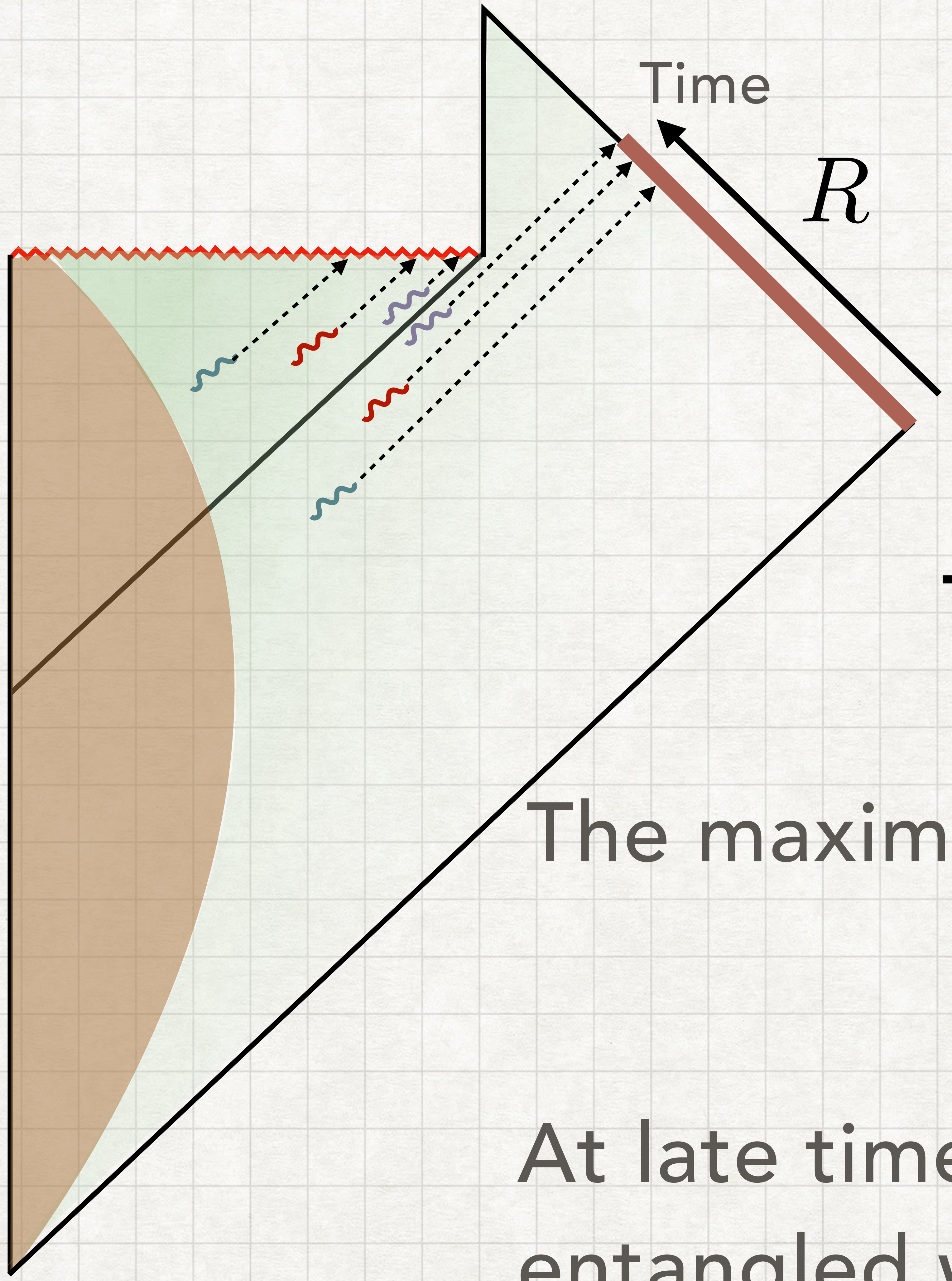


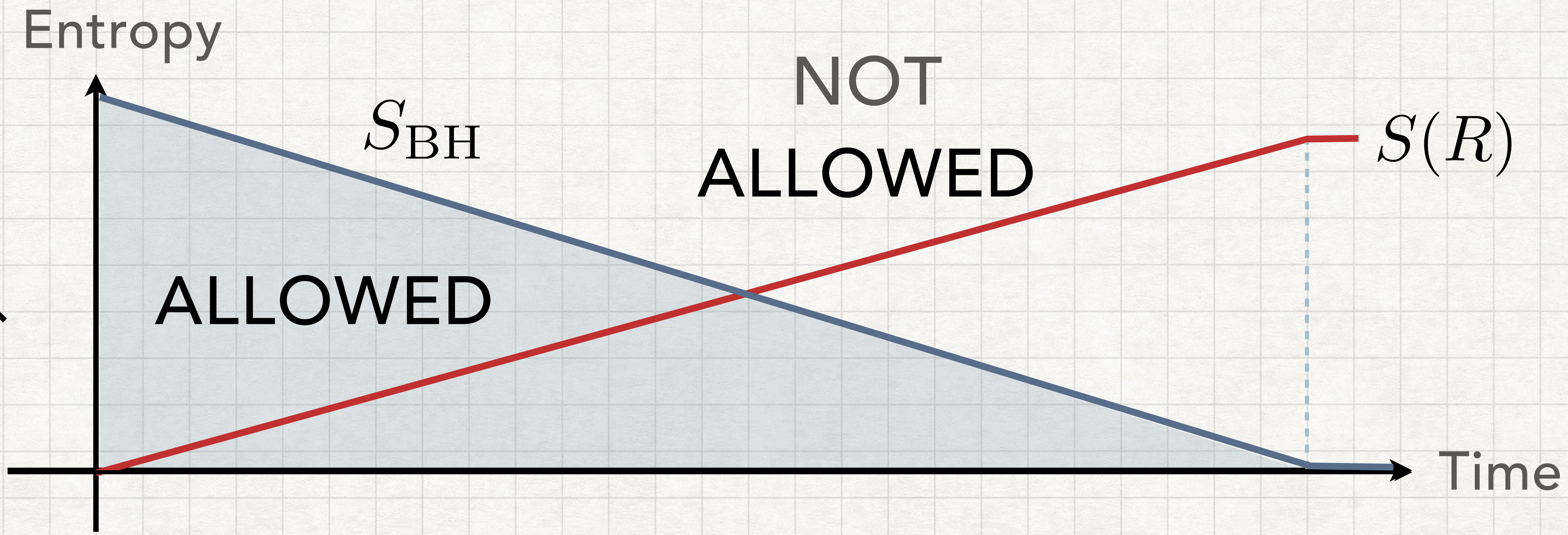
The entropy of Hawking radiation probes the amount of information of the BH interior lost from an outside observer

($\because$ Hawking radiation is entangled with the BH interior)

It keep growing until BH is fully evaporated

Black Hole Information Paradox





It causes the information paradox!

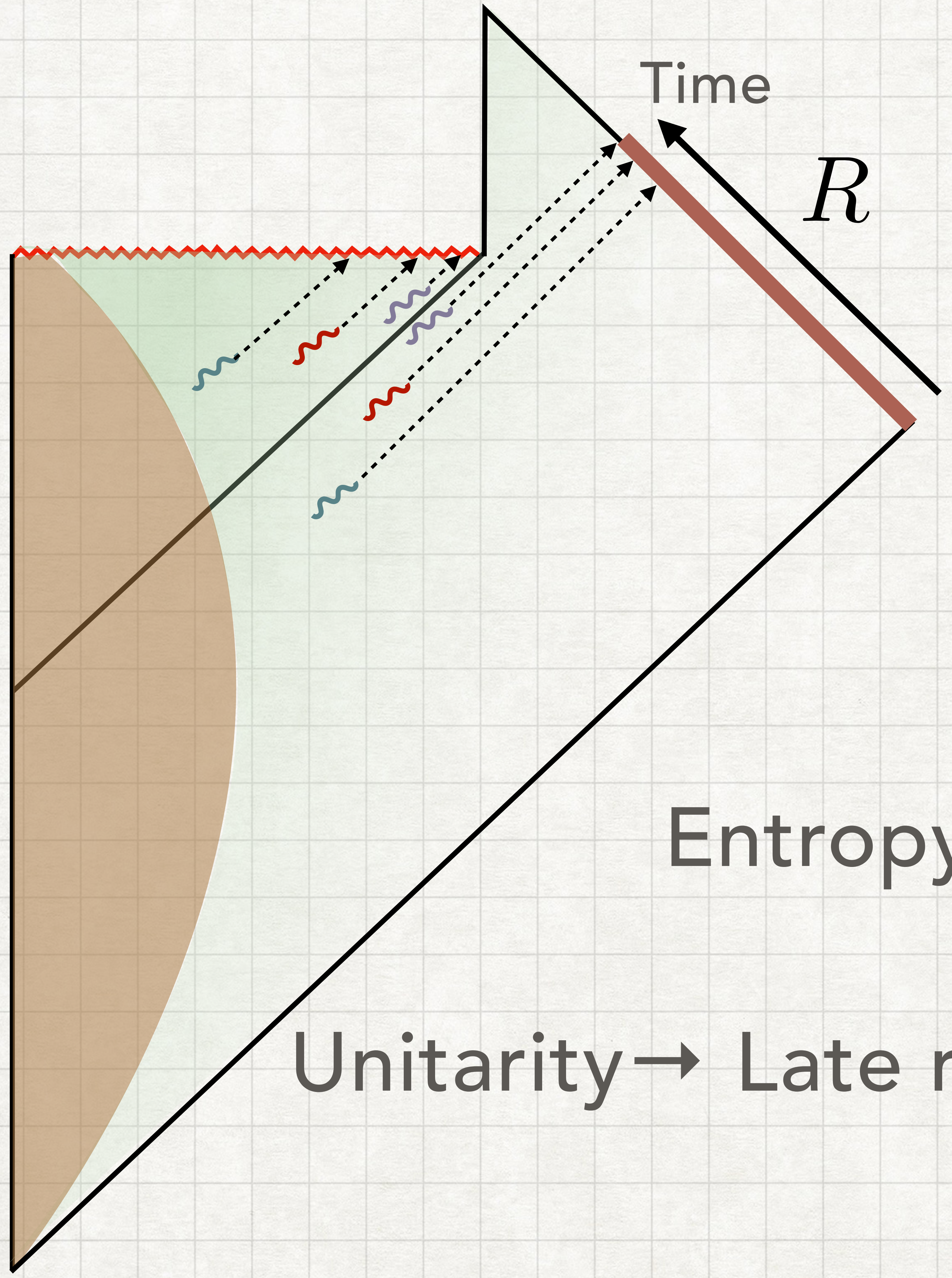
The maximum amount of information inside the BH is given by

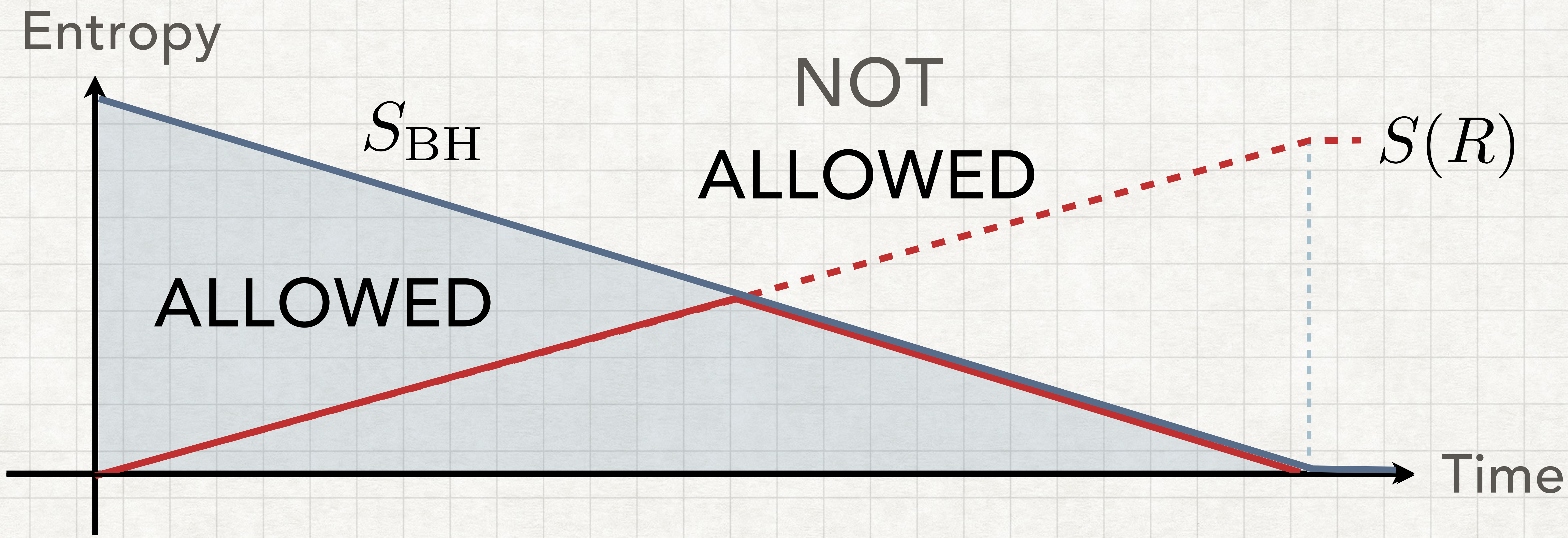
$$S_{\rm BH} = \frac{\rm Area}{4G_N}$$

At late times, the black hole does not have enough d.o.f to be entangled with Hawking radiation→ mixed state

Contradicts with unitarity!

Page Curve





Entropy of Hawing radiation should obey the Page curve

Unitarity→ Late radiation should be entangled with early radiation

Contradiction!

Equivalence principle→Radiation should be entangled with BH interior

Talk Plan

1. Brief intro of Entanglement entropy

2. Holographic principle

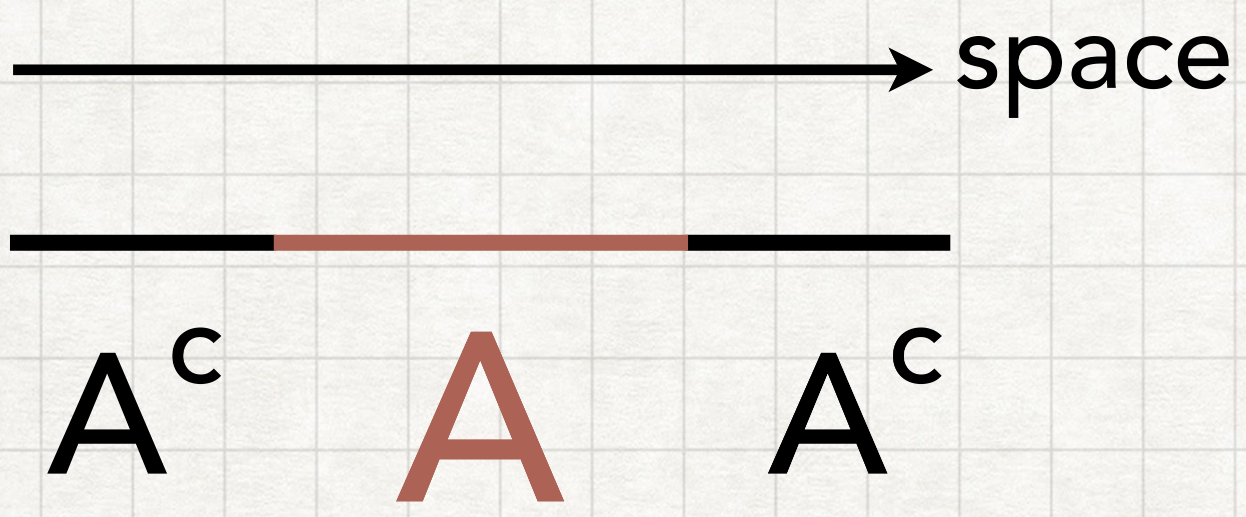
3. Holographic entanglement entropy and subregion duality

4. Recent developments in black hole information paradox

5. Gravitational replica calculation

Entanglement Entropy

Entropy of Hawking radiation can be computed as entanglement entropy



density matrix $\rho = |\Psi\rangle\langle\Psi|$

→reduced density matrix $\rho_A = \mathrm{tr}_{A^c}\rho$

only contains information inside A

How much amount of information is inside A ? →"Entanglement entropy"

$$S_A = -\mathrm{tr}\rho_A \log \rho_A$$

Total system is pure→ $S_{A \cup A^c} = 0$

Entanglement Entropy

system 1

A

system 2

A^c

Example: Thermofield double state

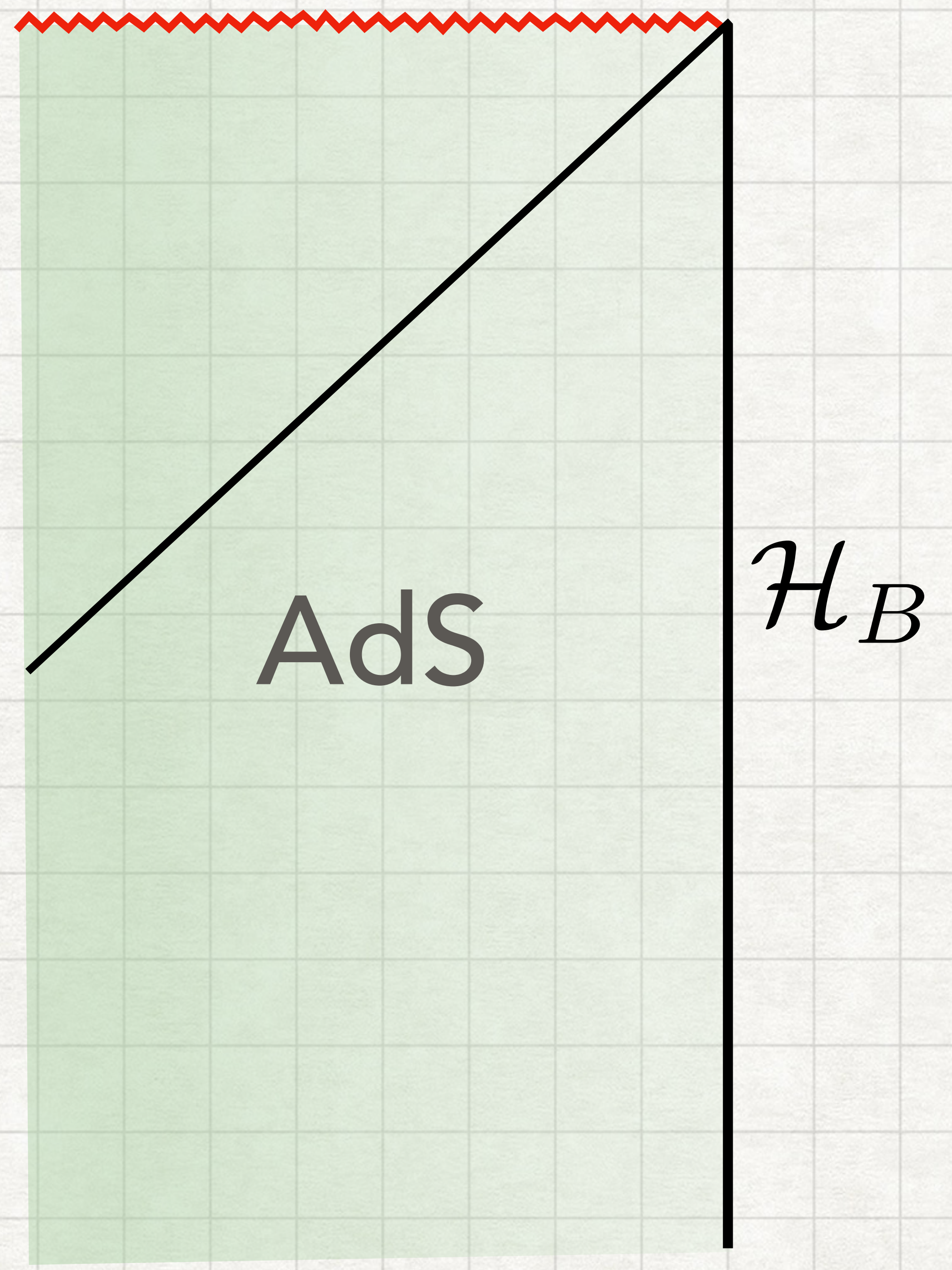
$$|\Psi_{\mathrm{TFD}}\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\beta E/2} |E\rangle_1 \otimes |E\rangle_2$$

reduced density matrix: thermal density matrix

$$\rho_A = \mathrm{tr}_A |\Psi_{\mathrm{TFD}}\rangle\langle\Psi_{\mathrm{TFD}}|$$
$$= \frac{e^{-\beta H}}{Z}$$

