Toward QCD-based description of dense baryonic matter

Yuki Fujimoto

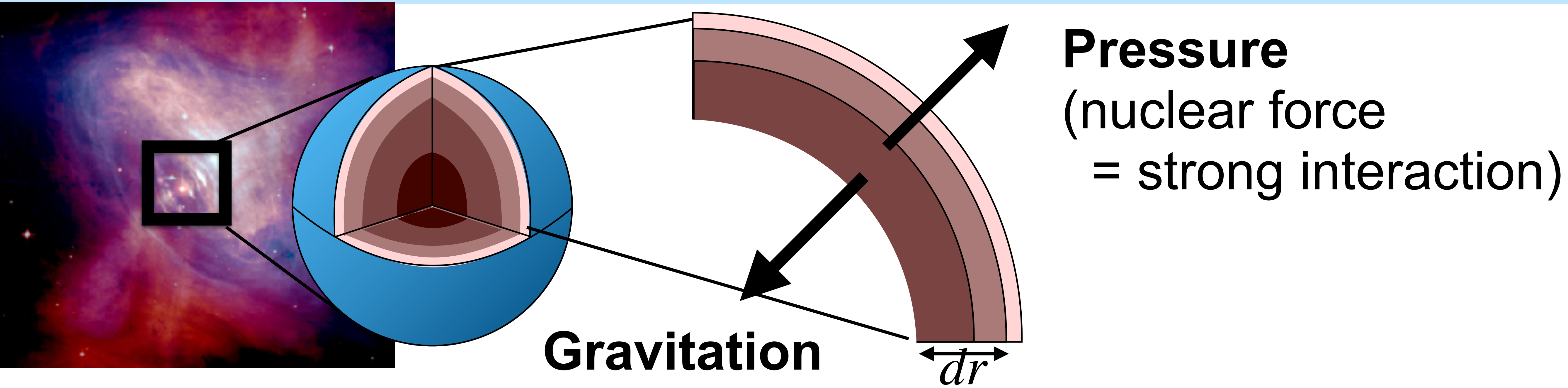
(The University of Tokyo)

Reference:

Y. Fujimoto, K. Fukushima, "Equation of state of cold and dense QCD matter in resummed perturbation theory," arXiv:2011.10891.

29 June 2021, Seminar @ Riken iTHEMS

Neutron star structure



Hydrostatic equilibrium (pressure = gravitation)

Tolman (1939)
Oppenheimer, Volkoff (1939)

$$\frac{dp(r)}{dr} = -G\frac{m(r)\varepsilon(r)}{r^2} \times \underbrace{\left(1+\frac{p}{\varepsilon}\right)\left(1+\frac{4\pi r^3 p}{m}\right)\left(1-\frac{2Gm}{r}\right)^{-1}}_{\text{General relativistic correction}}$$

← **TOV equation**

$$m(r) = \int_0^r dr 4\pi r^2 \varepsilon(r)$$

Unknown variables:
$p(r)$, $m(r)$ and $\varepsilon(r)$

One condition missing!

Equation of State (EoS)

$$p = p(\varepsilon)$$

Neutron star structure

TOV equation:

Tolman (1939)
Oppenheimer,Volkoff (1939)

$$\frac{dp(r)}{dr} = -G\frac{m(r)\varepsilon(r)}{r^2} \times \underbrace{\left(1+\frac{p}{\varepsilon}\right)\left(1+\frac{4\pi r^3 p}{m}\right)\left(1-\frac{2Gm}{r}\right)^{-1}}_{\text{General relativistic correction}}$$

$$m(r) = \int_0^r dr 4\pi r^2 \varepsilon(r)$$

Equation of State (EoS): $p = p(\varepsilon)$

Initial value problem

Initial:

$r = 0$

$\varepsilon(r=0) = \varepsilon_c$ free parameter

$p(r=0) = p(\varepsilon_c) = p_c$

$m(r=0) = 0$

Final:

$r = \boldsymbol{R}$ M-R relation

$\varepsilon(r=R) = 0$

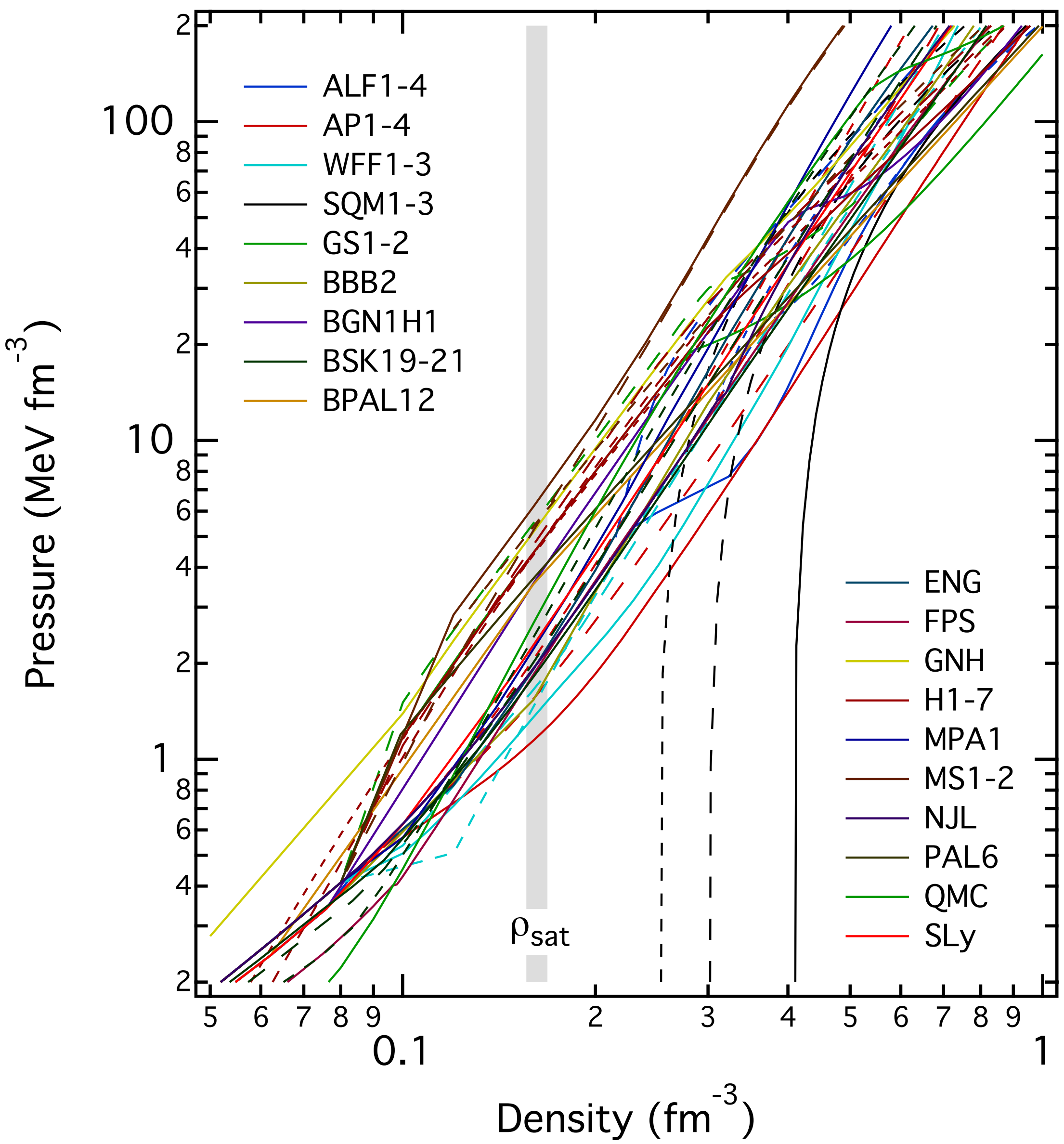
$p(r=R) = 0$

$m(r=R) = \boldsymbol{M}$

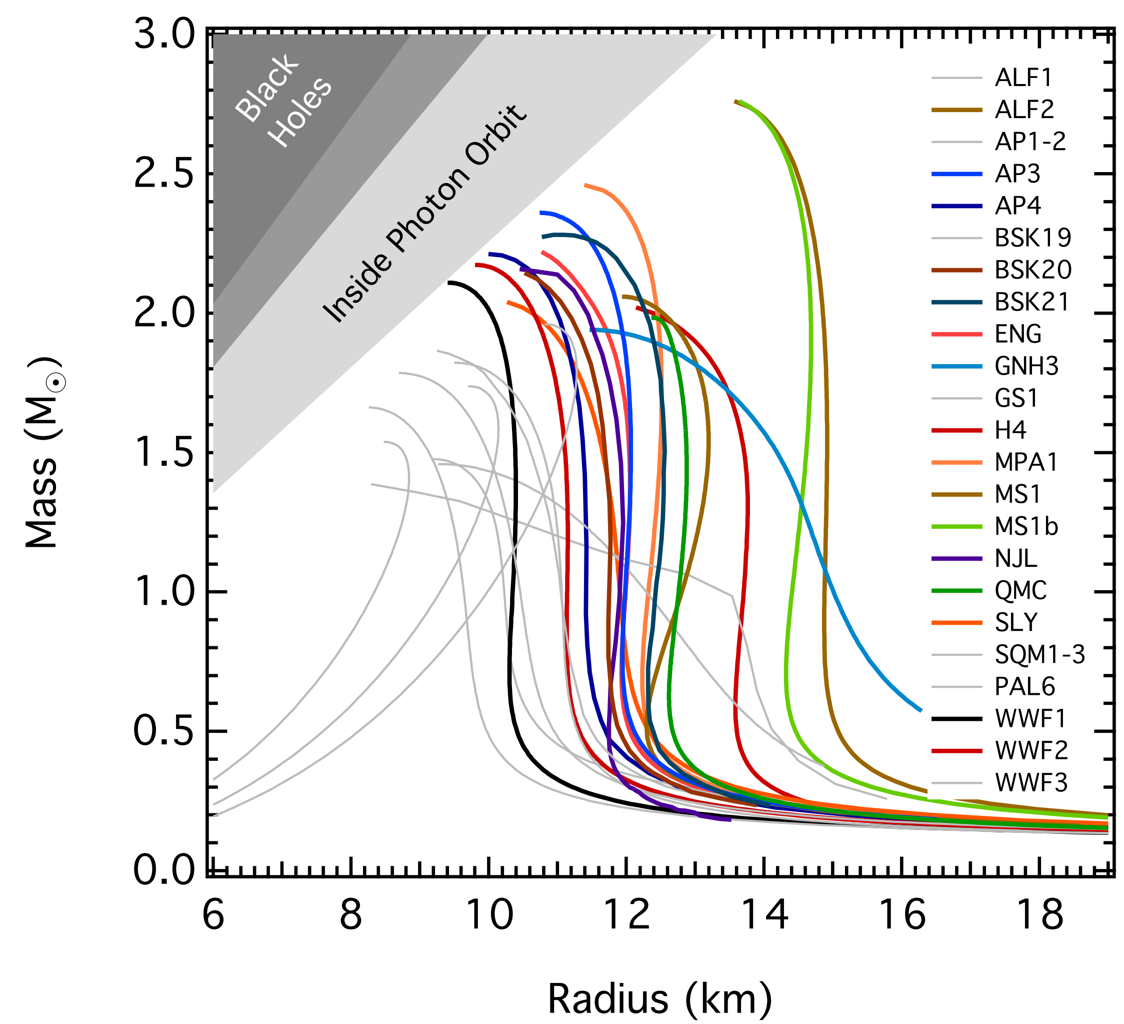
Equation of state (EoS)

EoS: pressure function $p(n_B)$, $p(\varepsilon)$, or $p(\mu_B)$

(n_B: baryon density, ε: energy density, μ_B: chemical potential)



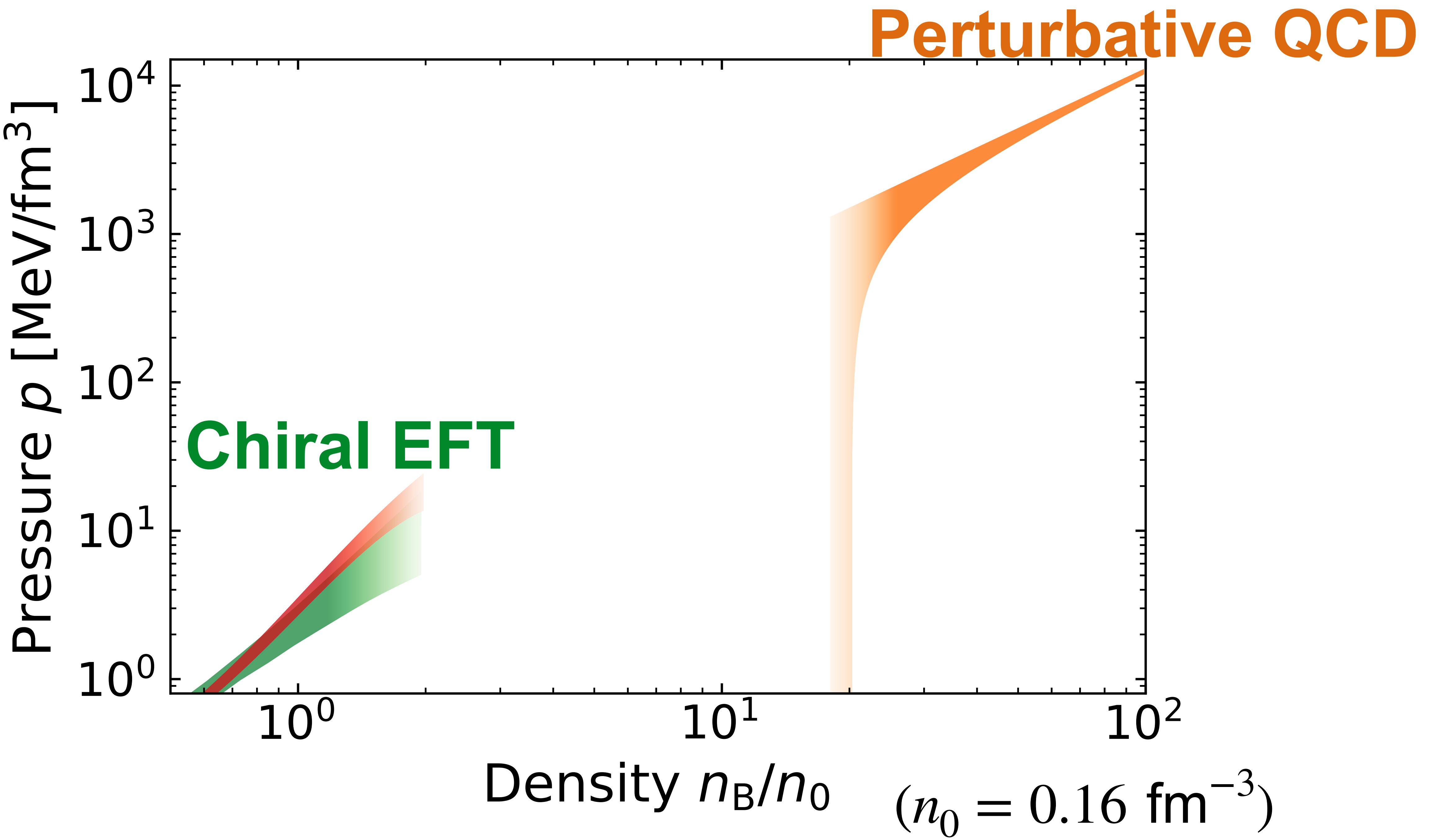
Özel,Freire (2016)



"Spaghetti diagram", a little messy…

A QCD physicist's view on the NS EoS

Constraints from QCD point of view:

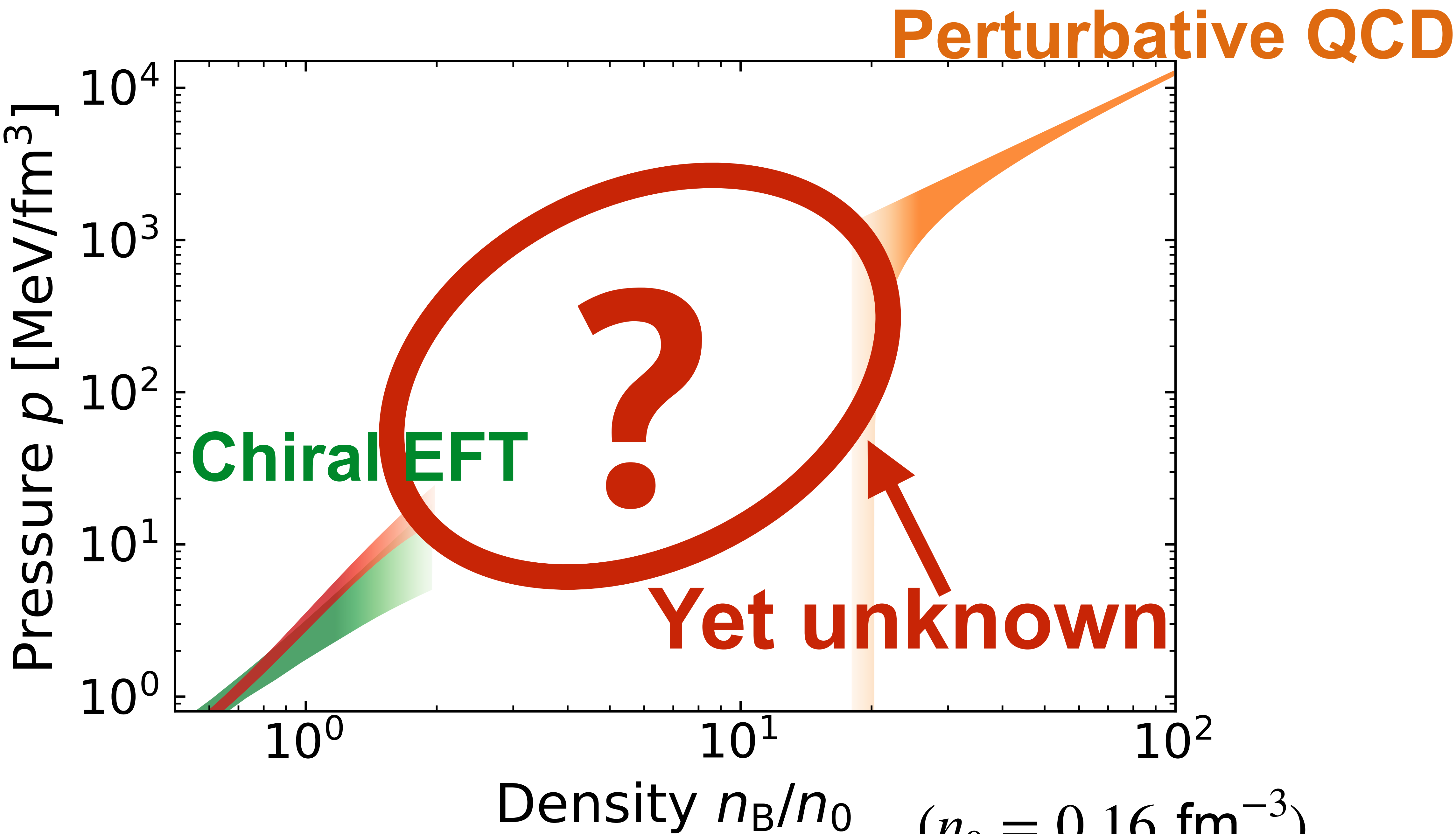


ChEFT: Tews,Carlson,Gandolfi,Reddy (2018); Drischler,Furnstahl,Melendez,Phillips (2020)

pQCD: Freedman,McLerran (1978); Baluni (1979); Kurkela,Romatschke,Vuorinen,Gorda,Sappi (2009-)

A QCD physicist's view on the NS EoS

Constraints from QCD point of view:



$(n_0 = 0.16\ \mathrm{fm}^{-3})$

ChEFT: Tews,Carlson,Gandolfi,Reddy (2018);
Drischler,Furnstahl,Melendez,Phillips (2020)

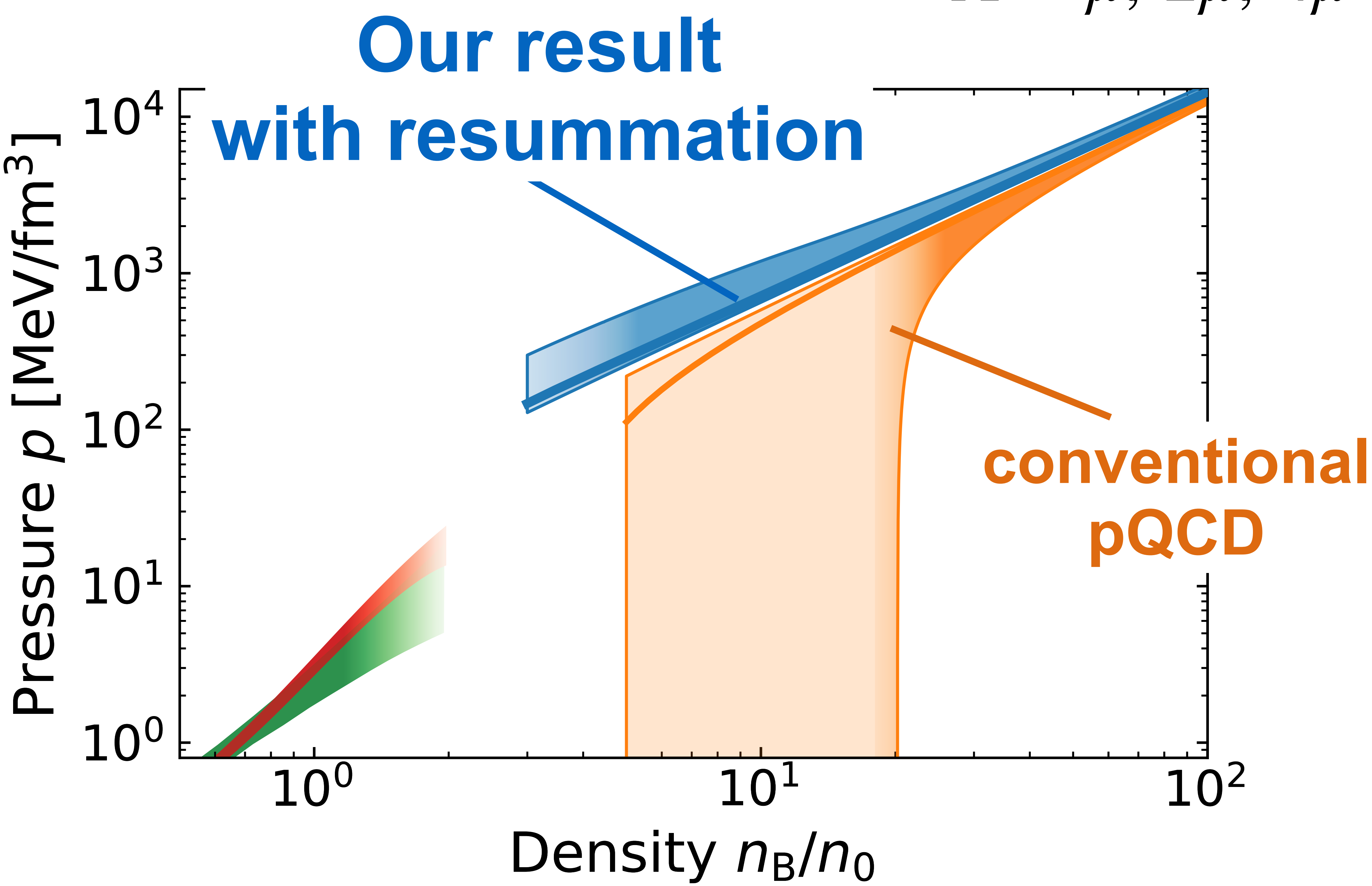
pQCD: Freedman,McLerran (1978); Baluni (1979);
Kurkela,Romatschke,Vuorinen,Gorda,Sappi (2009-)

Our result in a nutshell

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \log(\bar{\Lambda}^2/\Lambda^2_{\overline{\mathrm{MS}}})}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar{\Lambda}^2/\Lambda^2_{\overline{\mathrm{MS}}})}{\log(\bar{\Lambda}^2/\Lambda^2_{\overline{\mathrm{MS}}})}\right]$$