Entanglement entropy becomes the thermal entropy $S_A = S_{\mathrm{thermal}}$

AdS/CFT correspondence

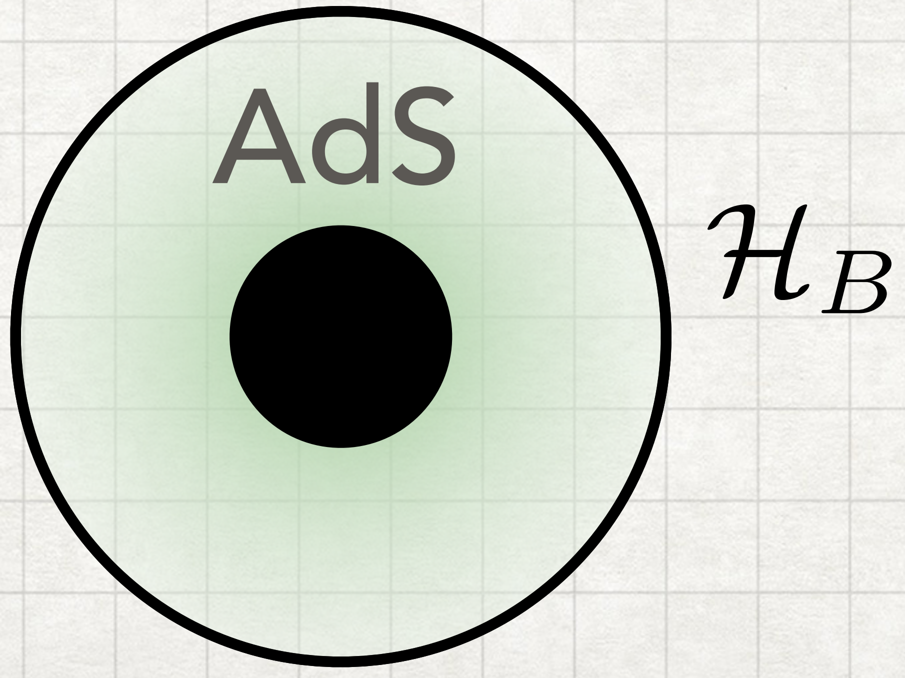


AdS/CFT correspondence:

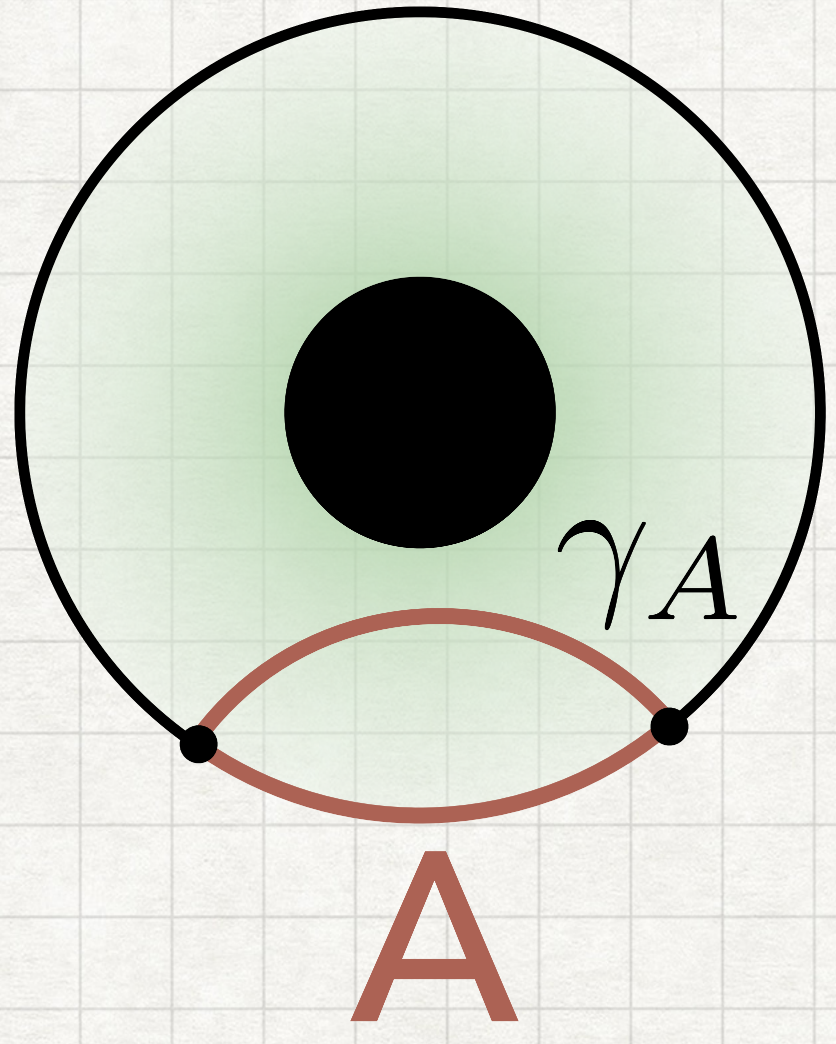
Information of quantum gravity in AdS is encoded in the boundary system (CFT) at infinity

$$\mathcal{H}^{\mathrm{QG}}_{\mathrm{Bulk}} \simeq \mathcal{H}_B$$

Boundary system is defined on $\mathbb{S}^{d-1} \times \mathbb{R}$



Holographic Entanglement Entropy

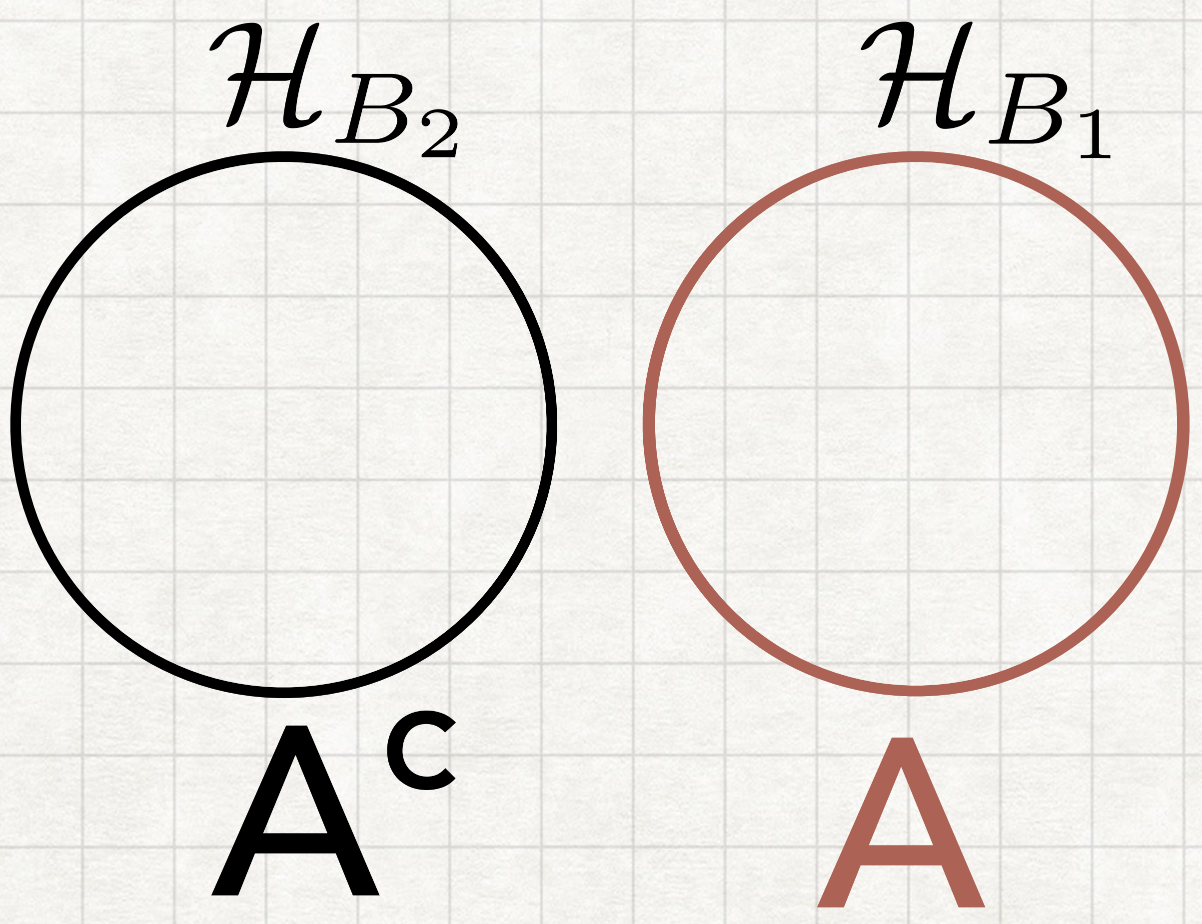


Ryu-Takayanagi formula:

$$S_A = \mathrm{ext}_{\gamma_A} \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$

Entanglement entropy can be calculated by the area of the extremal surface anchored at ∂A

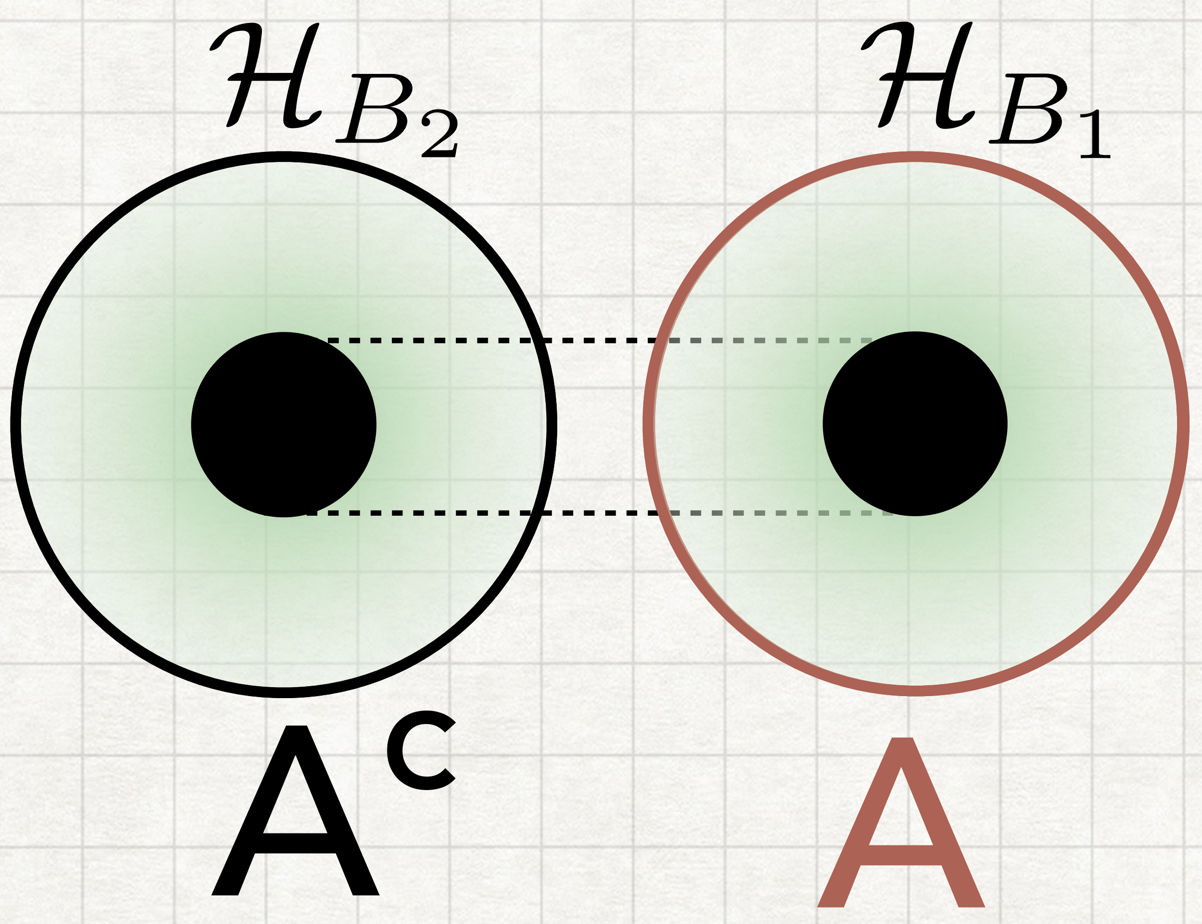
Holographic Entanglement Entropy



Example: Thermofield double state

$$|\Psi_{\mathrm{TFD}}\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\beta E/2} |E\rangle_1 \otimes |E\rangle_2$$

Holographic Entanglement Entropy



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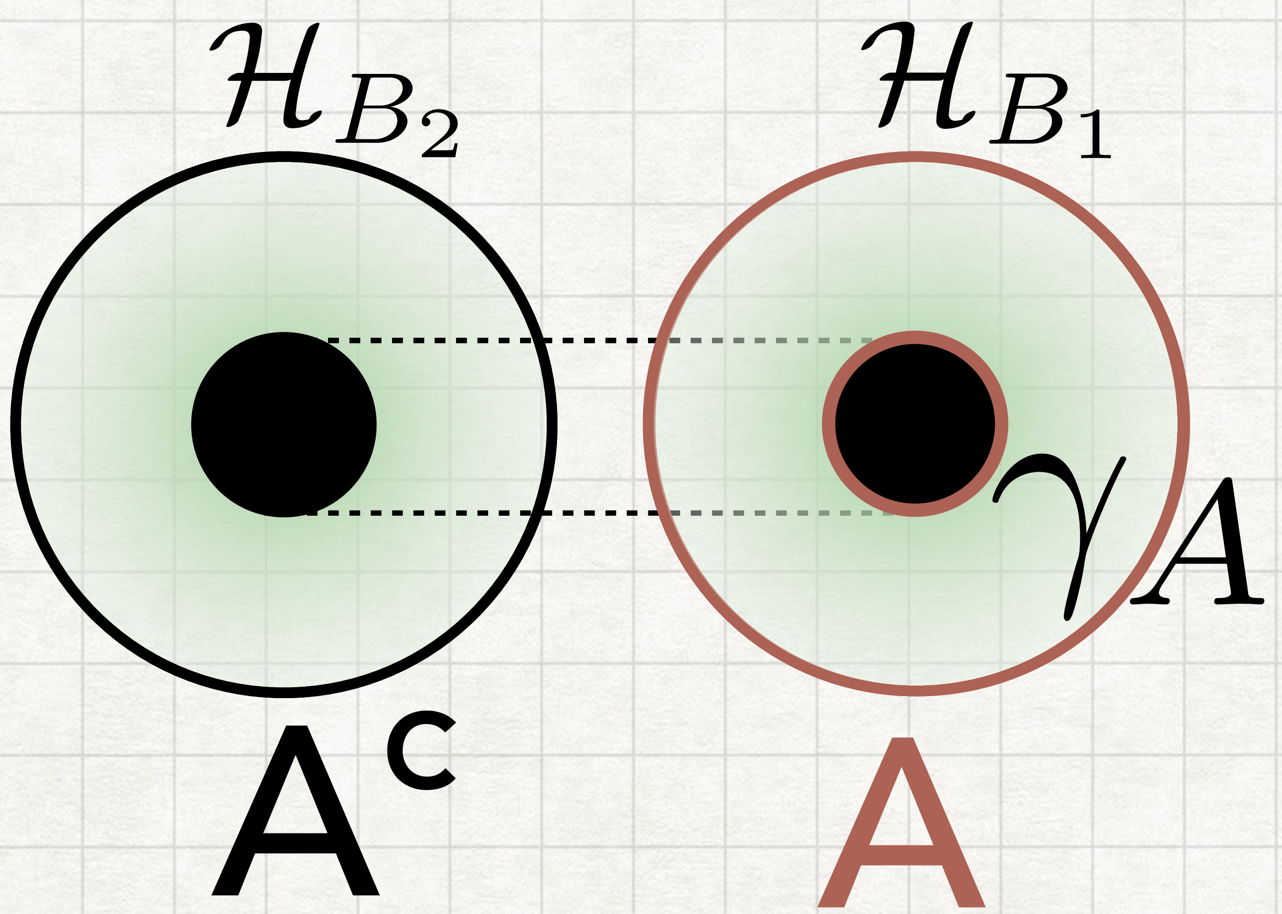

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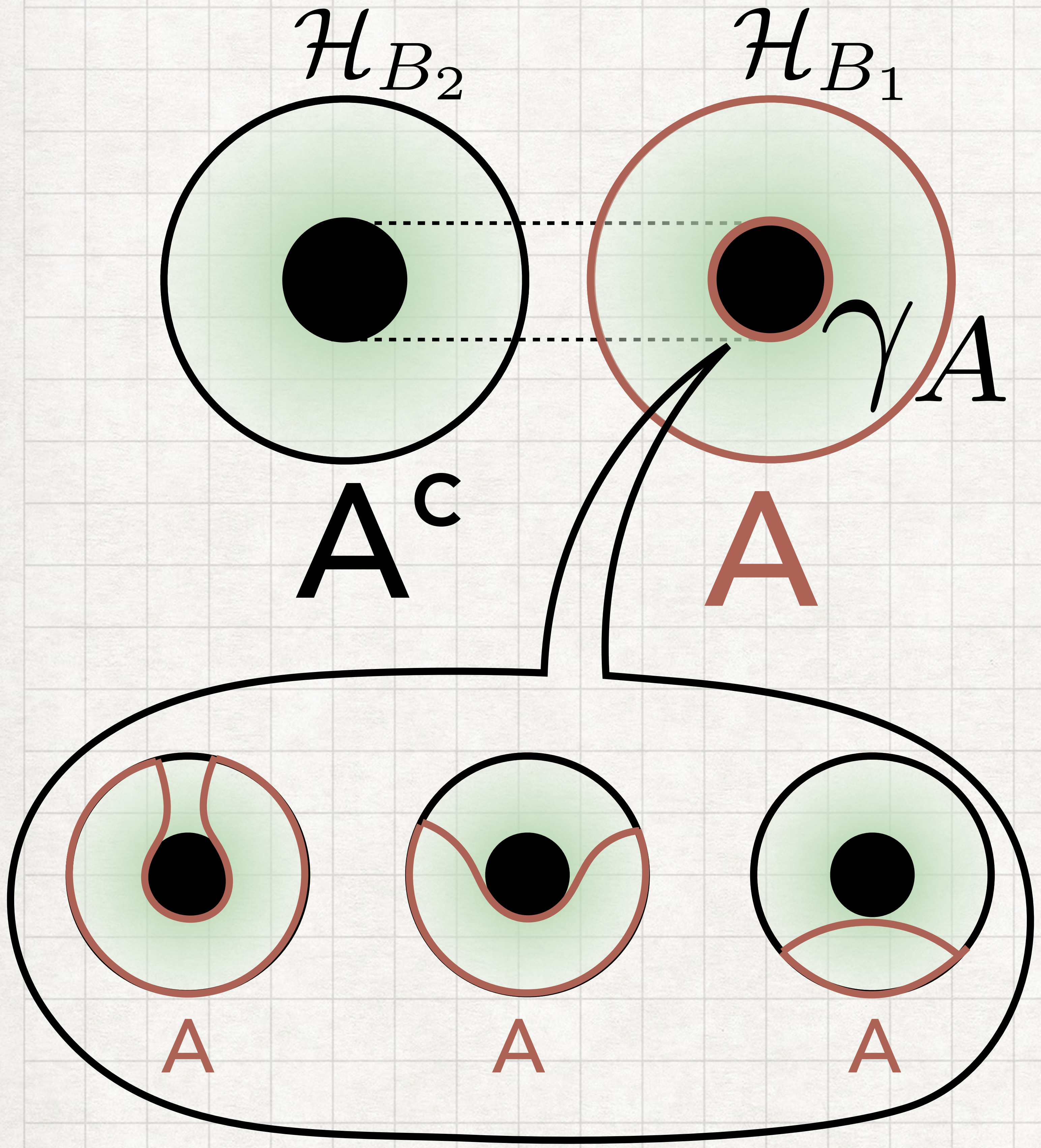
$$|\Psi_{\rm TFD}\rangle = \frac{1}{\sqrt{Z}} \sum_E e^{-\beta E/2} |E\rangle_1 \otimes |E\rangle_2$$