uncertainty: originate from scale variation

$$\bar{\Lambda} = \mu,\ 2\mu,\ 4\mu$$

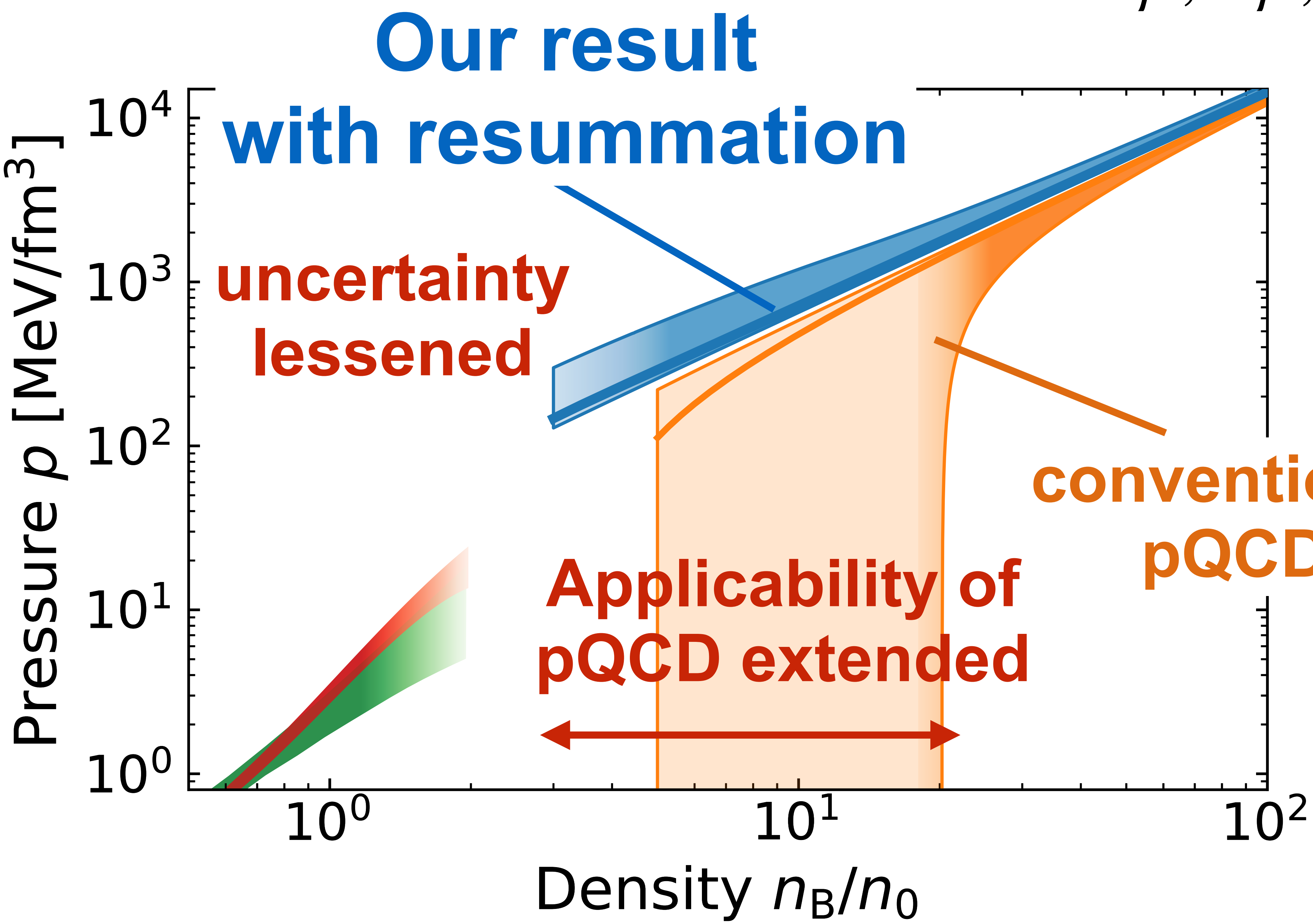


Our result in a nutshell

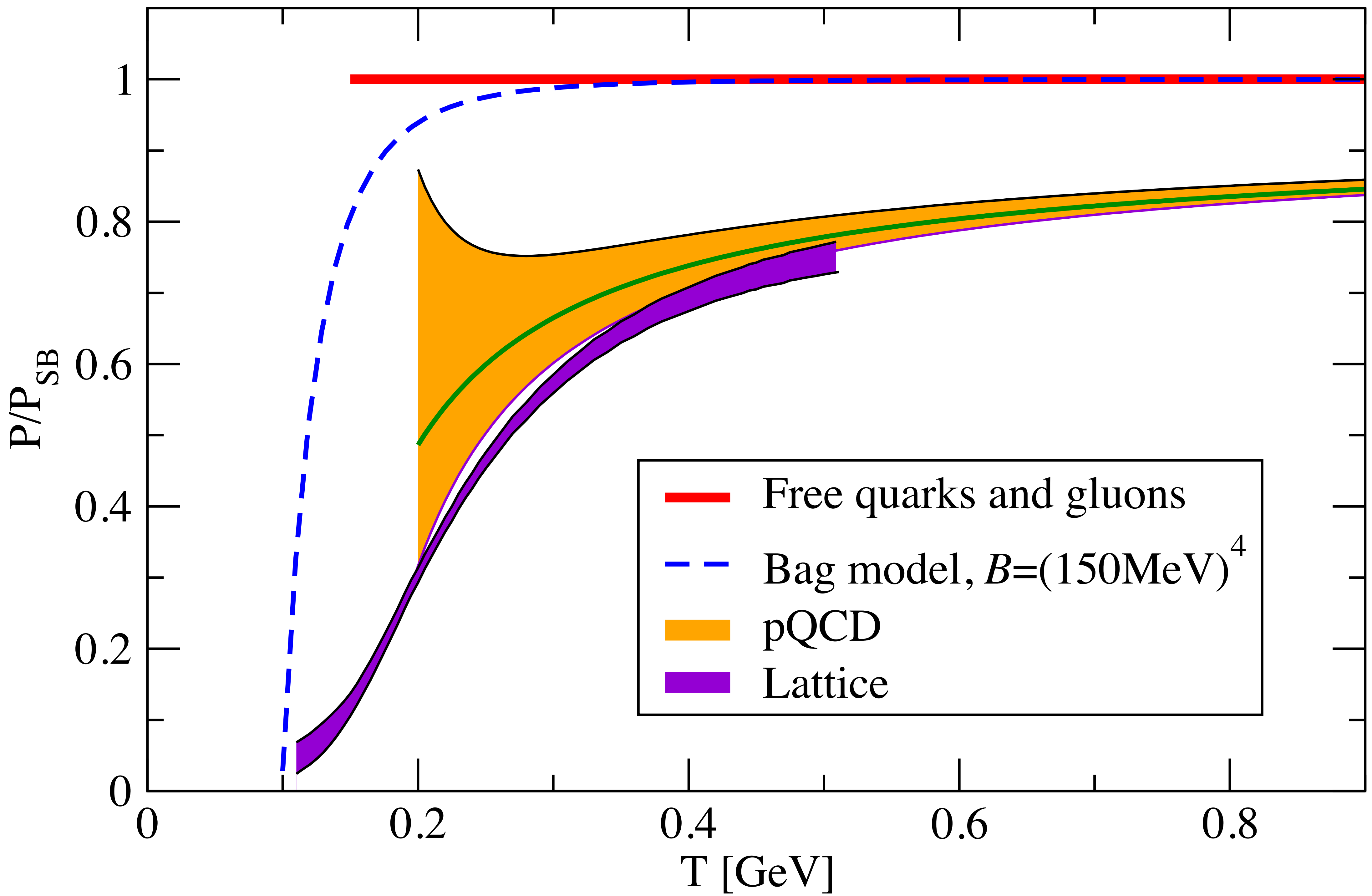
$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \log(\bar{\Lambda}^2/\Lambda^2_{\overline{\rm MS}})}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar{\Lambda}^2/\Lambda^2_{\overline{\rm MS}})}{\log(\bar{\Lambda}^2/\Lambda^2_{\overline{\rm MS}})}\right]$$

uncertainty: originate from scale variation

$$\bar{\Lambda} = \mu,\ 2\mu,\ 4\mu$$



Analogy with high temperature case



Fraga,Kurkela,Vuorinen (2013)

Outline of the talk

- Introduction

- Hard Dense Loop (HDL) resummation and
  the setup of calculations

- The result for the EoS and
  its phenomenological implication

- Summary

Outline of the talk

- Introduction

- **Hard Dense Loop (HDL) resummation and the setup of calculations**

- The result for the EoS and its phenomenological implication

- Summary

Hard Thermal Loops (HTL)

- The problem of the gauge-dependent **gluon damping rate** at finite temperature (two scales: g & T) :

$$\gamma_g = a\frac{g^2T}{8\pi}$$

Heinz,Kajantie,Toimela,… (1987)

$a = 1$ (in Coulomb gauge), $a = -5$ (in covariant gauge)

- **Hard thermal loop (HTL) resummation:**
resum certain kinds of diagrams called HTLs, and use effective resummed propagator

Braaten,Pisarski (1990)

$a \simeq 6.635$ (both in Coulomb and covariant gauge)

Need of resummation

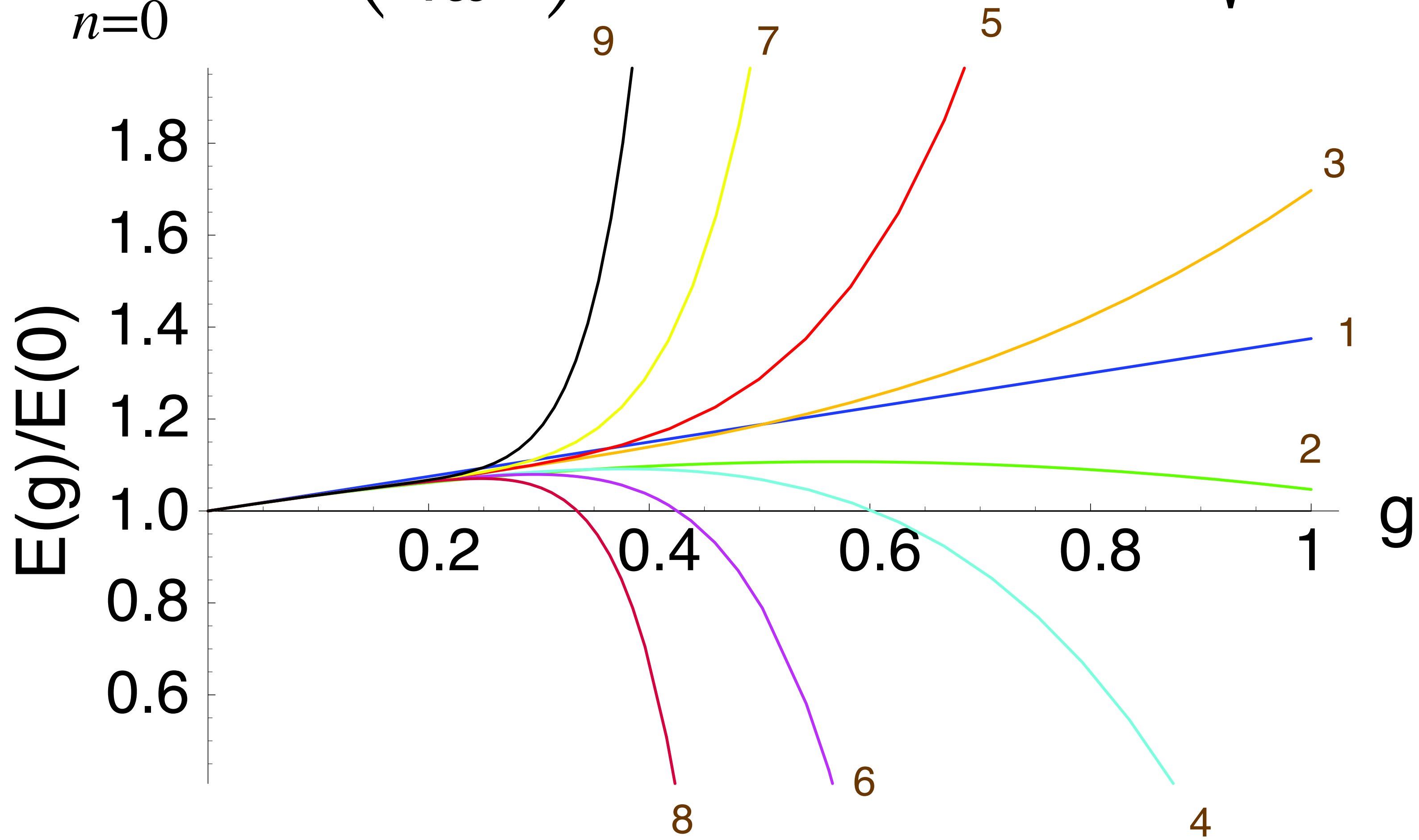
Prototypical example: an **anharmonic oscillator**

$$V(x) = \frac{\omega^2}{2}x^2 + \frac{g}{4}x^4$$

ground state energy:

Bender,Wu (1968)

$$E(g) = \omega \sum_{n=0}^{\infty} c_n^{\mathrm{BW}} \left(\frac{g}{4\omega^3}\right)^n, \; c_n^{\mathrm{BW}} \xrightarrow{n\to\infty} (-1)^{n+1}\sqrt{\frac{6}{\pi^3}}3^n \left(n-\frac{1}{2}\right)!$$



poor convergence…

Review by Su(2012)

Need of resummation

Feynman,Kleinert,… (1986)

Variational perturbation theory (VPT):

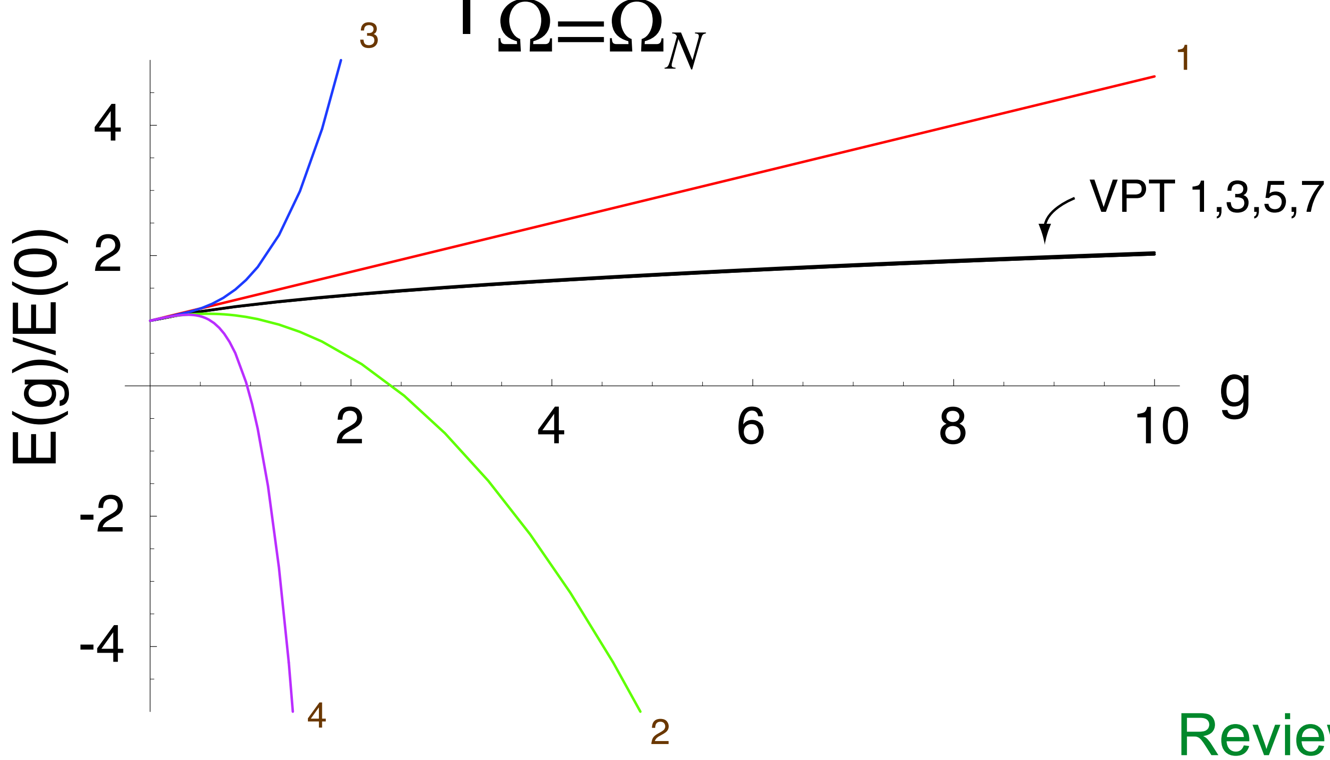
$$\omega^2 \to \Omega^2 + (\omega^2 - \Omega^2)$$

$$V(x) = \frac{\omega^2}{2}x^2 + \frac{g}{4}x^4 \to V(x) = \frac{\Omega^2}{2}x^2 + \frac{g}{4}(rx^2 + x^4)$$

$$(r := 2(\omega^2 - \Omega^2)/g)$$

minimize ground state energy w.r.t. Ω:

$$\left.\frac{\partial E_N}{\partial \Omega}\right|_{\Omega=\Omega_N} = 0, \left(E_N(g,r) = \Omega \sum_{n=0}^{N} c_n(r) \left(\frac{g}{4\Omega^3}\right)^n \right)$$