γ_A : surface enclosing the BH horizon

Entanglement entropy= Black hole entropy

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N} = S_{\rm BH}$$

Holographic Entanglement Entropy



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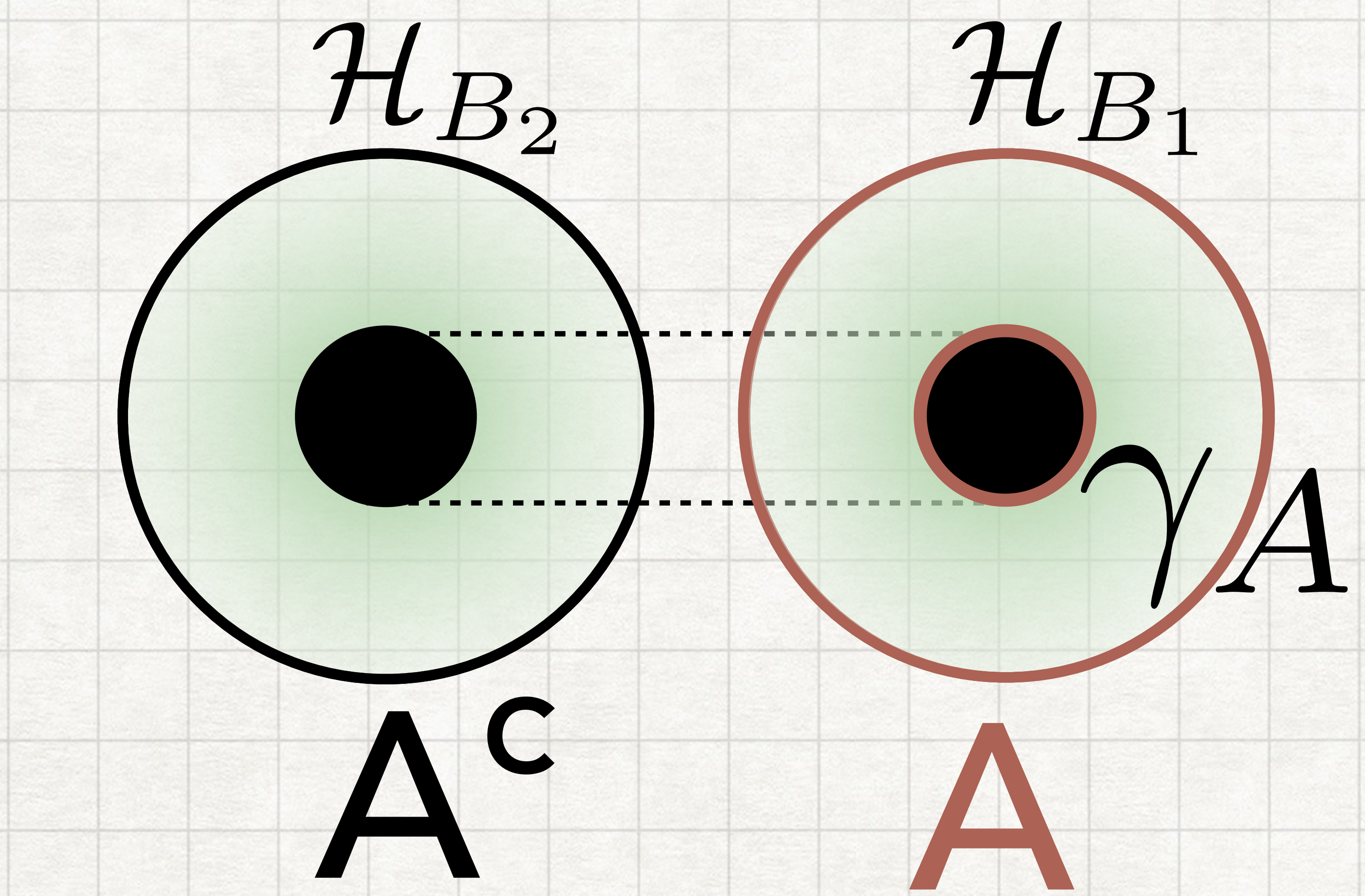

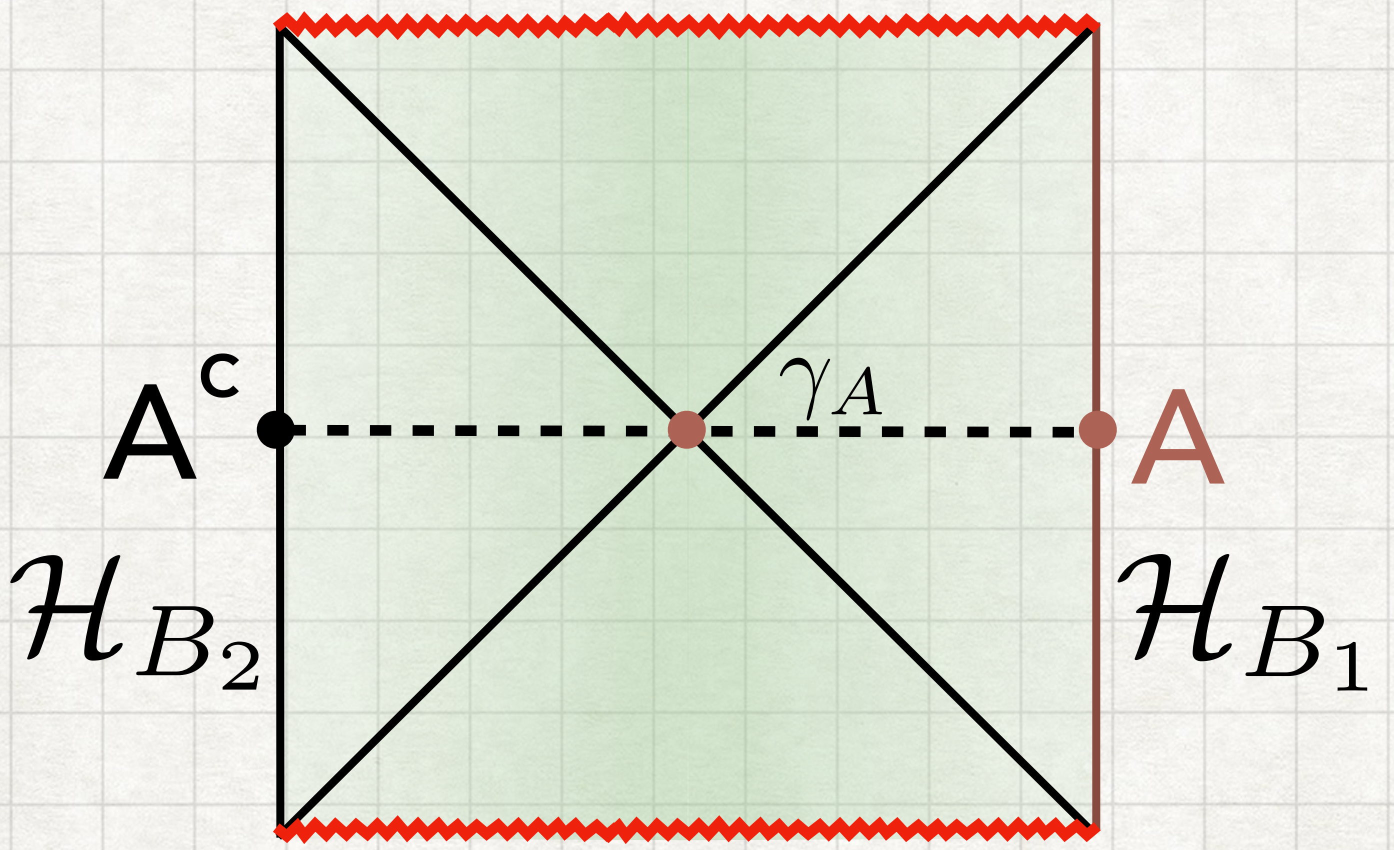

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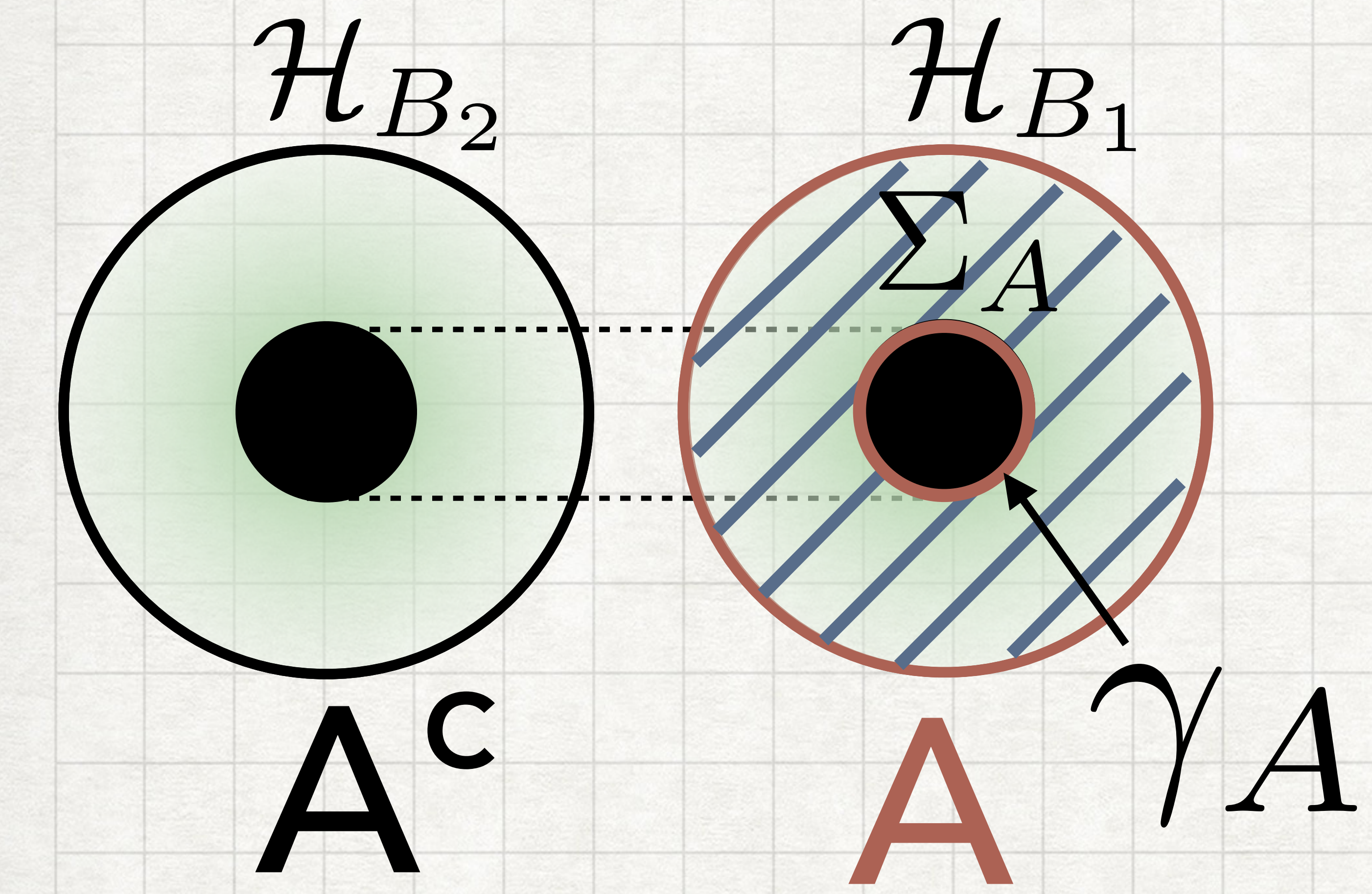
γ_A : surface enclosing the BH horizon

Entanglement entropy= Black hole entropy

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N} = S_{\mathrm{BH}}$$

Subregion duality

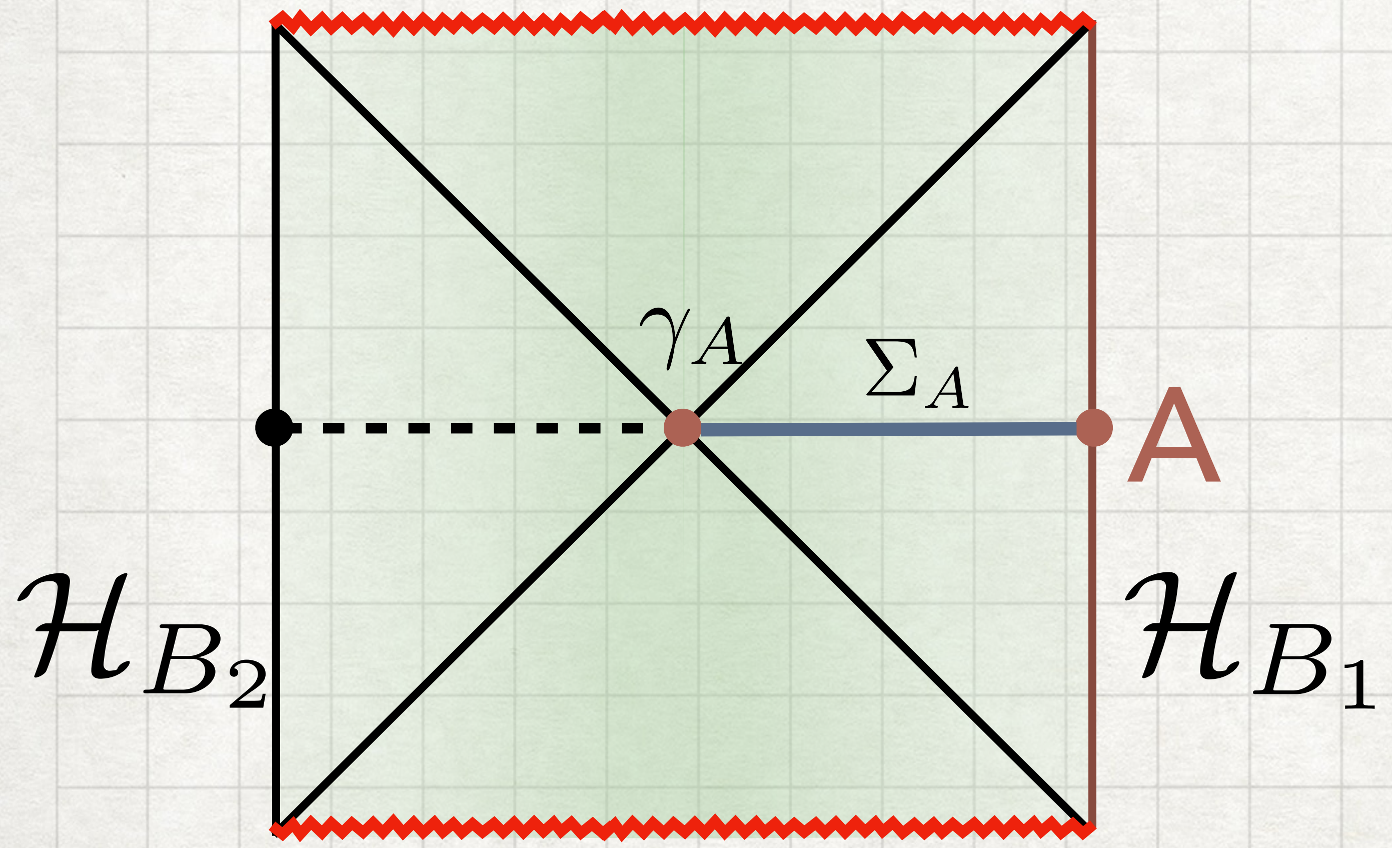
Which part of the bulk is encoded in which part of the boundary system?



Subregion duality

A part of the bdy system: A encodes the information of the bulk region called *entanglement wedge of A*

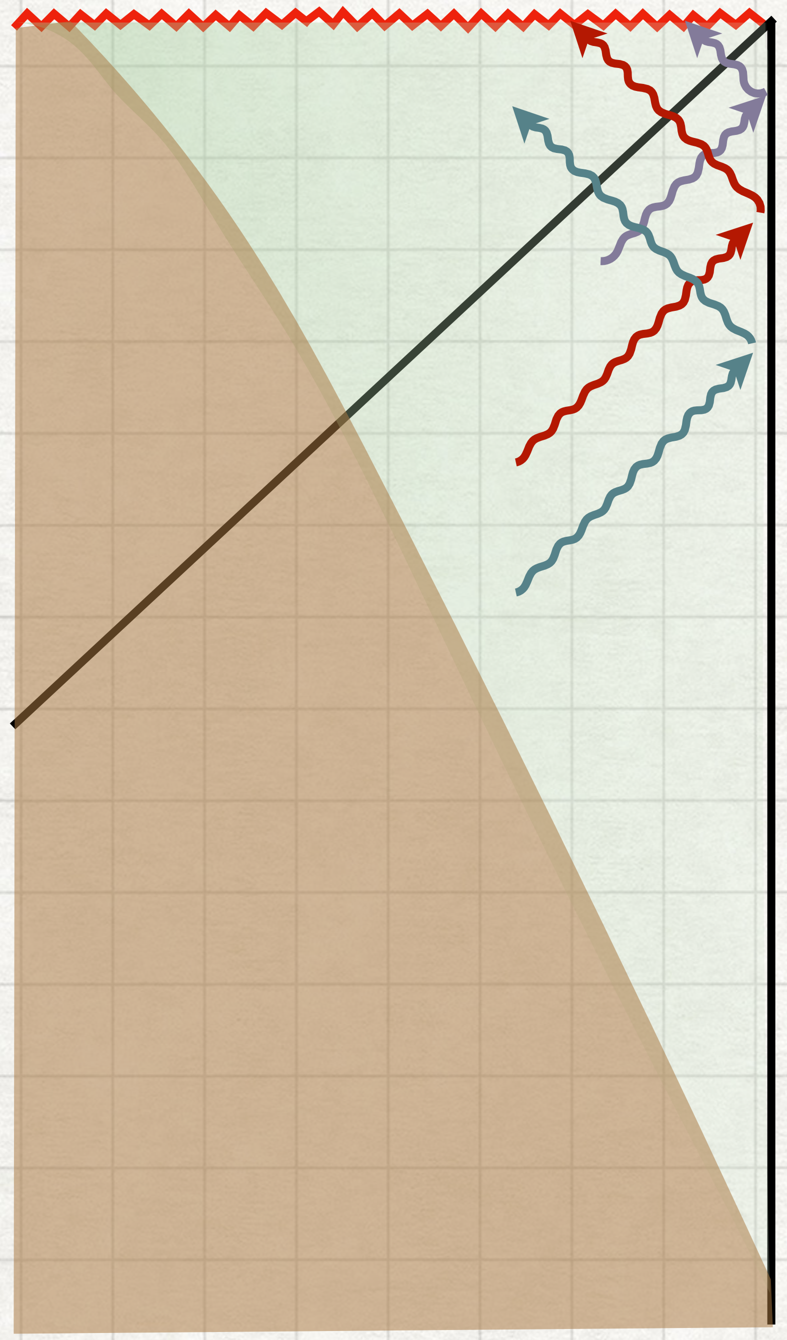
Entanglement wedge of A: Σ_A
bulk region enclosed by A and γ_A



TFD case: A encodes the information outside BH

Recent developments

Penington, Almheiri-Engelhardt-Marolf-Maxfield (AEMM) '19



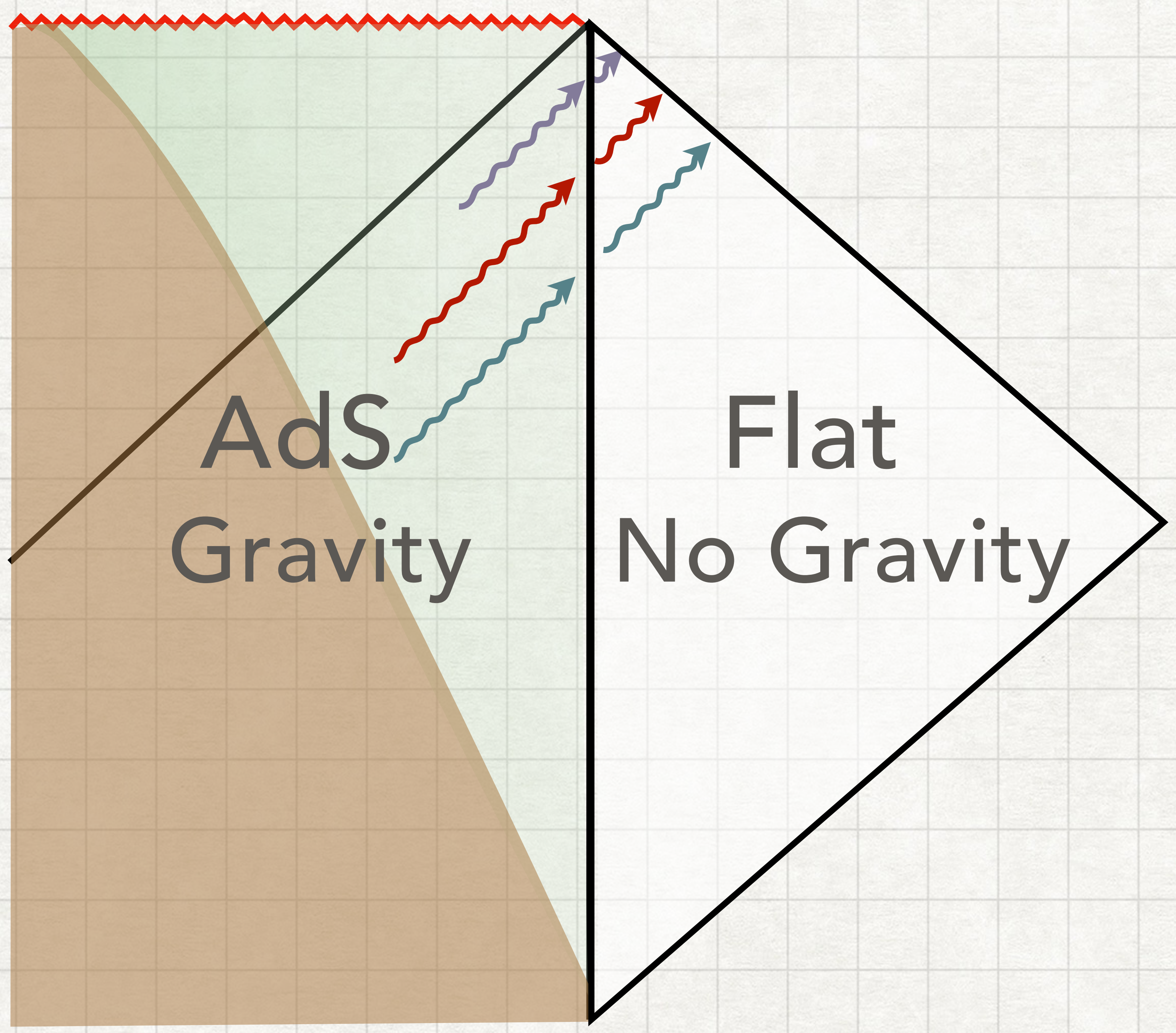
AdS Black hole formed by the gravitational collapse:

Hawking radiation is reflected at bdy and absorbed by BH

→ never evaporate!