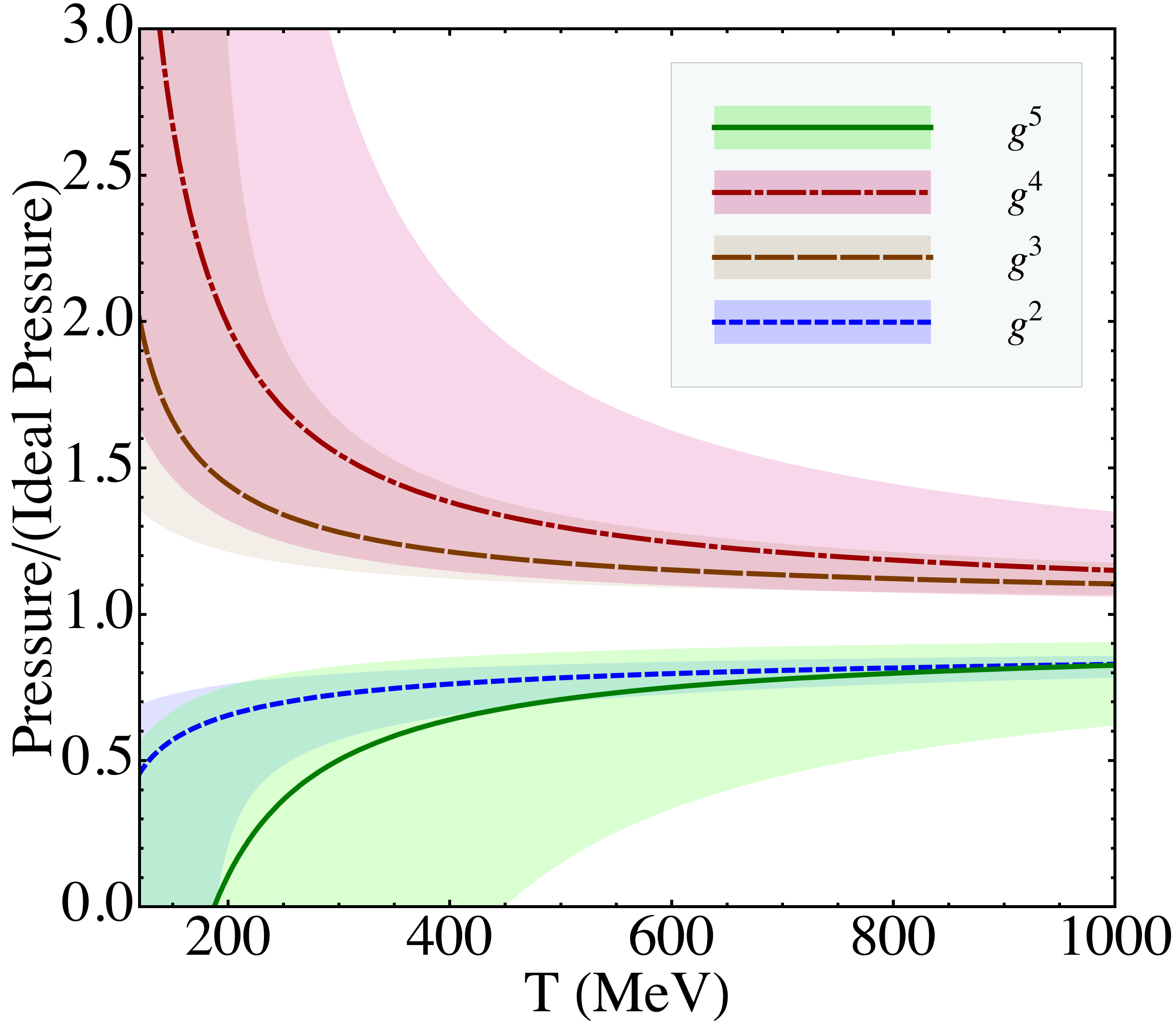


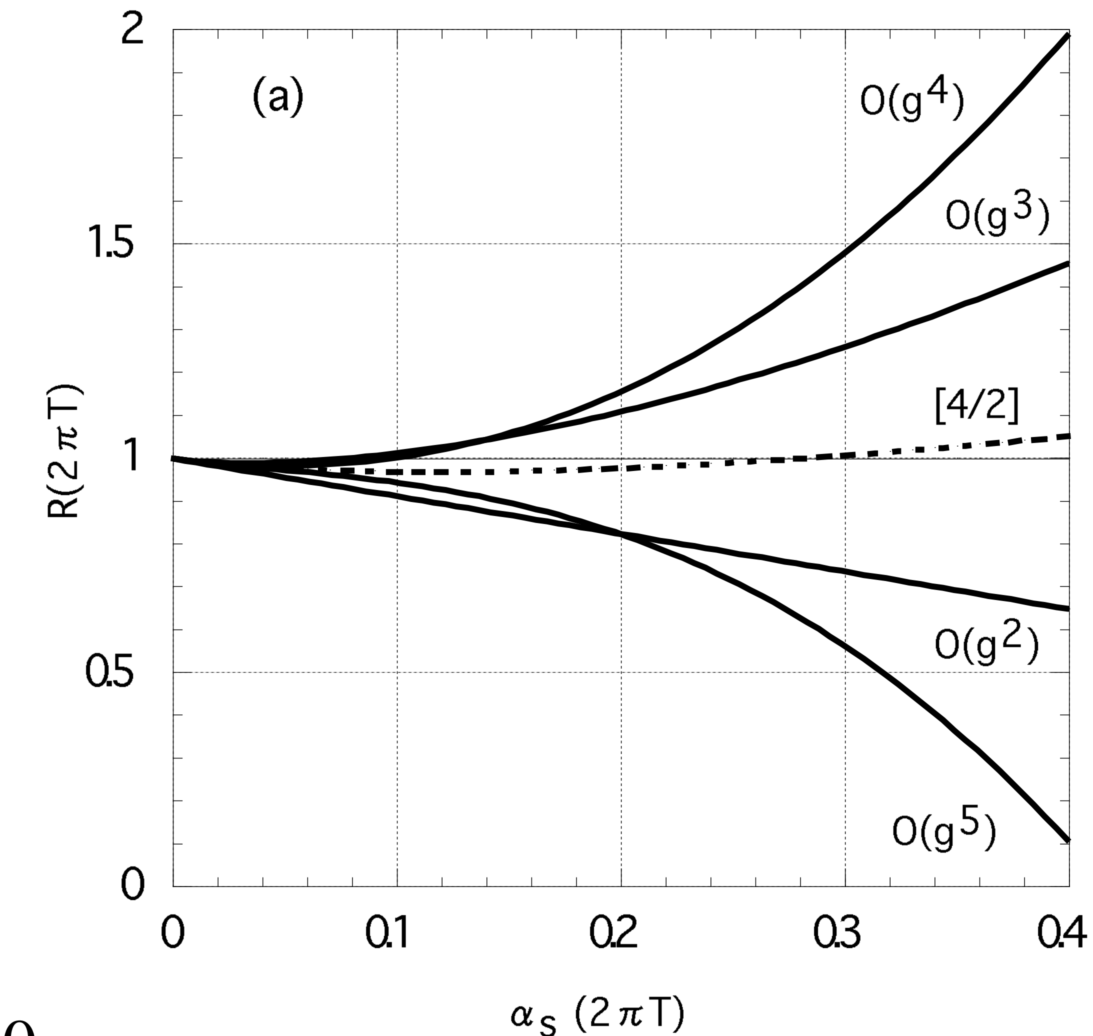
Effective resummation by VPT cures the convergence problem

Review by Su(2012)

QCD thermodynamics at high T

- The same problem of poor convergence resides in the QCD EoS at high T:





Hatsuda (1997)

Review by Su(2012)

HTL perturbation theory

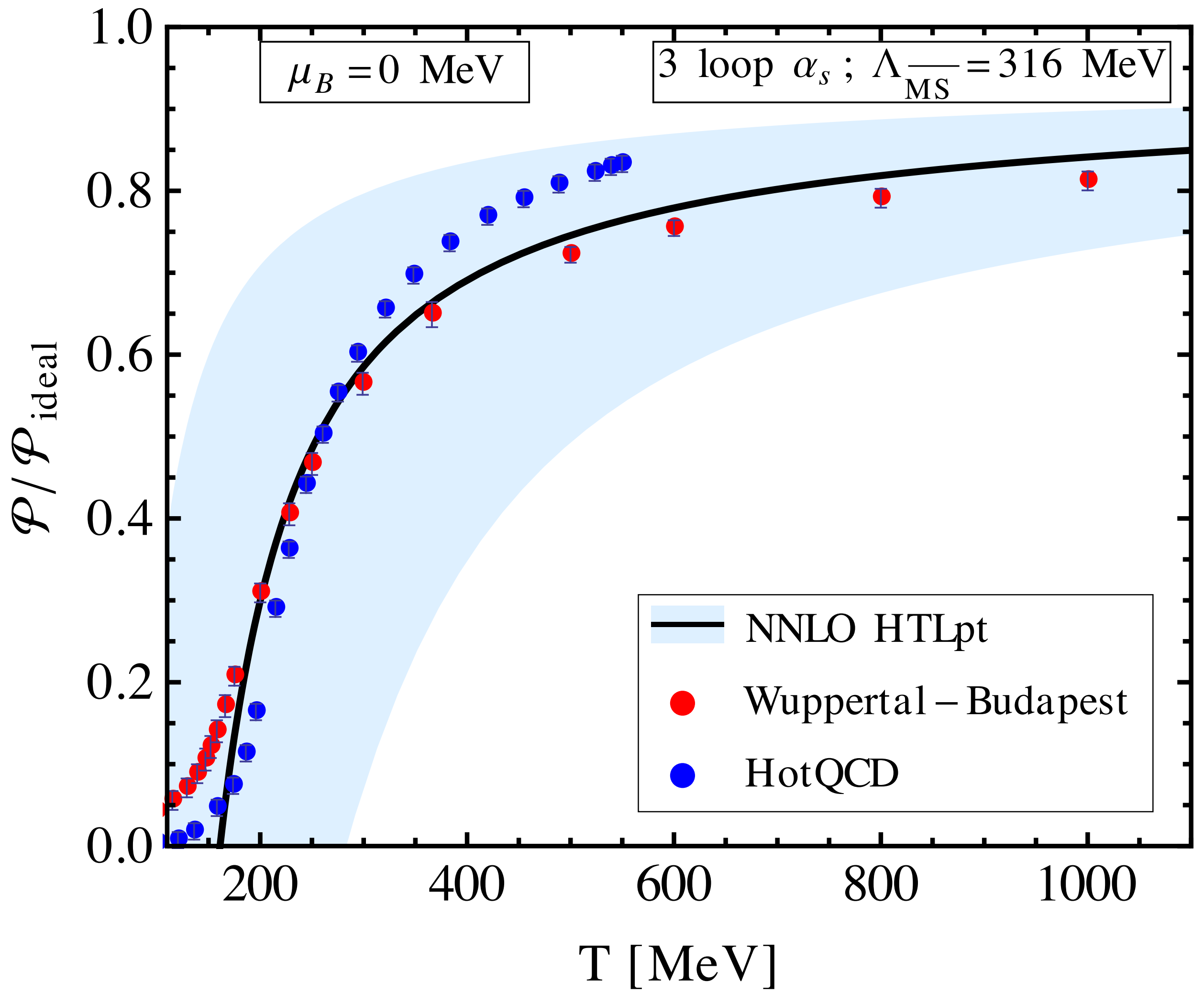
- HTL perturbation theory (sort of VPT) is one way to implement HTL resummation

Andersen,Braaten,Strickland,… (1998)

cf) Pade: Hatsuda (1997), Optimized PT: Chiku,Hatsuda (1998), EQCD: Blaizot,Iancu,Rebhan (2003), Laine,Schroder (2006)

- Hot QCD EoS with improved convergence confronting lattice data:

Haque,Bandyopadhyay,Andersen, Mustafa,Strickland,Su (2014)



Hard Dense Loops (HDLs)

- **Hard dense loops (HDLs):**
  $T = 0$ and $\mu > 0$ counterpart of the HTLs
  cf. thermal quark mass:

$$m_q^2 = \frac{g^2}{6}\left(\underline{T^2} + \underline{\frac{\mu^2}{\pi^2}}\right)$$

- Parallelism between $T \leftrightarrow \mu$ has been studied Manuel (1996)

- **EoS calculations at $T = 0$:**
  - Baier, Redlich: hep-ph/9908372
  - Andersen, Strickland: hep-ph/0206196