Recent developments

Penington, Almheiri-Engelhardt-Marolf-Maxfield (AEMM) '19



Attaching a flat space to AdS:

Hawking radiation escapes into the flat region

→an evaporating AdS black hole!

Recent developments

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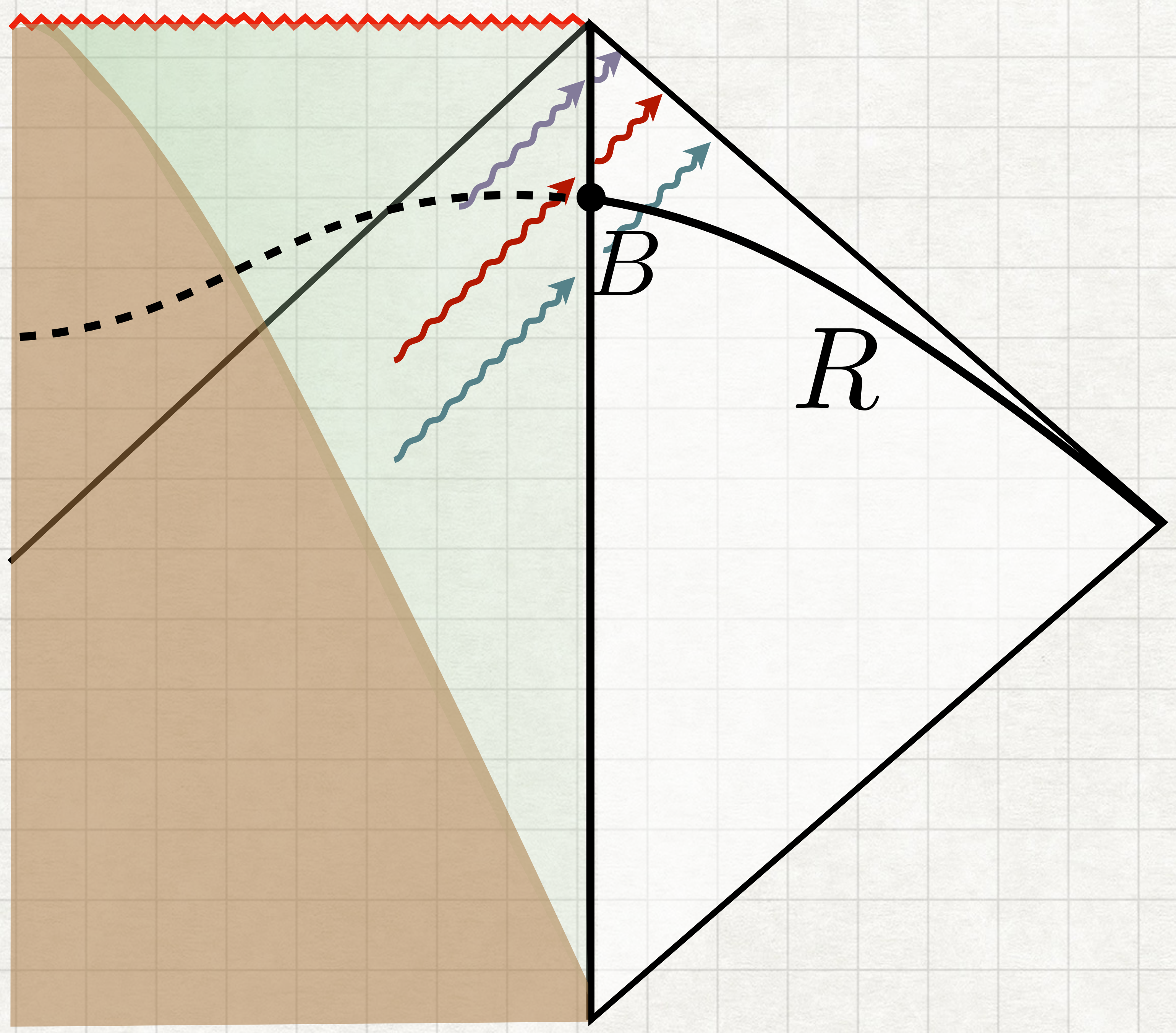
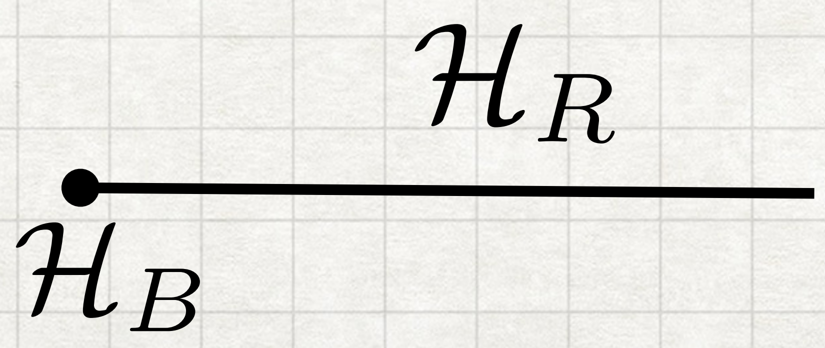


The bdy system



Attaching a flat space to AdS:
Hawking radiation escapes into the flat region

→an evaporating AdS black hole!

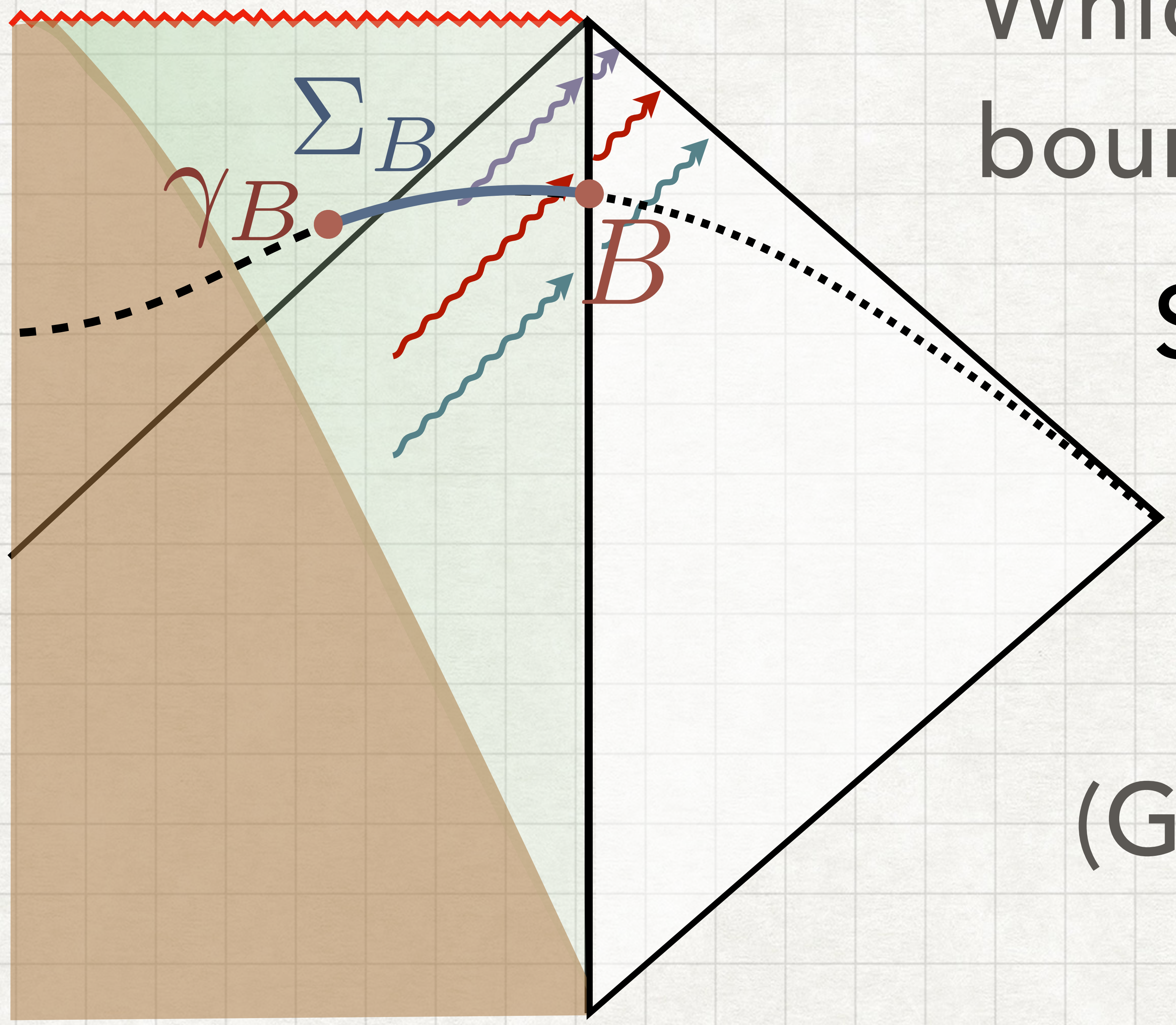
The bdy system can be decomposed:

$$\mathcal{H}_B \otimes \mathcal{H}_R$$

Region R can collects the Hawking radiation
→ $\mathcal{H}_R$ plays a role of d.o.f. of Hawking radiation

Recent developments

Penington, Almheiri-Engelhardt-Marolf-Maxfield (AEMM) ‘19



The bdy system

$\mathcal{H}_R$

$\mathcal{H}_B$

Which part of the bulk is encoded in a part of the boundary system B?

Subregion duality

A part of the bdy system: B encodes the information of the bulk region called *entanglement wedge of B*

(Generalized Ryu-Takayanagi formula)

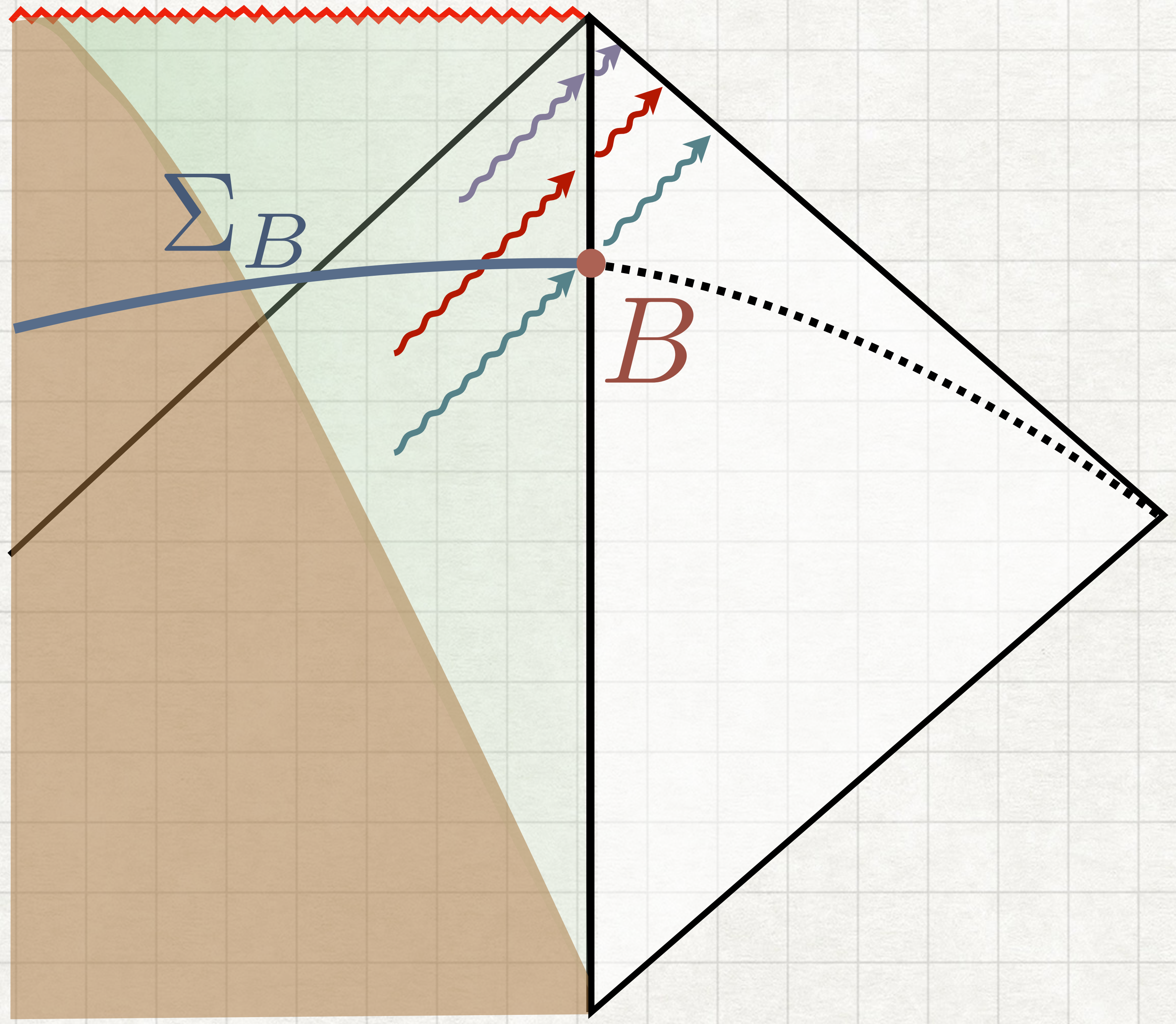
$$S_B = \min_{\gamma_B} \operatorname{ext} \left[\frac{\text{Area}(\gamma_B)}{4G_N} + \underline{S_{\text{matter}}(\Sigma_B)} \right]$$

Bulk matter entanglement entropy

γ_B: quantum extremal surface (QES)

Recent developments

Penington, Almheiri-Engelhardt-Marolf-Maxfield (AEMM) '19



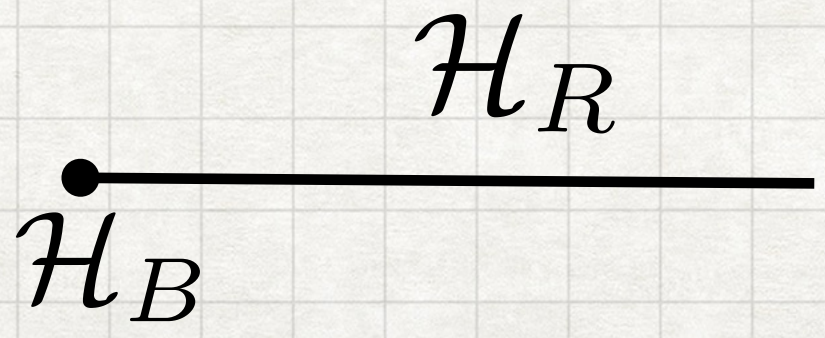
(Generalized Ryu-Takayanagi formula)

$$S_B = \min_{\gamma_B} \operatorname{ext} \left[\frac{\mathrm{Area}(\gamma_B)}{4G_N} + S_{\mathrm{matter}}(\Sigma_B) \right]$$

At early times,

No non-trivial QES→
$\mathcal{H}_B$ describes the entire bulk region

The bdy system



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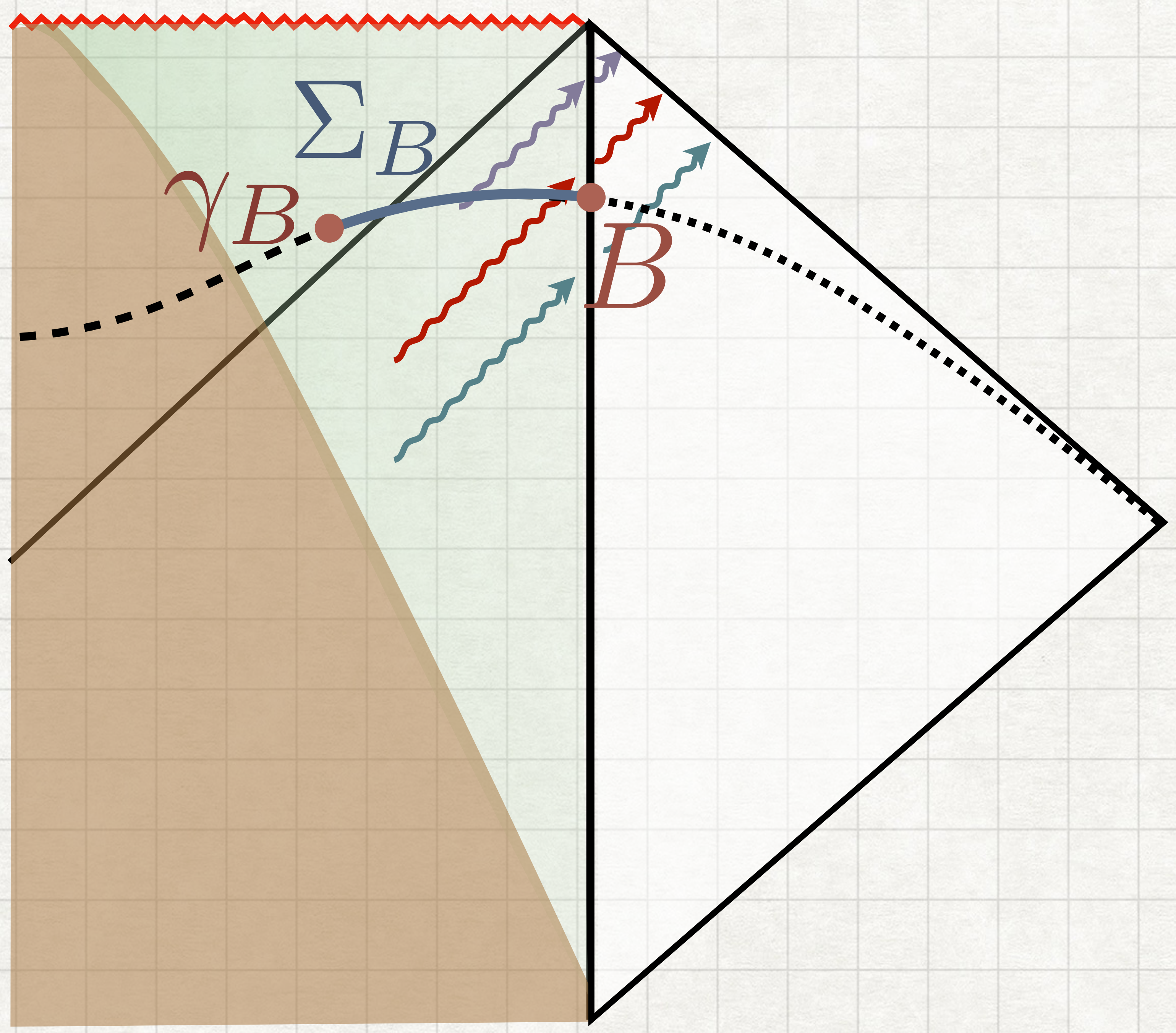


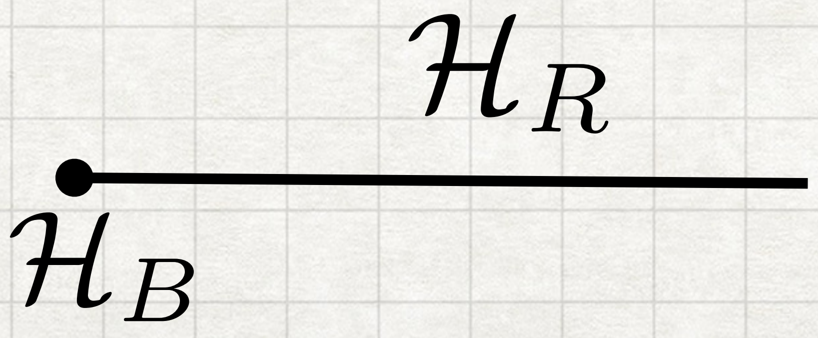
(Generalized Ryu-Takayanagi formula)

$$S_B = \min_{\gamma_B} \text{ext} \left[\frac{\text{Area}(\gamma_B)}{4G_N} + S_{\text{matter}}(\Sigma_B) \right]$$

At Late times when the paradox arises:

Non-trivial QES exists near the horizon,
a large part of the BH interior does not belong to the EW of B!

The bdy system



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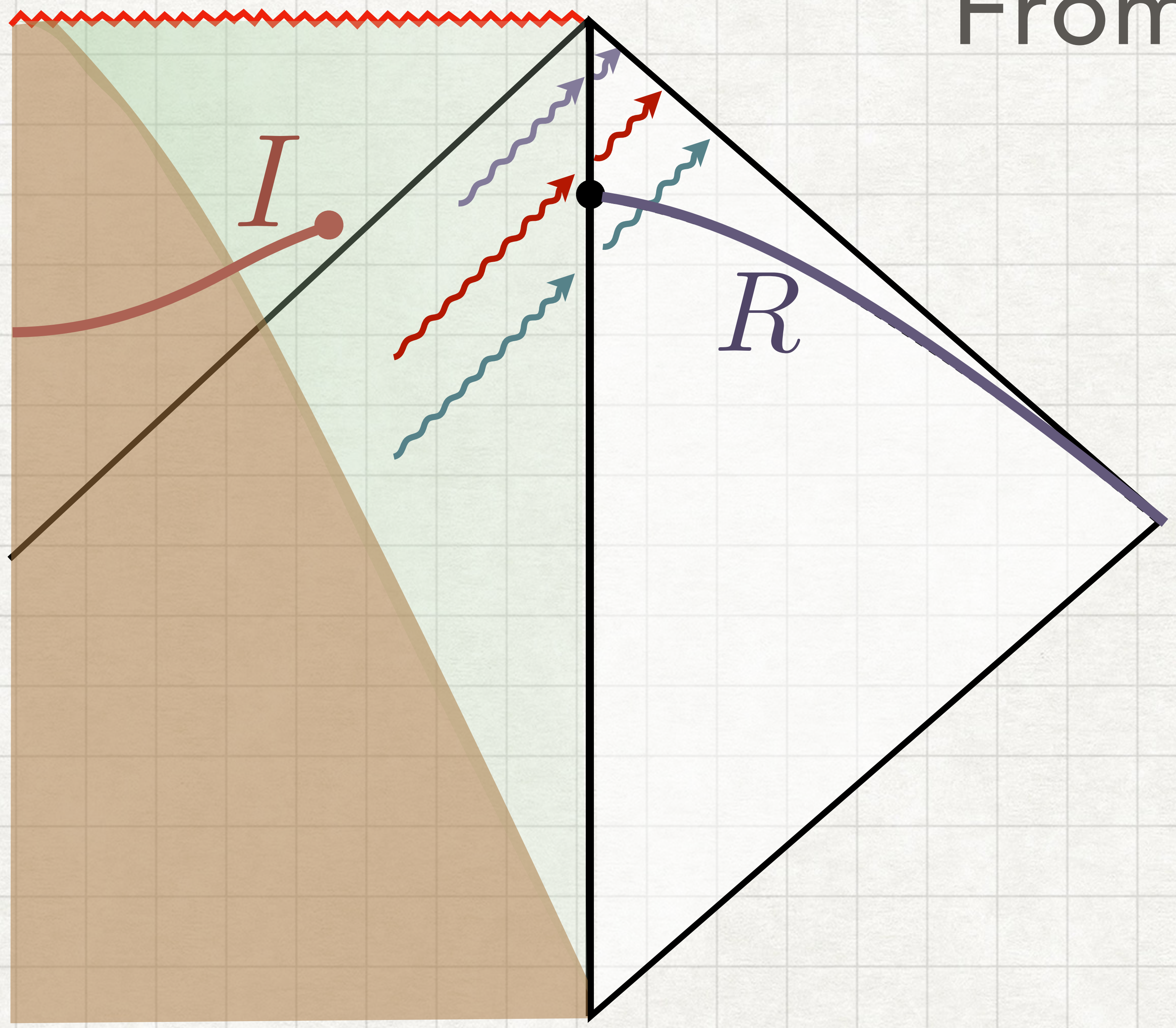


From the consistency, the BH interior is encoded in $\mathcal{H}_R$

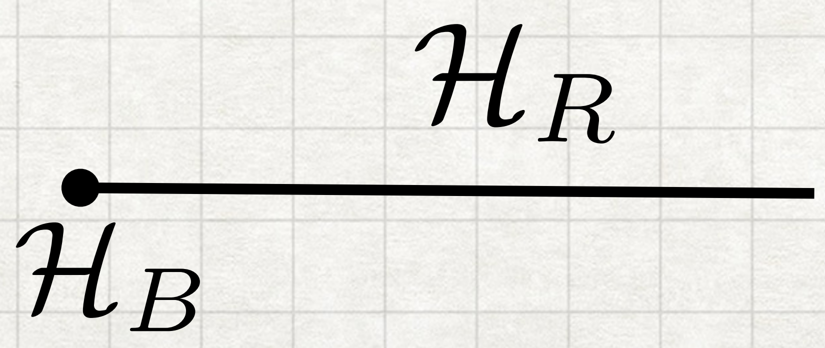
The entanglement wedge of R is $R \cup \underline{I}$!

"Island"

Island formula :

$$S_R = \min_I \operatorname{ext} \left[\frac{\text{Area}(\partial I)}{4G_N} + S_{\text{matter}}(R \cup I) \right]$$

The bdy system



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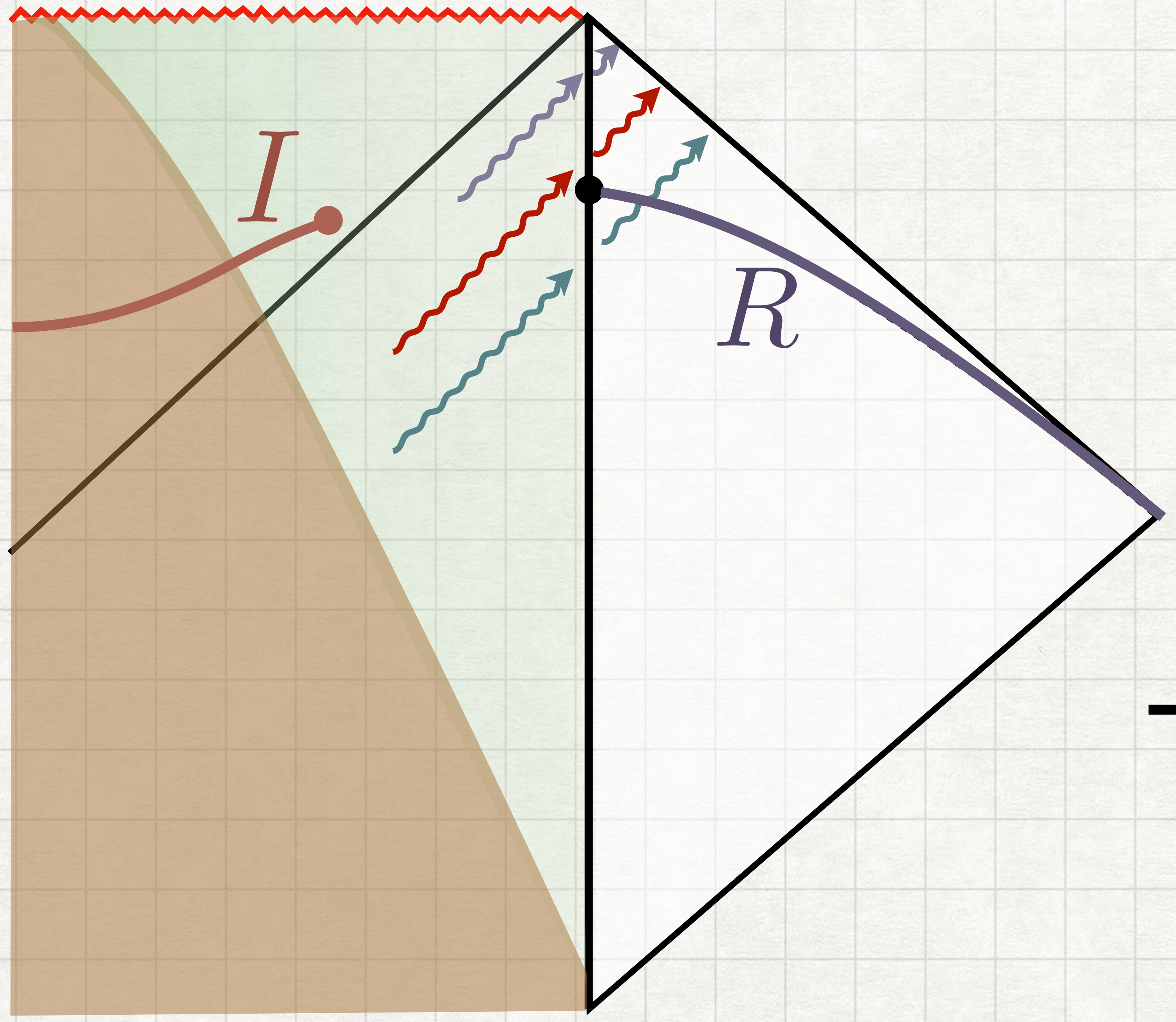
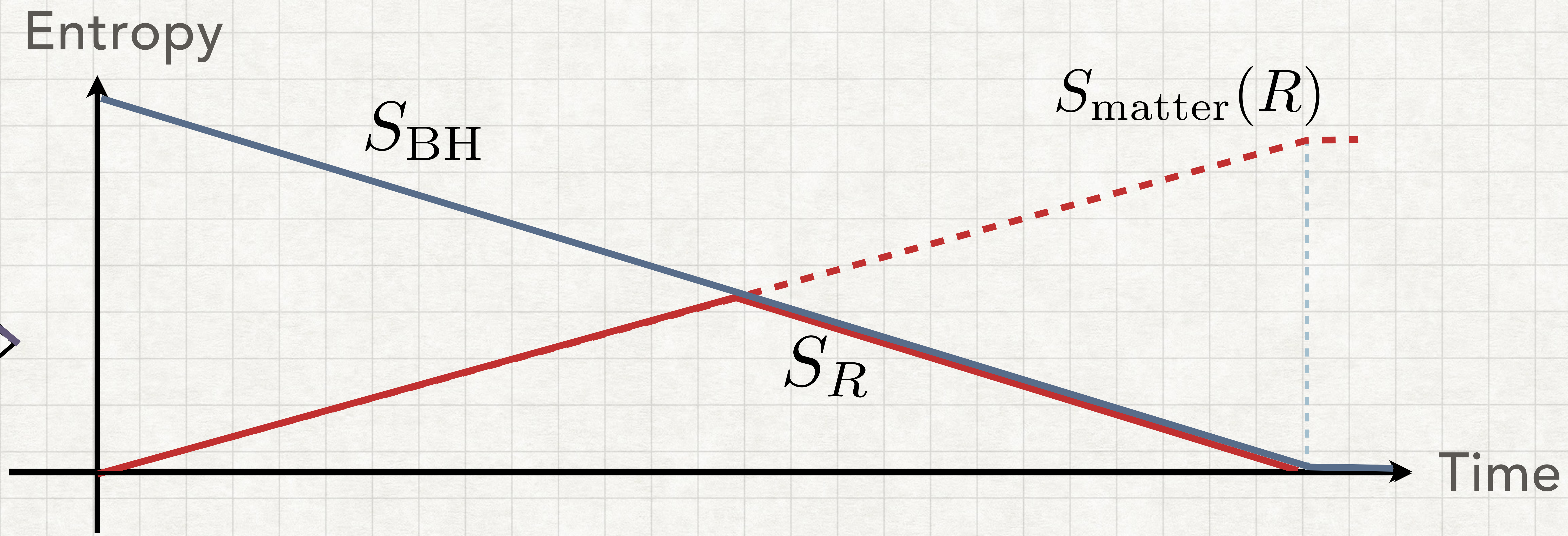




$$S_R = \min_I \operatorname{ext} \left[\frac{\mathrm{Area}(\partial I)}{4G_N} + S_{\mathrm{matter}}(R \cup I) \right]$$

At Late times, the island becomes a part of Hawking radiation, which leads to the unitary Page curve

Replica calculation in a gravitational system

The goal: perform the replica calculation for the entropy of Hawking radiation and derive the island formula

KG-Hartman-Tajdini (see also Almheiri-Hartman-Maldacena-Shagoulian-Tajdini)

Replica trick:

$$S_R = -\mathrm{tr}_R \hat{\rho}_R \log \hat{\rho}_R$$
$$= (1 - n\partial_n) Z_n \big|_{n=1}$$

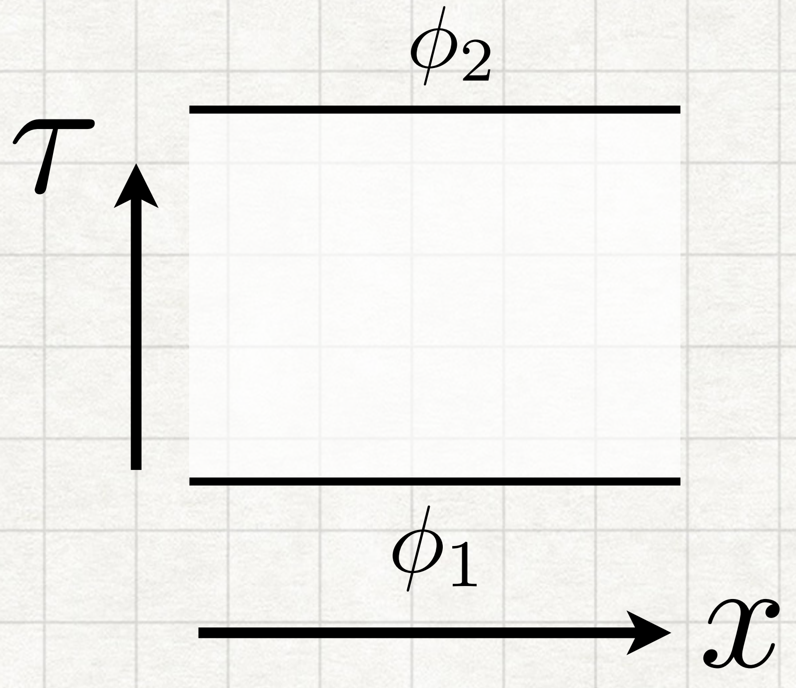
$$\hat{\rho}_R = \frac{\rho_R}{\mathrm{tr}\rho_R}$$

The replica calculation of an entanglement entropy amounts to computing the replica partition function: $Z_n = \mathrm{tr}\rho_R^n$

↑can be computed using the path-integral

Path-integral in QFT

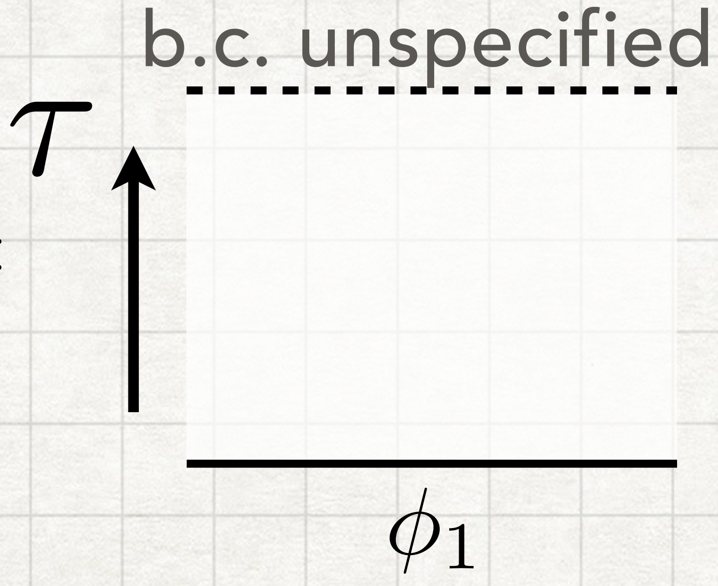
$$\langle\phi_2(x)|e^{-\tau H}|\phi_2(x)\rangle = \int_{\phi(0)=\phi_1}^{\phi(\tau)=\phi_2} \mathcal{D}\phi\, e^{-I_E[\phi]} =$$



“number”

Path-integral with one open cut defines a quantum state

$$|\Psi\rangle = e^{-\tau H}|\phi_1\rangle =$$



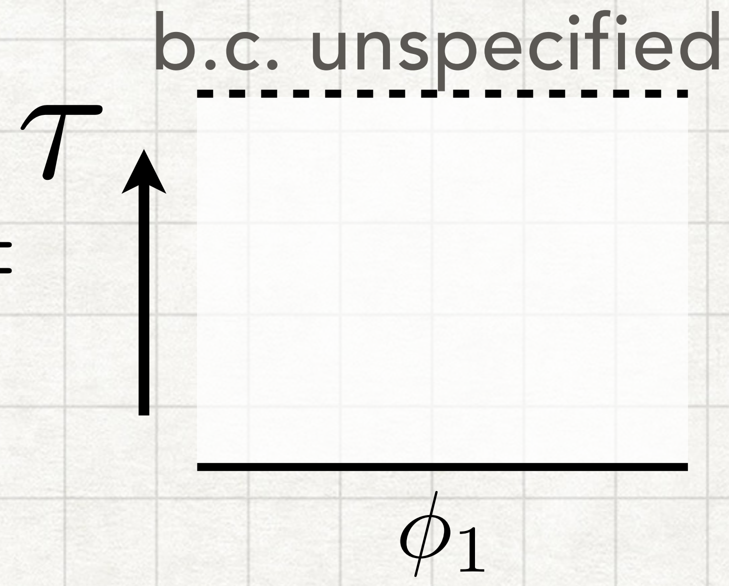
“state”

$$\Psi[\phi_2] = \langle\phi_1|\Psi\rangle = \langle\phi_1|e^{-\tau H}|\phi_1\rangle$$

Wave functional

Path-integral in QFT

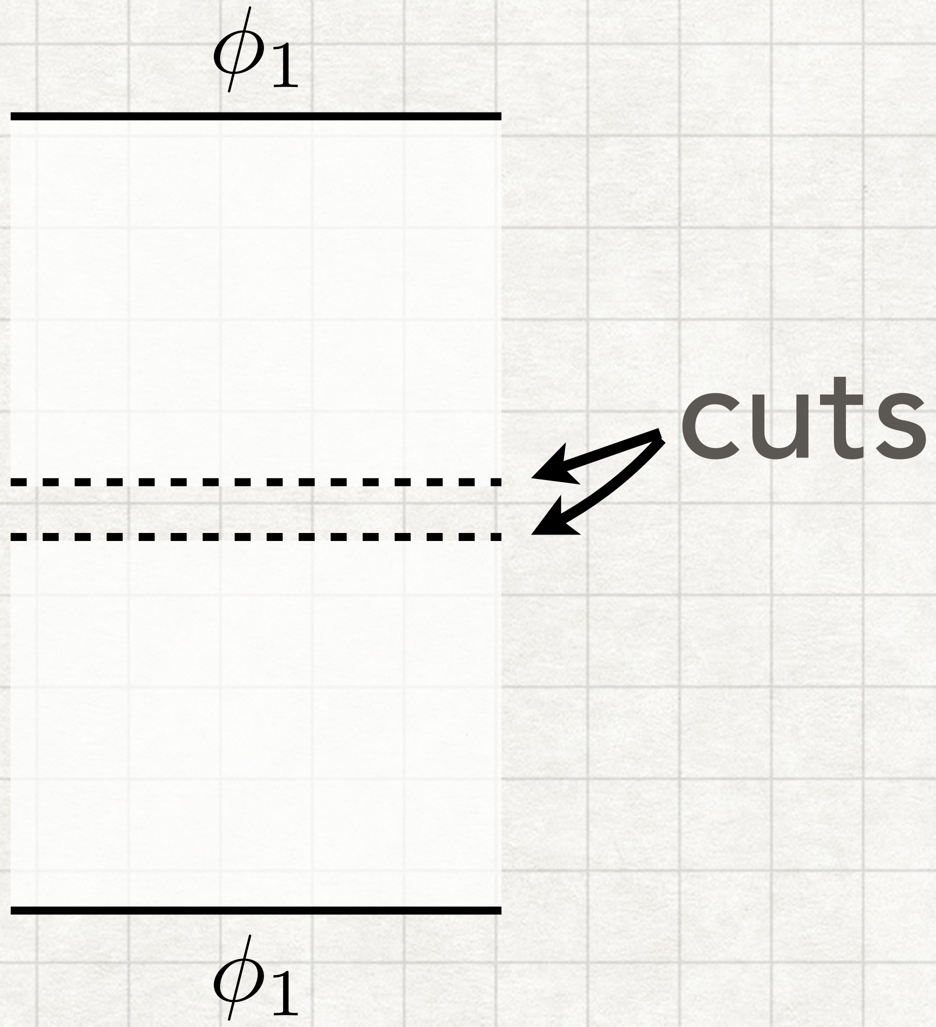
$$|\Psi\rangle = e^{-\tau H}|\phi_1\rangle =$$



"state"

Path-integral with two open cuts defines an operator

density matrix: $\rho = |\Psi\rangle\langle\Psi| =$

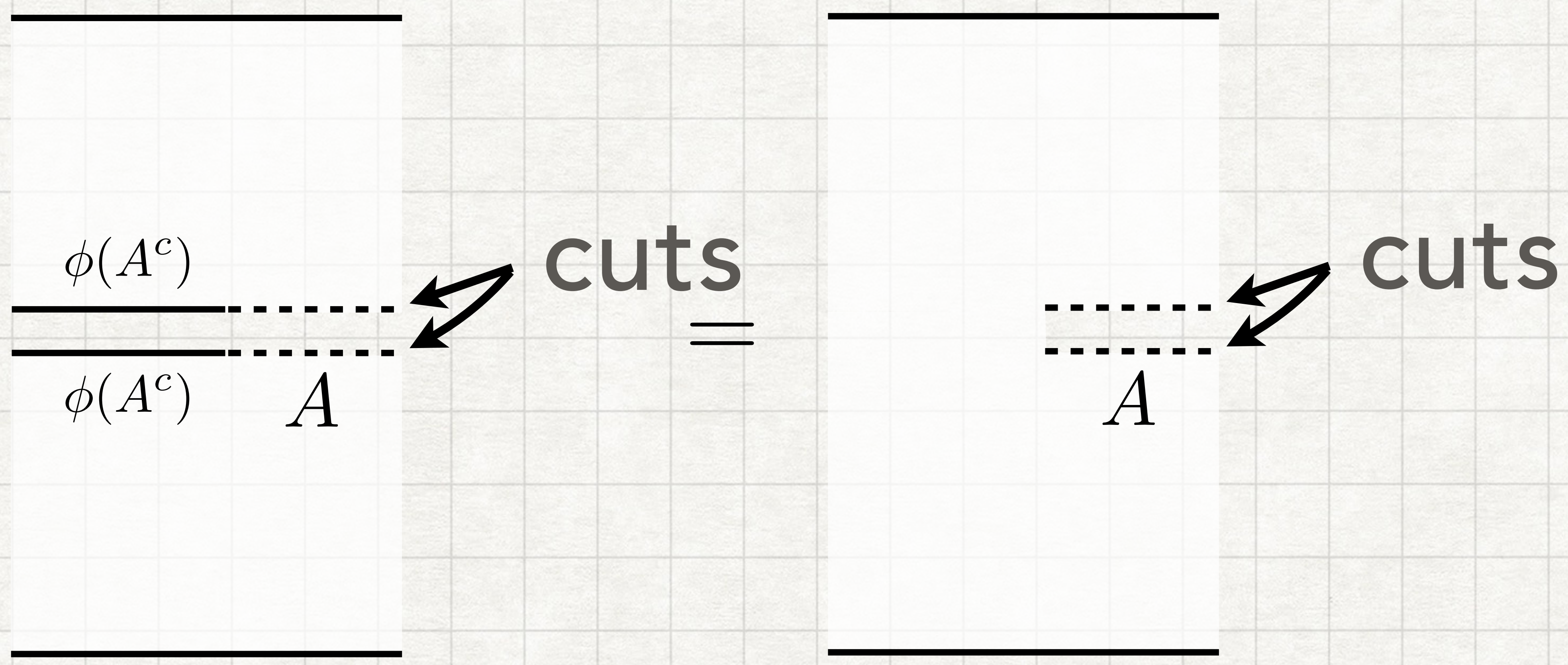


"operator"

Path-integral in QFT

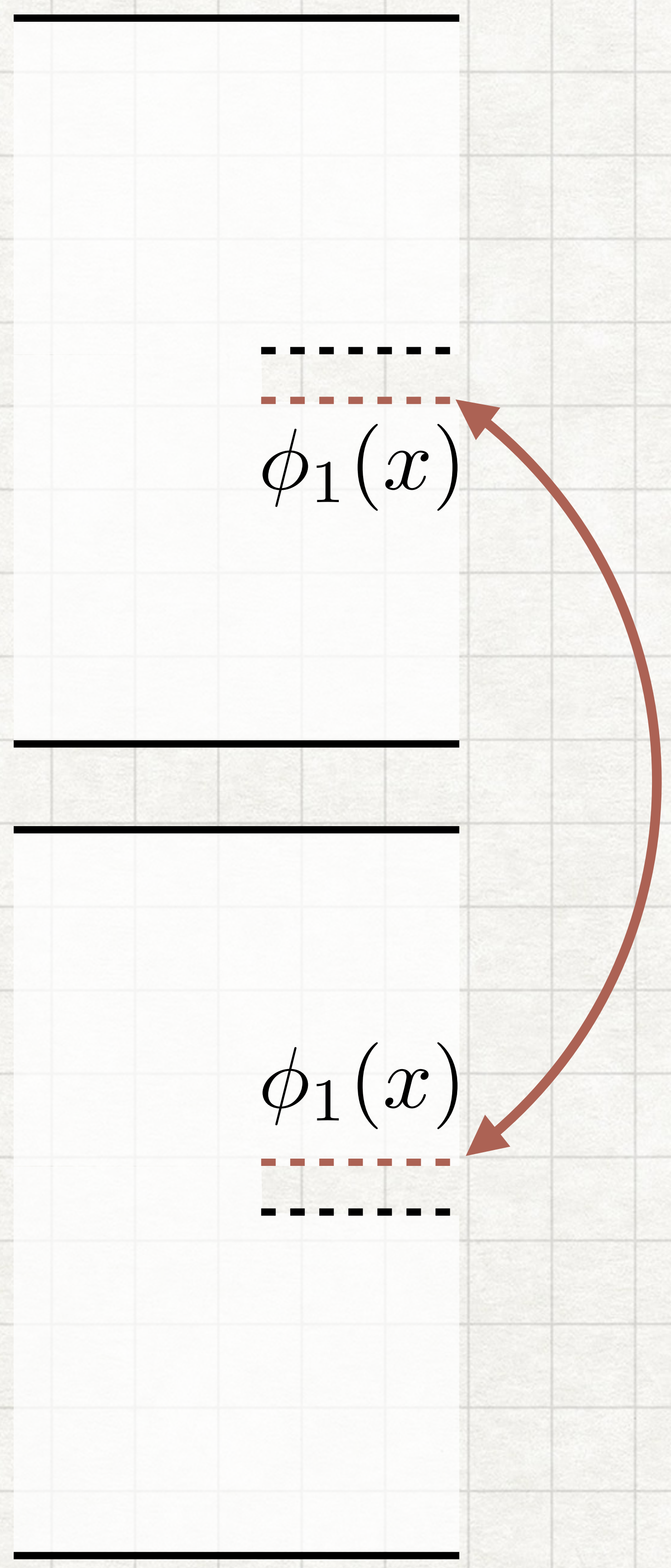
reduced density matrix:

$$\rho_A = \mathrm{tr}_{A^c}\rho = \int D\phi(A^c)$$



Path-integral in QFT

$$\rho_A^2 = \int D\phi_1(x)\ \rho_A|\phi_1(x)\rangle\langle\phi_1(x)|\rho_A = \int D\phi_1(x)$$

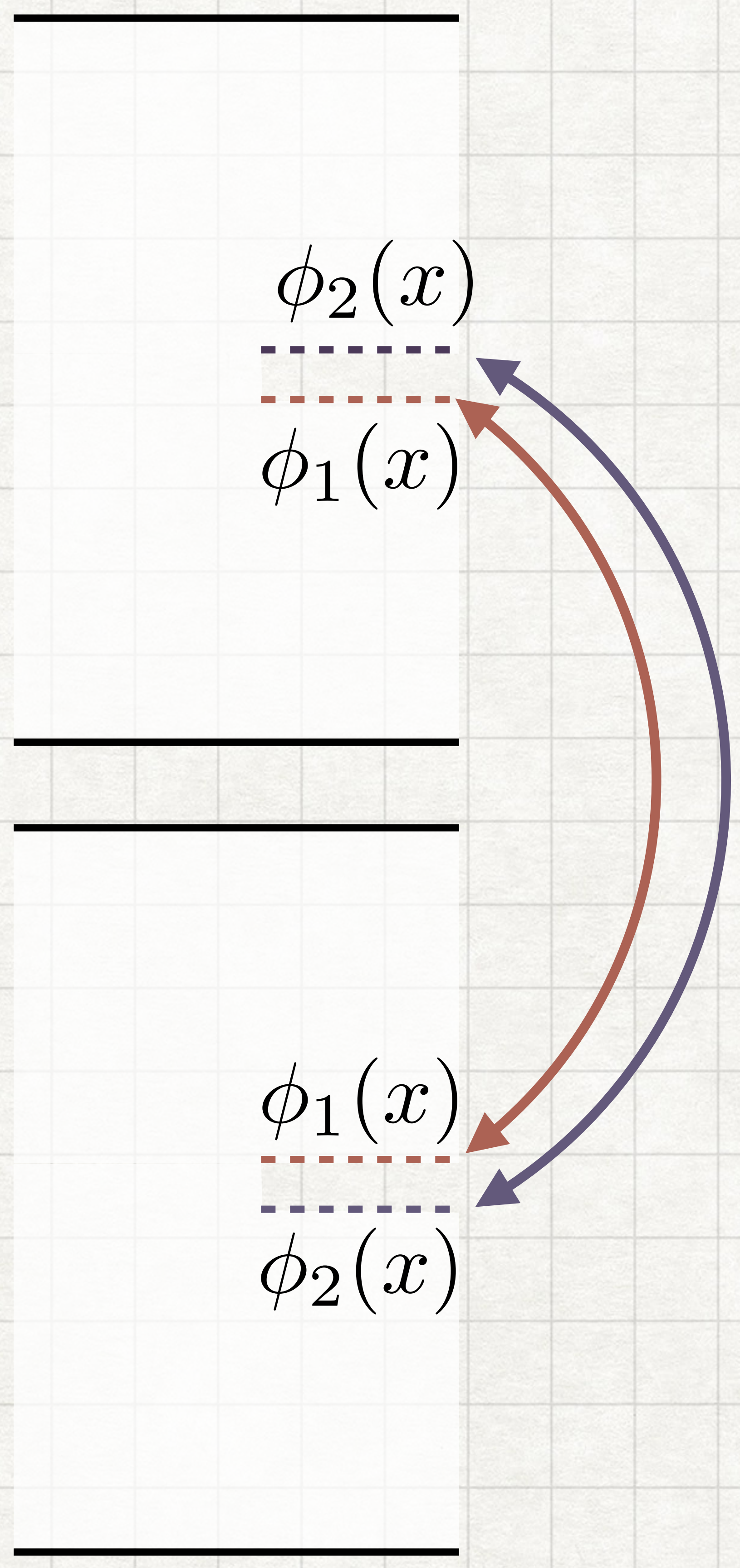


Insert "1"

$$1 = \int D\phi_1(x)|\phi_1(x)\rangle\langle\phi_1(x)|$$

Path-integral in QFT

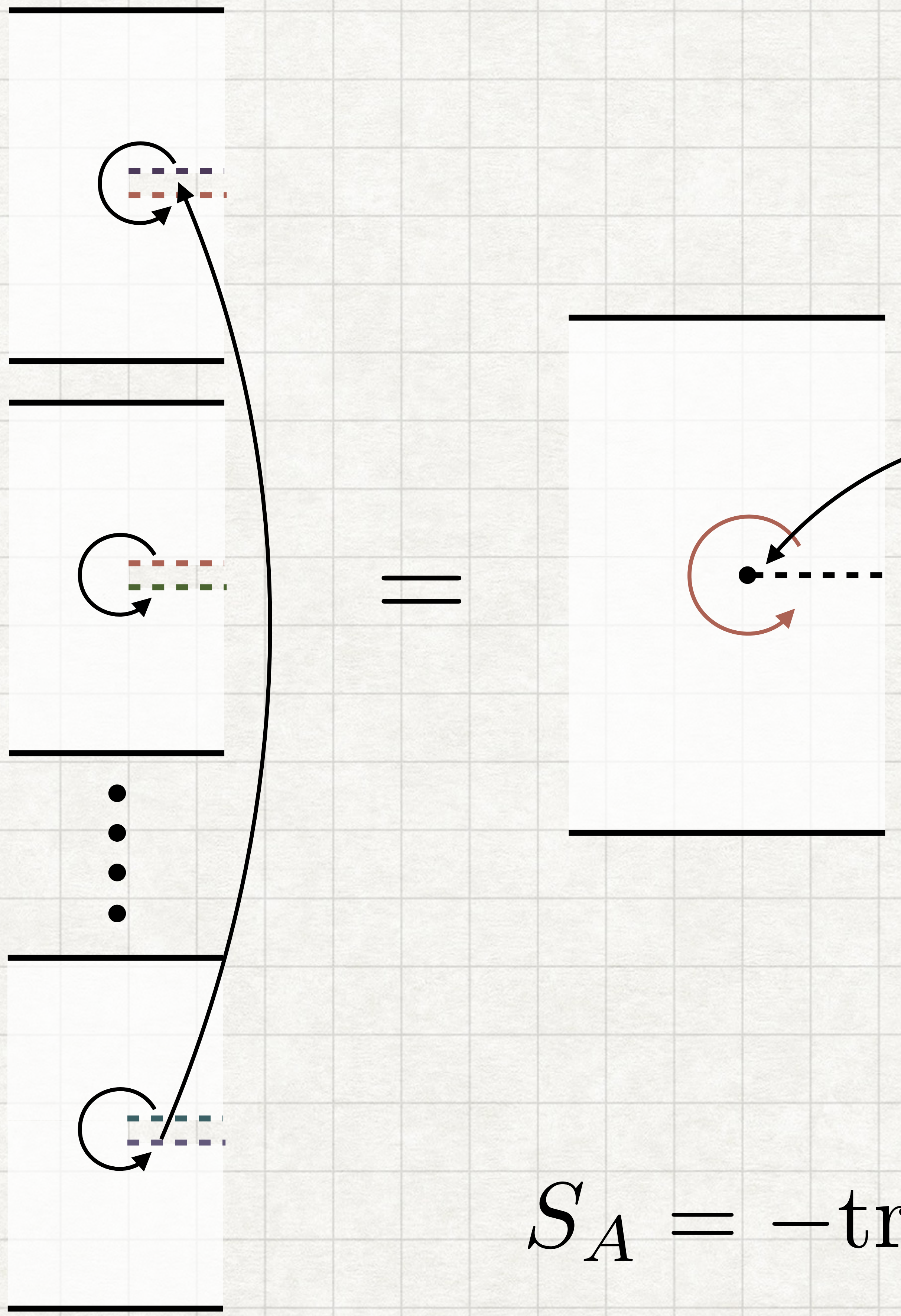
$$\mathrm{tr}\rho_A^2 = \int D\phi_2(x)\ \langle\phi_2(x)|\rho_A^2|\phi_2(x)\rangle = \int D\phi_2(x)$$



Path-integral in QFT

$Z_n = \mathrm{tr}\rho_A^n$ can also be computed by n-copies of the original geometry

$$Z_n = \mathrm{tr}\rho_A^n =$$



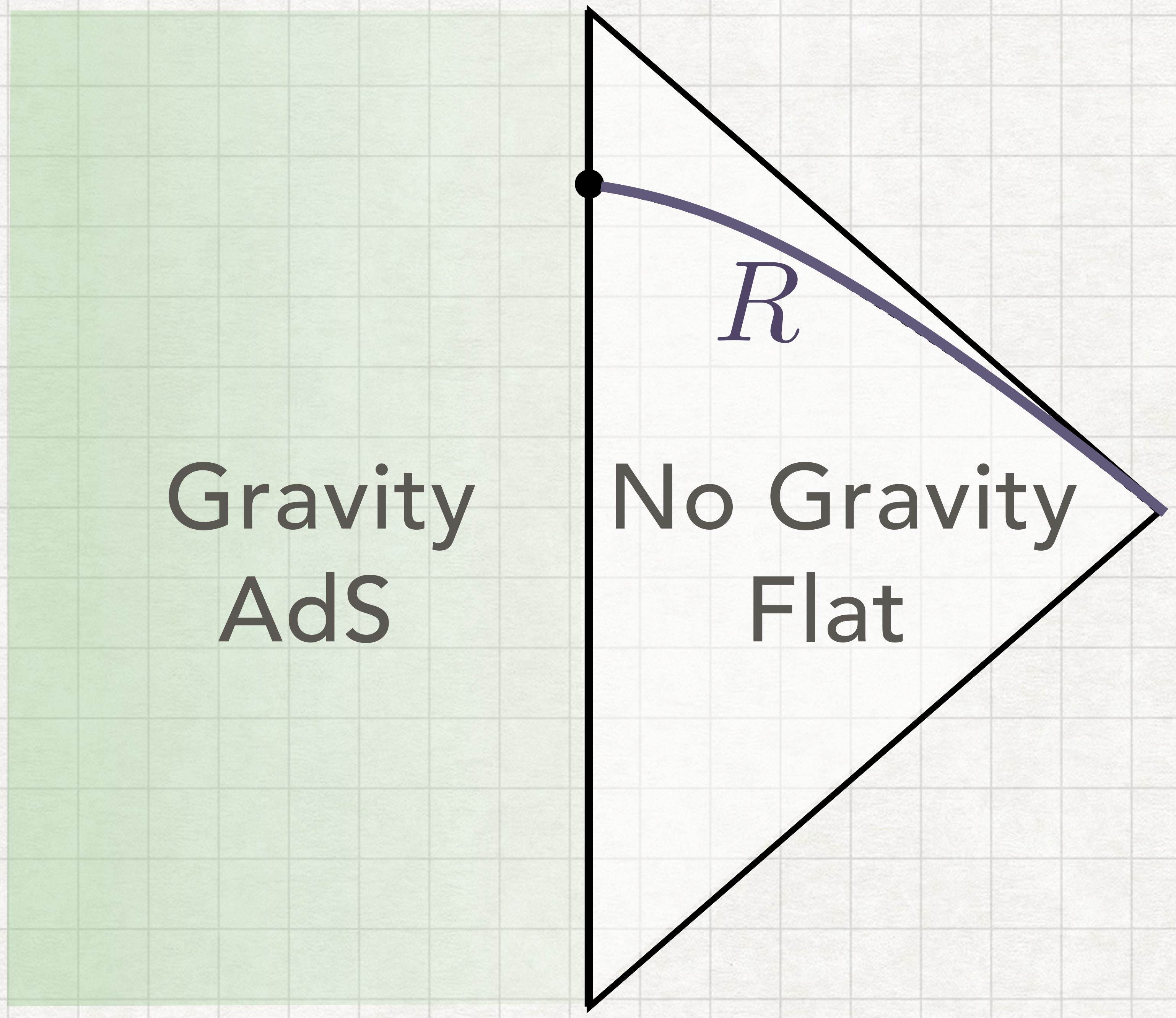
branch point

=conical excess → metric singular

$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A$$
$$= (1 - n\partial_n) Z_n|_{n=1}$$

Path-integral in Gravity

We compute the entanglement entropy in a system with gravity



Difference from QFT:

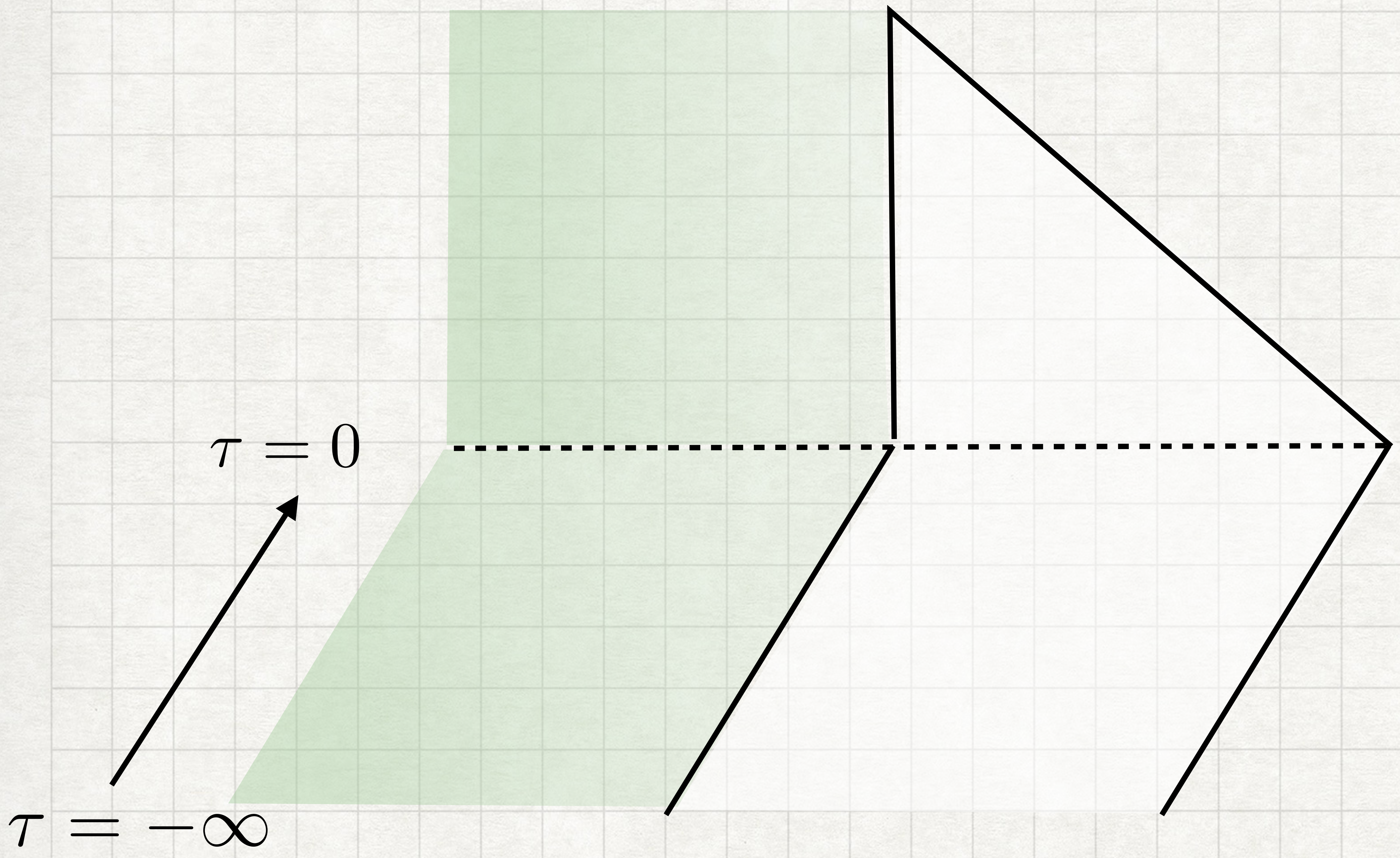
1. Geometry should satisfy the Einstein eq.

   →branch-point becomes smooth

2. Need to consider various geometries that satisfies the b.c.

Path-integral in Gravity

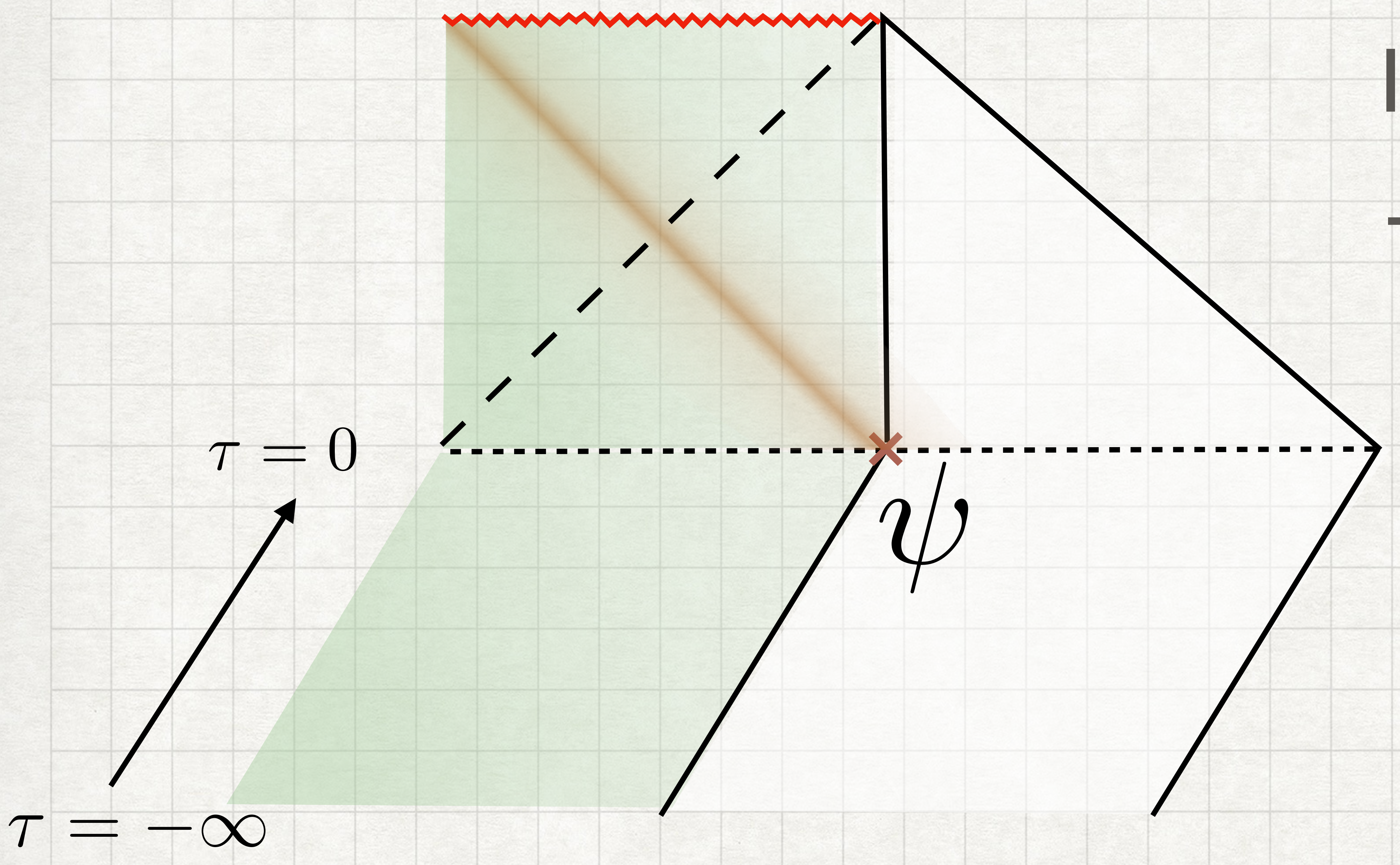
Prepare an evaporating BH using the Euclidean path-integral



“Vacuum state”

Path-integral in Gravity

Prepare an evaporating BH using the Euclidean path-integral



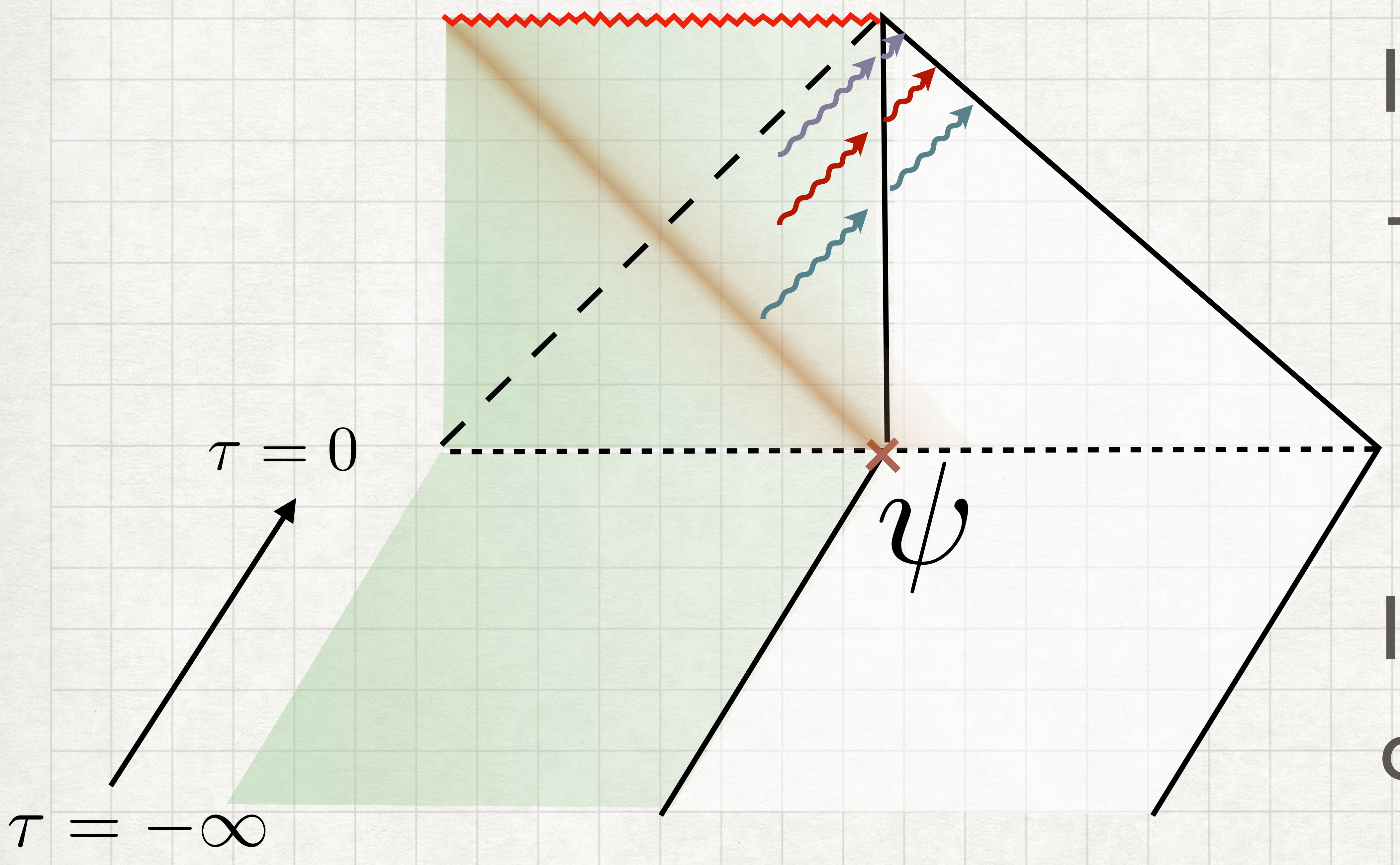
Evaporating BH

Insert a local operator during path-integral

→creates a matter shockwave after Lorentzian time-evolution

Path-integral in Gravity

Prepare an evaporating BH using the Euclidean path-integral



Evaporating BH

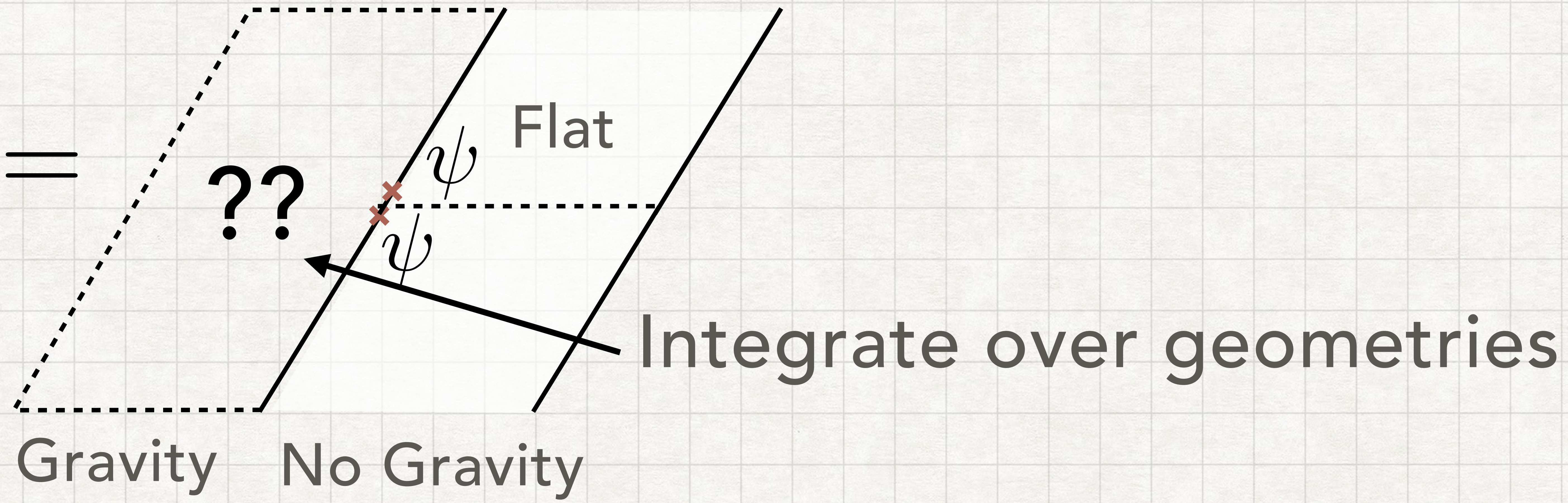
Insert a local operator during path-integral

→creates a matter shockwave after Lorentzian time-evolution

In Lorentzian regime, an evaporating BH is created by the "gravitational collapse"

Path-integral in Gravity

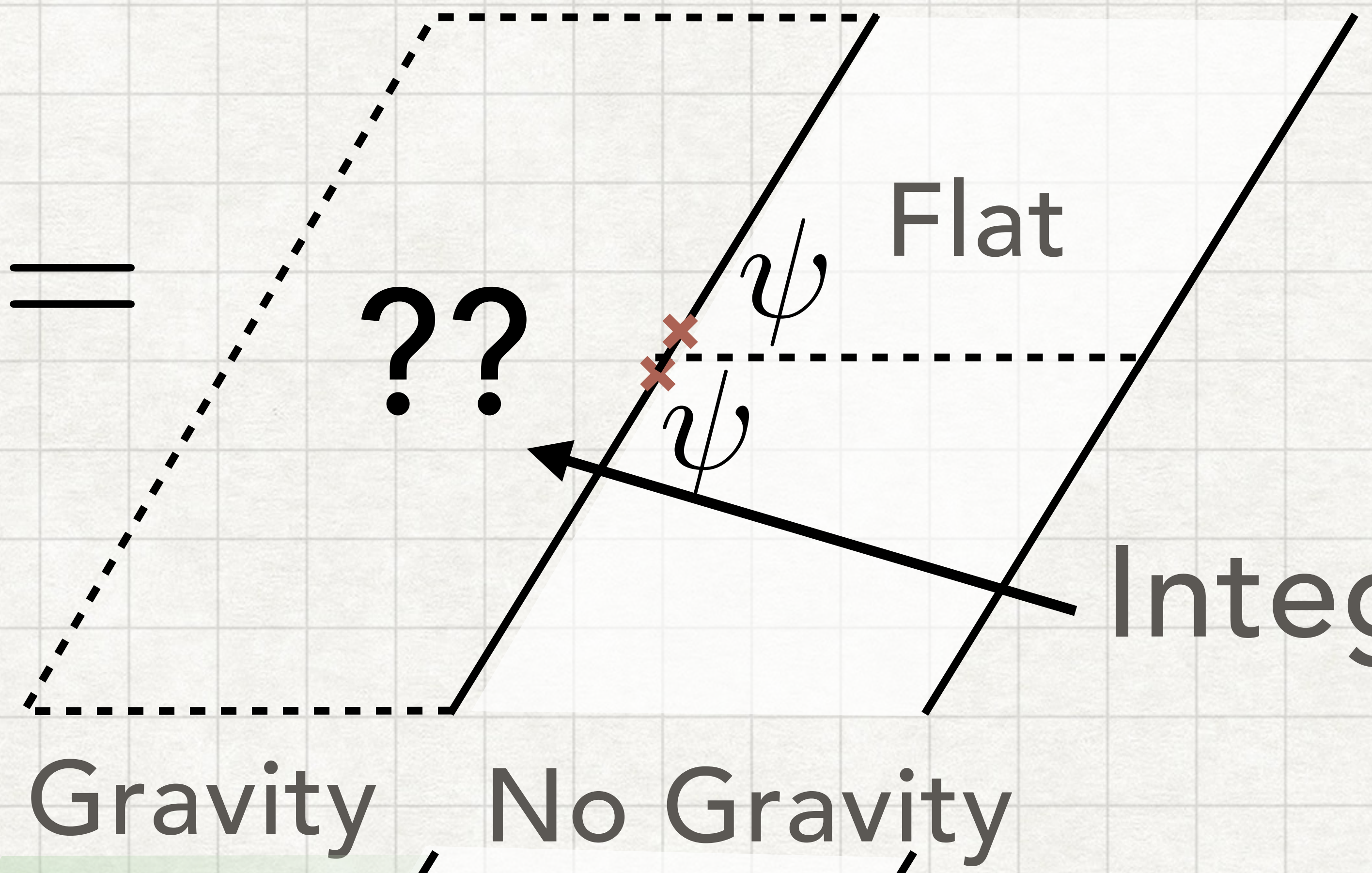
$$Z = \mathrm{tr}\rho = \int_{\partial\mathcal{M}=\text{fixed}} \mathcal{D}g_{\mu\nu}\mathcal{D}\phi \; e^{-I_{\text{grav}}[g]-I_{\text{matter}}[g,\phi]}$$

$=$ 

Path-integral in Gravity

$$Z = \mathrm{tr}\rho = \int_{\partial\mathcal{M}=\text{fixed}} \mathcal{D}g_{\mu\nu}\mathcal{D}\phi \; e^{-I_{\text{grav}}[g]-I_{\text{matter}}[g,\phi]}$$

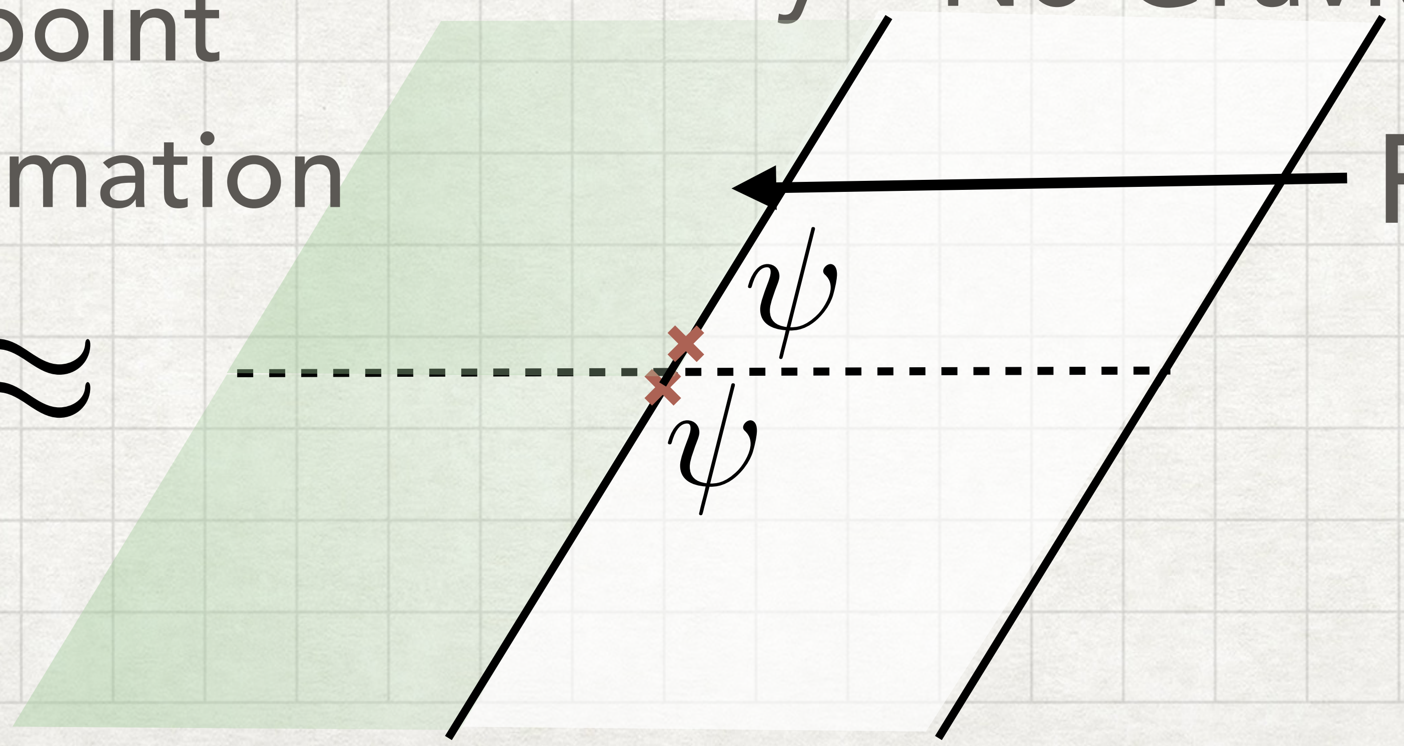
$=$



Integrate over geometries

Saddle point approximation

$\approx$



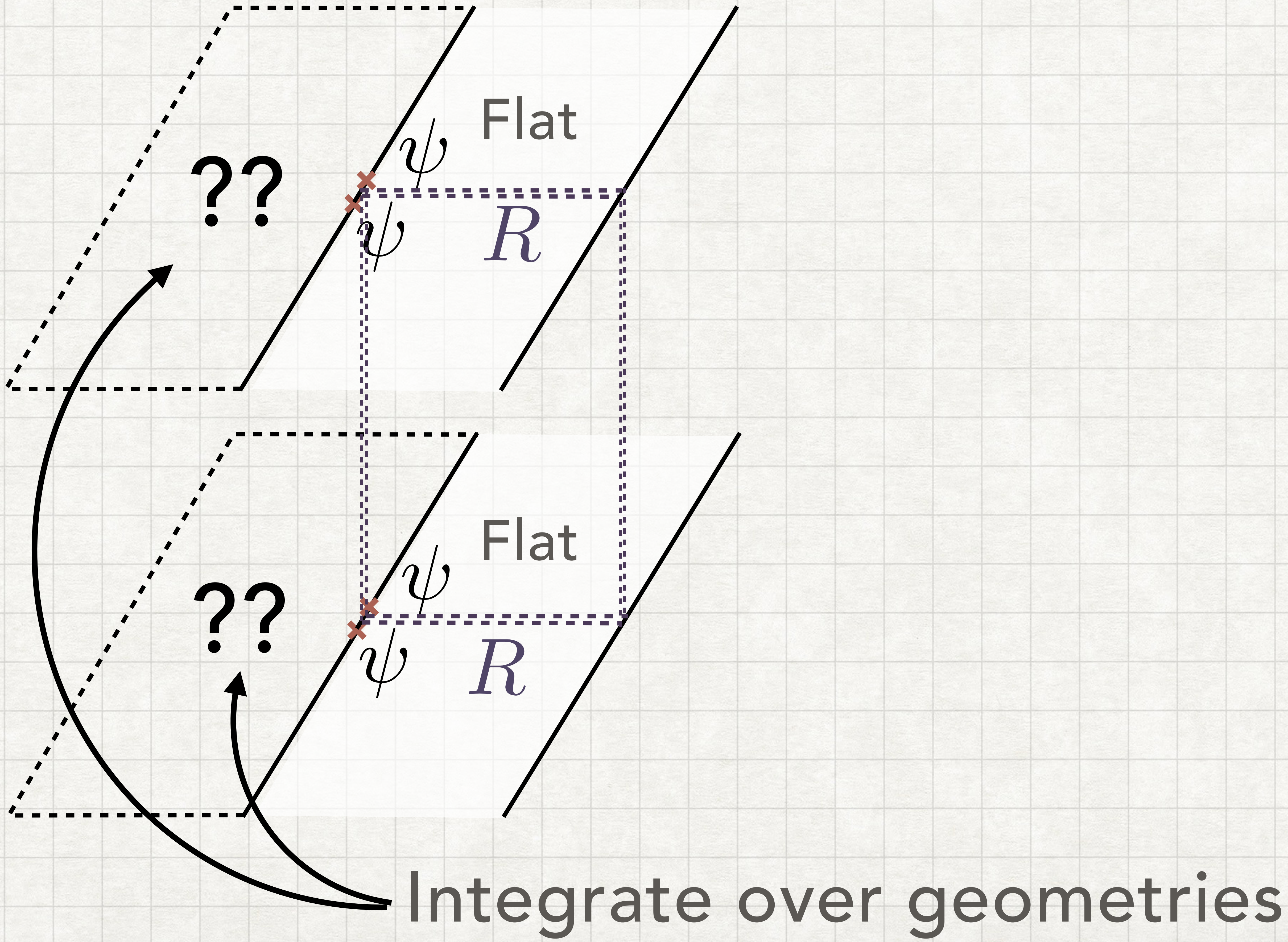
Fill in with a geometry that solves Einstein eq.

After analytic continuation, we obtain the Lorentzian evaporating black hole

Replica Calculation in Gravity

The replica calculation of the entropy of Hawking radiation S_R amounts to computing the replica partition function: $Z_n = \mathrm{tr}\rho_R^n$

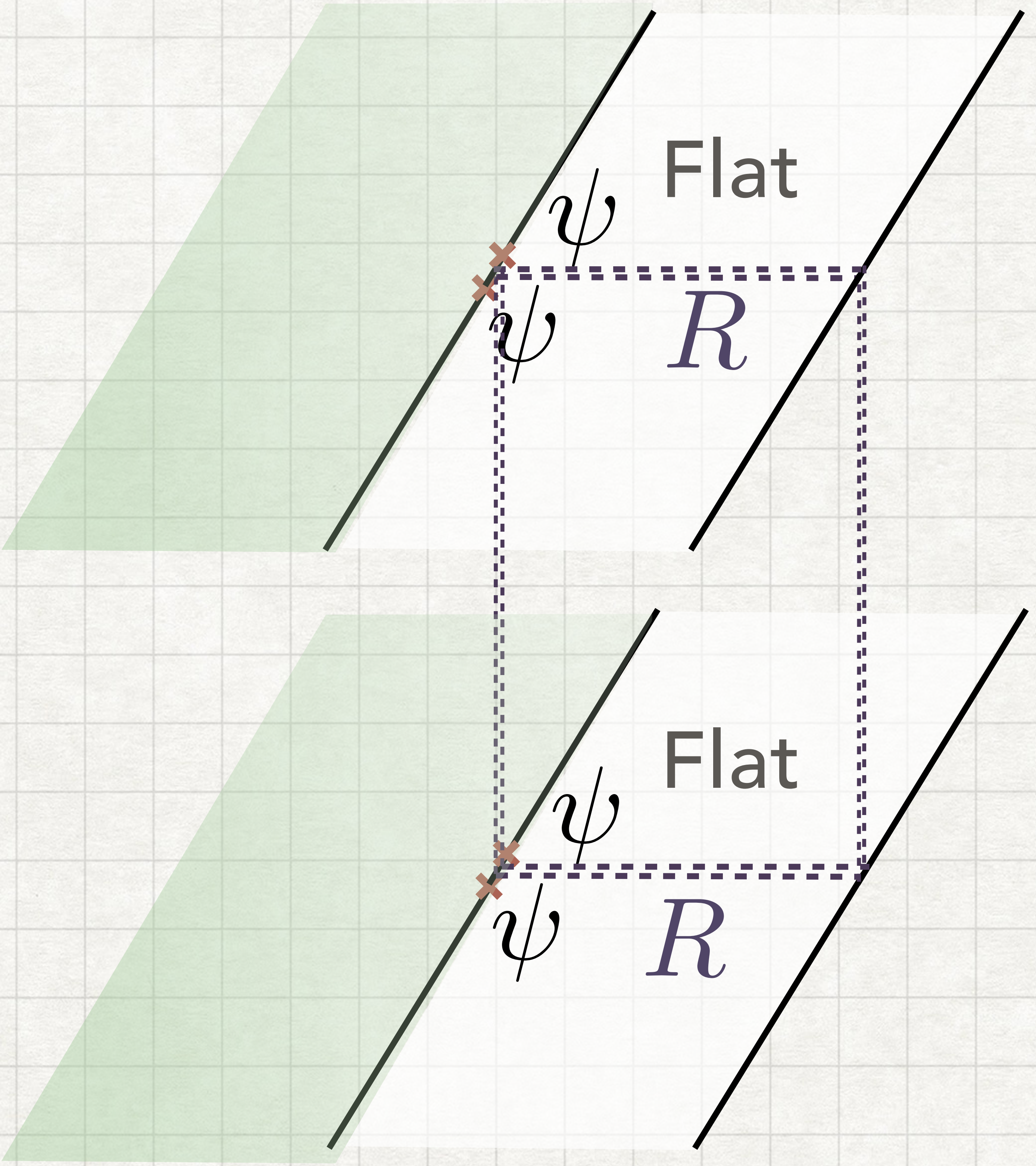
$$Z_2 = \mathrm{tr}\rho_R^2 =$$



Replica Calculation in Gravity

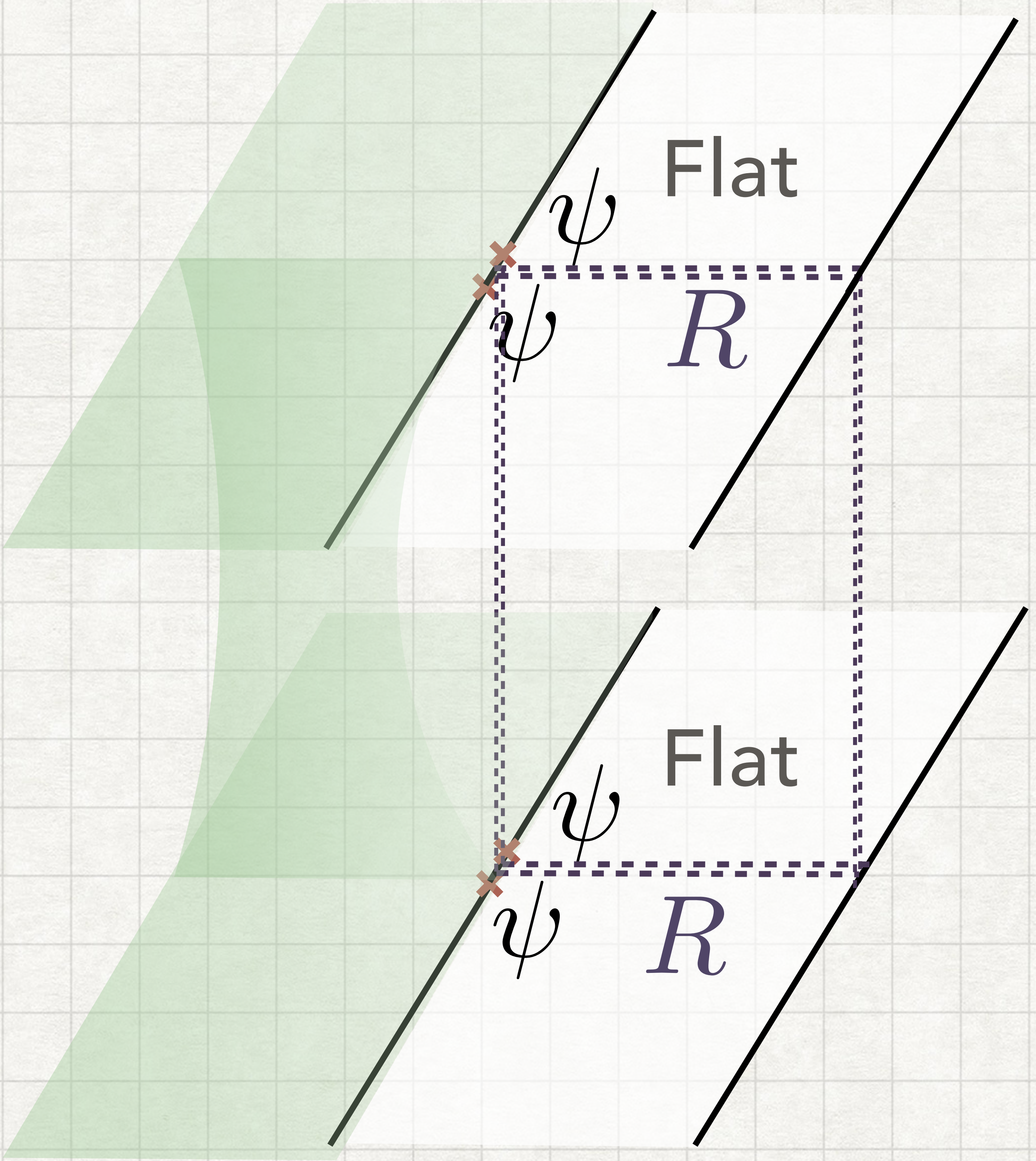
The replica calculation of the entropy of Hawking radiation S_R amounts to computing the replica partition function: $Z_n = \mathrm{tr}\rho_R^n$

$$Z_2 = \mathrm{tr}\rho_R^2 \approx$$



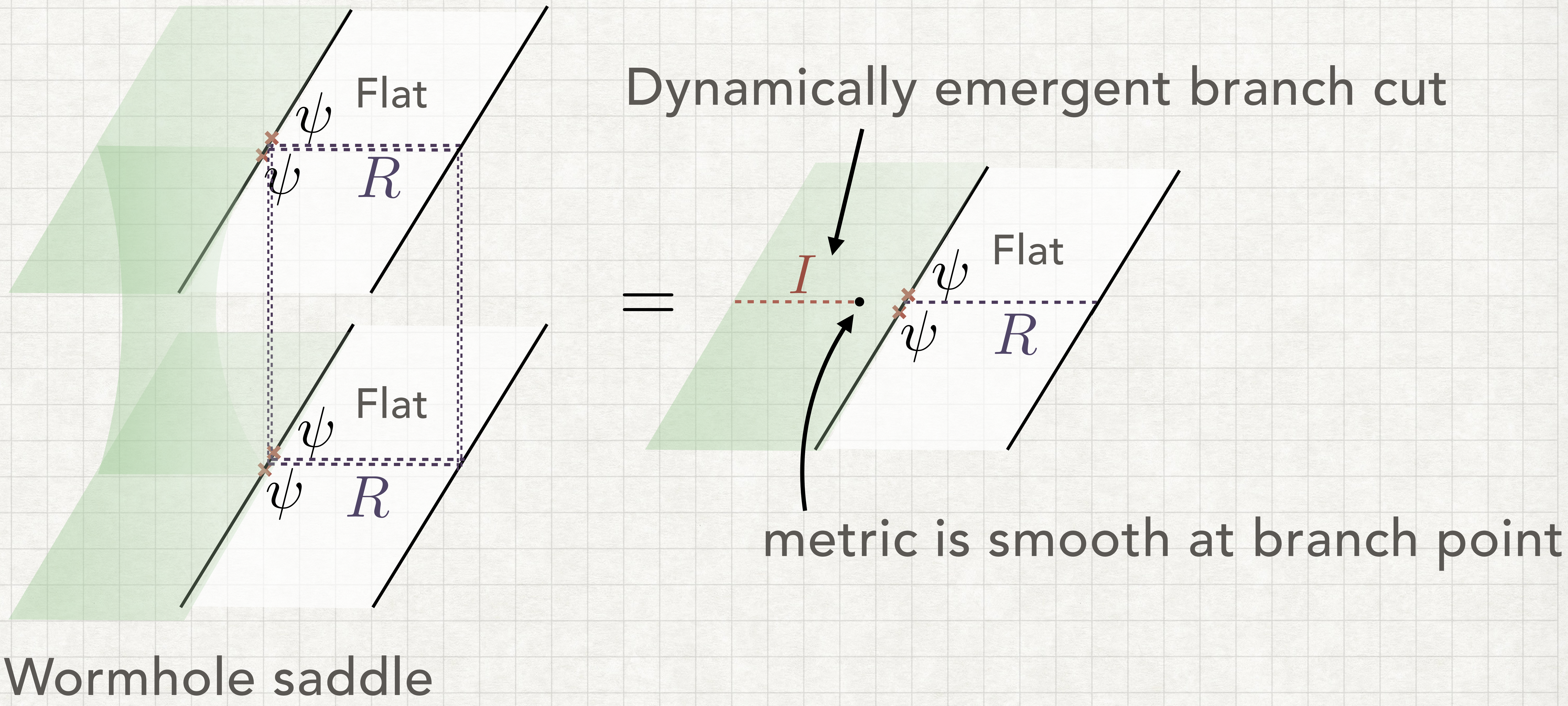
~Usual computation on a fixed geometry

$+$



Wormhole saddle

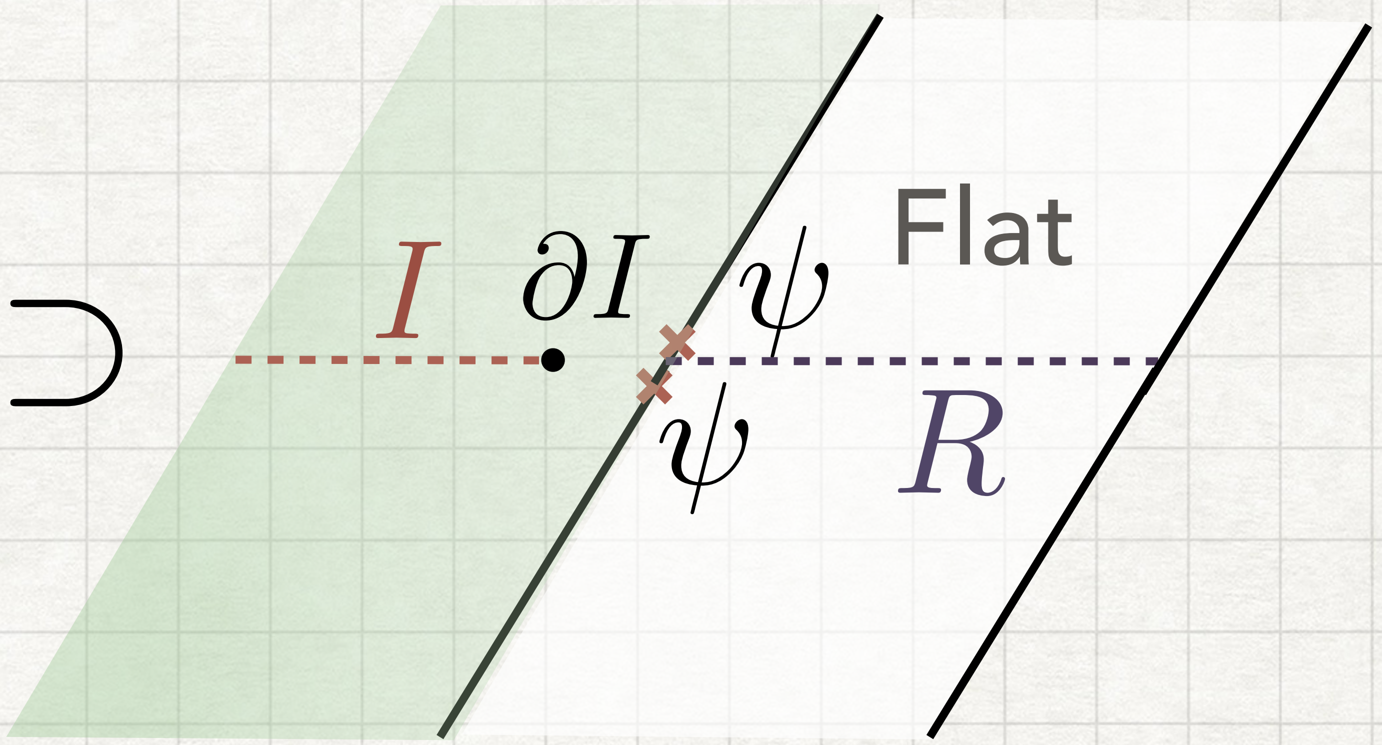
Replica Calculation in Gravity



Replica Calculation in Gravity

Wormhole solution exists at any *n*

$$Z_n = \mathrm{tr}_R \rho_R^n \supset$$

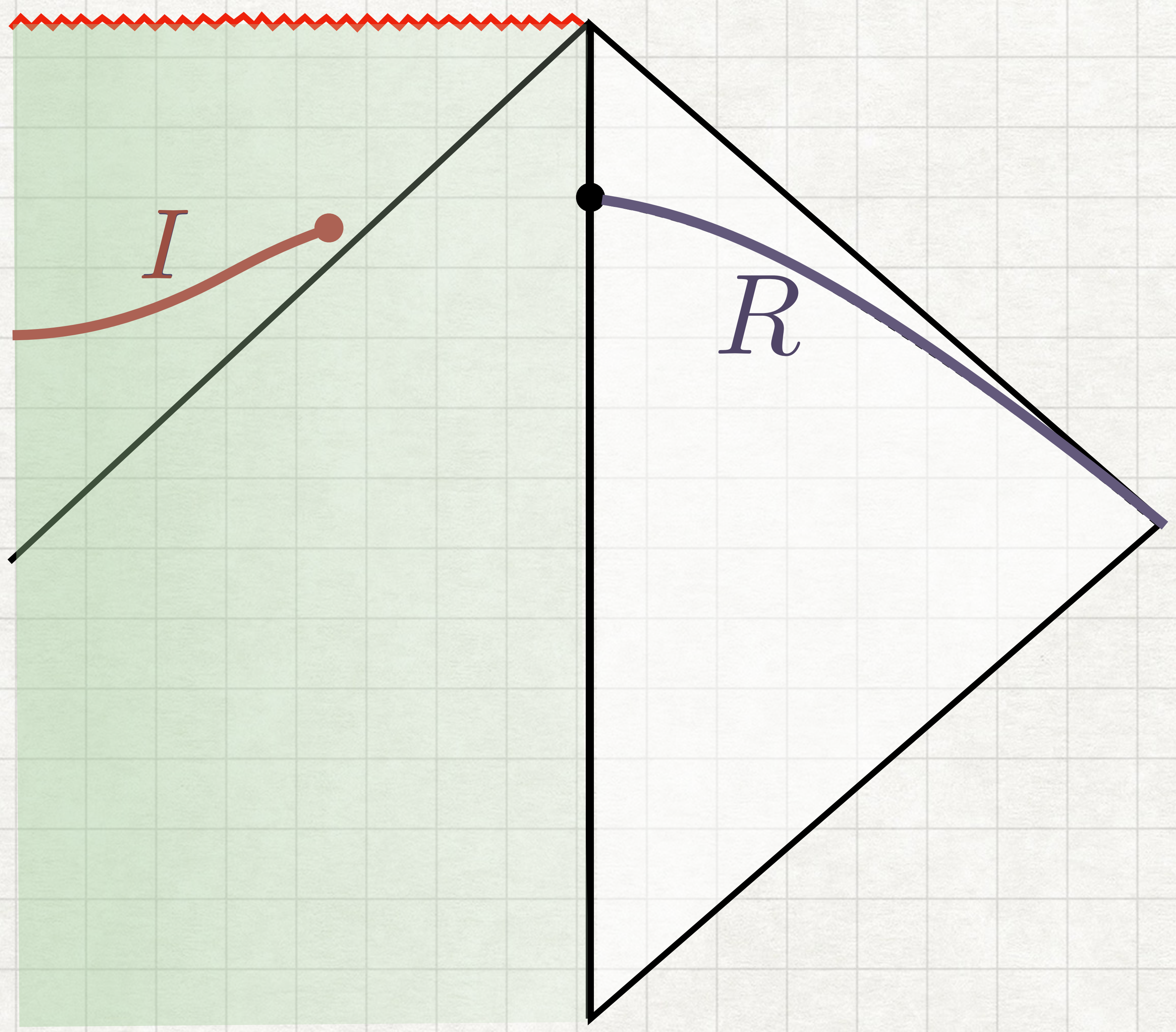


$$\simeq e^{-I_{\mathrm{grav}}[g^{(n)}_{\mathrm{wormhole}}]} Z_{\mathrm{matter}}[g^{(n)}_{\mathrm{wormhole}}]$$

$$\underset{n\to 1}{\to} \exp\left[\overbrace{-I_{\mathrm{grav}}[g^{n=1}_{\mathrm{saddle}}] - (n-1)\left[\underline{\frac{\mathrm{Area}(\partial I)}{4G_N}}}\; + S_{\mathrm{matter}}(R\cup I)\right]\right]$$

from the back reaction at ∂I

Replica Calculation in Gravity



$$S_R = -\mathrm{tr}_R \hat{\rho}_R \log \hat{\rho}_R$$
$$= (1 - n\partial_n) Z_n \big|_{n=1}$$

Island formula is derived:

$$S_R = \min_I \mathrm{ext} \left[\frac{\mathrm{Area}(\partial I)}{4G_N} + S_{\mathrm{matter}}(R \cup I) \right]$$

Einstein equation of the wormhole

Back-reaction of the dynamical branch point

Matter entropy from the wormhole geometry