What we calculate here

Quark contribution to the pressure p:

$$p(\mu) = \mathrm{Tr}\log S^{-1}$$

HDL resummed full propagator

$$= \sum\!\!\!\!\!\!\!\int_{\{K\}} \ln\det\left[\not{k} - M_f - \Sigma(i\tilde{\omega}_n + \mu_f, k)\right]$$

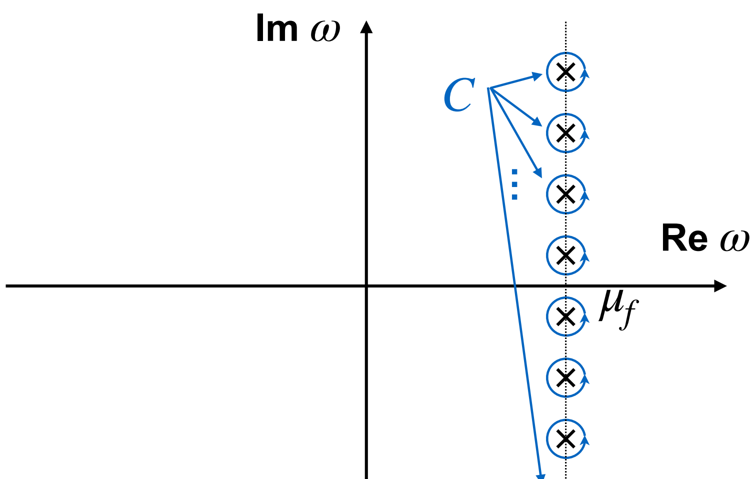
Strange quark mass is included

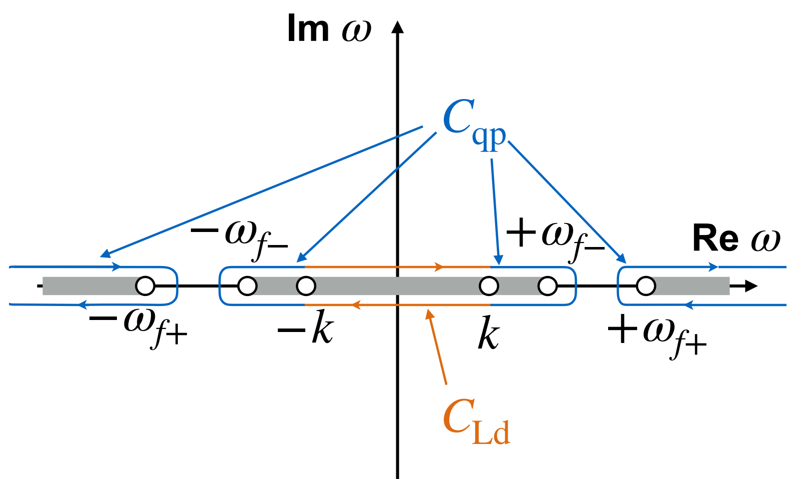
Self-energy Σ in HDL approximation:

$$\Sigma \equiv \frac{m_q^2}{k}\gamma^0 Q_0\left(\frac{k_0}{k}\right) + \frac{m_q^2}{k}\boldsymbol{\gamma}\cdot\hat{\boldsymbol{k}}\left[1 - \frac{k_0}{k}Q_0\left(\frac{k_0}{k}\right)\right]$$

(Legendre function: $Q_0(x) = \dfrac{1}{2}\log\dfrac{x+1}{x-1}$)

Integration contour deformation:

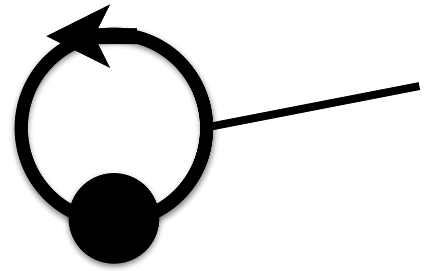




tedious calculation…

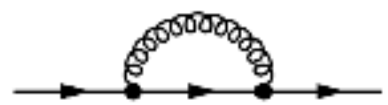
Diagrammatic argument

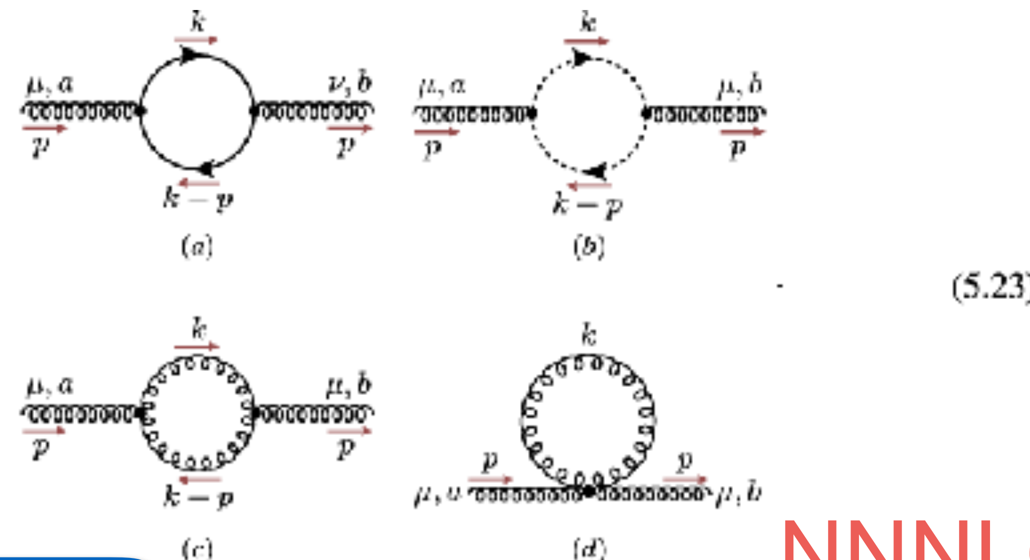
What we calculate:



full propagator

Hard dense loops:

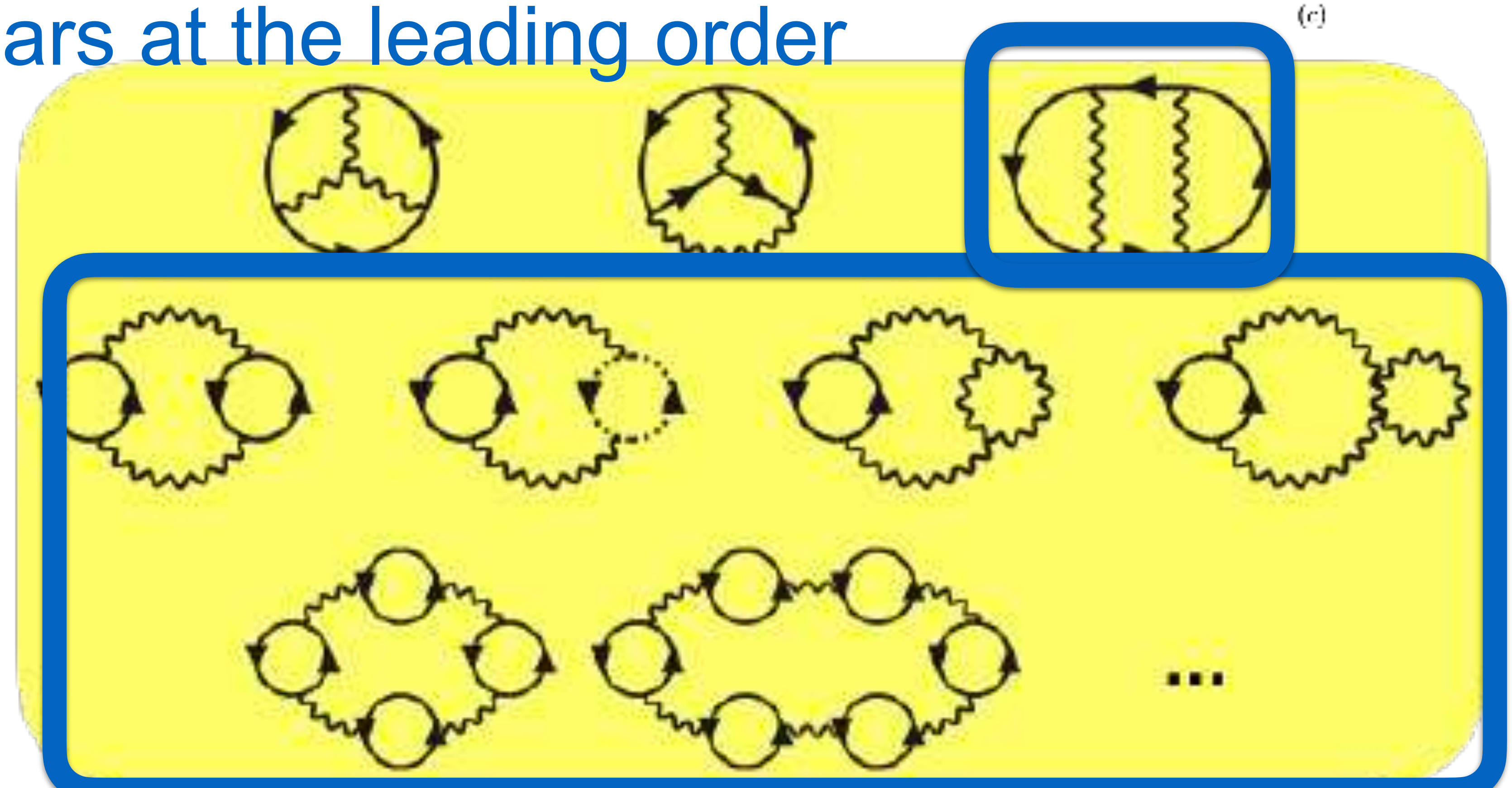


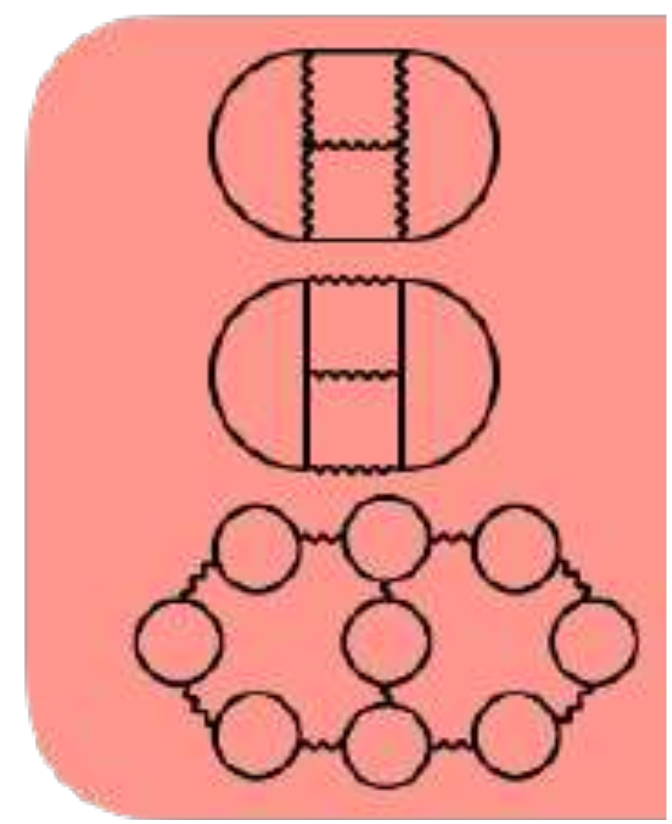


higher order diagrams already appears at the leading order

NNNLO

NLO





NNLO

Taken from A. Kurkela's slide

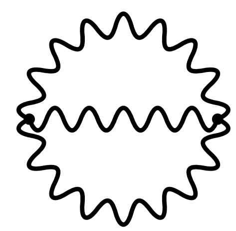
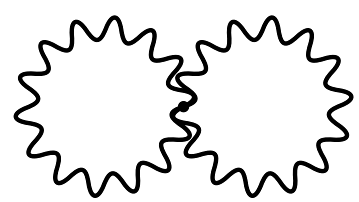
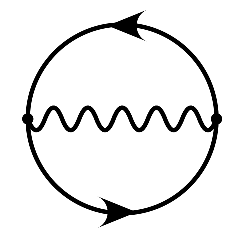
In terms of the 2PI language

- **2PI (CJT) formalism:**

Cornwall,Jackiw,Tomboulis(1974)
Blaizot,Iancu,Rebhan (1999-)

$$p[D,S] = -\frac{1}{2}\mathrm{Tr}\,\log D^{-1} + \frac{1}{2}\mathrm{Tr}\,\Pi D$$
$$+\mathrm{Tr}\,\log S^{-1} - \mathrm{Tr}\,\Sigma S - \Phi_2[D,S]$$

(D: gluon full propagator)

$\Phi[D,S] =$ -1/12  -1/8  +1/2  + ...

- **2PI formalism** employs propagator with the self-energy insertion → quasi-particle approximation

- May not be a systematic expansion in the coupling α_s

$\bar{\Lambda}$ (renormalization scale) dependence

- Use perturbative expression for screening mass m_q in Σ:

$$m_q^2 = \frac{4\alpha_s(\bar{\Lambda})}{3\pi}\mu^2 \qquad \left(\Sigma \equiv \frac{m_q^2}{k}\gamma^0 Q_0\left(\frac{k_0}{k}\right) + \frac{m_q^2}{k}\boldsymbol{\gamma}\cdot\hat{\boldsymbol{k}}\left[1 - \frac{k_0}{k}Q_0\left(\frac{k_0}{k}\right)\right]\right)$$

- Renormalization scale $\bar{\Lambda}$ dependence enters the theory through the running coupling $\alpha_s(\bar{\Lambda})$:

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \log(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)} \times \left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}{\log(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}\right]$$

$$(\Lambda_{\overline{\rm MS}} = 378\,\text{MeV})$$

- $\bar{\Lambda}$: **only undetermined const.**

  set to the typical scale of system:

$$\bar{\Lambda} = 2\mu$$

  “uncertainty” of $\bar{\Lambda}$ is evaluated as:

$$\bar{\Lambda} \in [\mu, 4\mu]$$

Outline of the talk

- Introduction

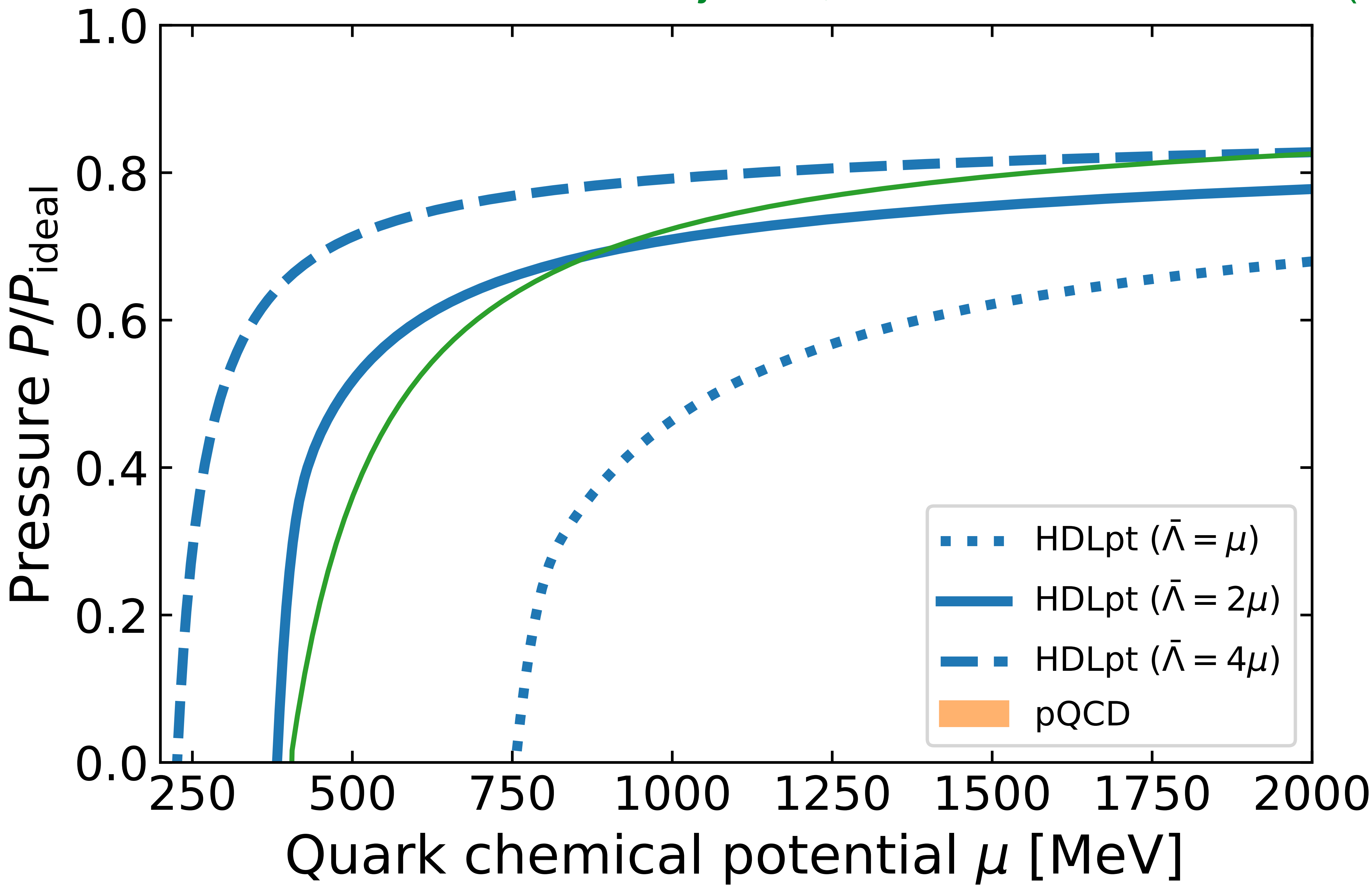
- Hard Dense Loop (HDL) resummation and the setup of calculations

- **The result for the EoS and its phenomenological implication**

- Summary

Result from the HDL resummed QCD

Fujimoto, Fukushima: 2011.10891 (2020)



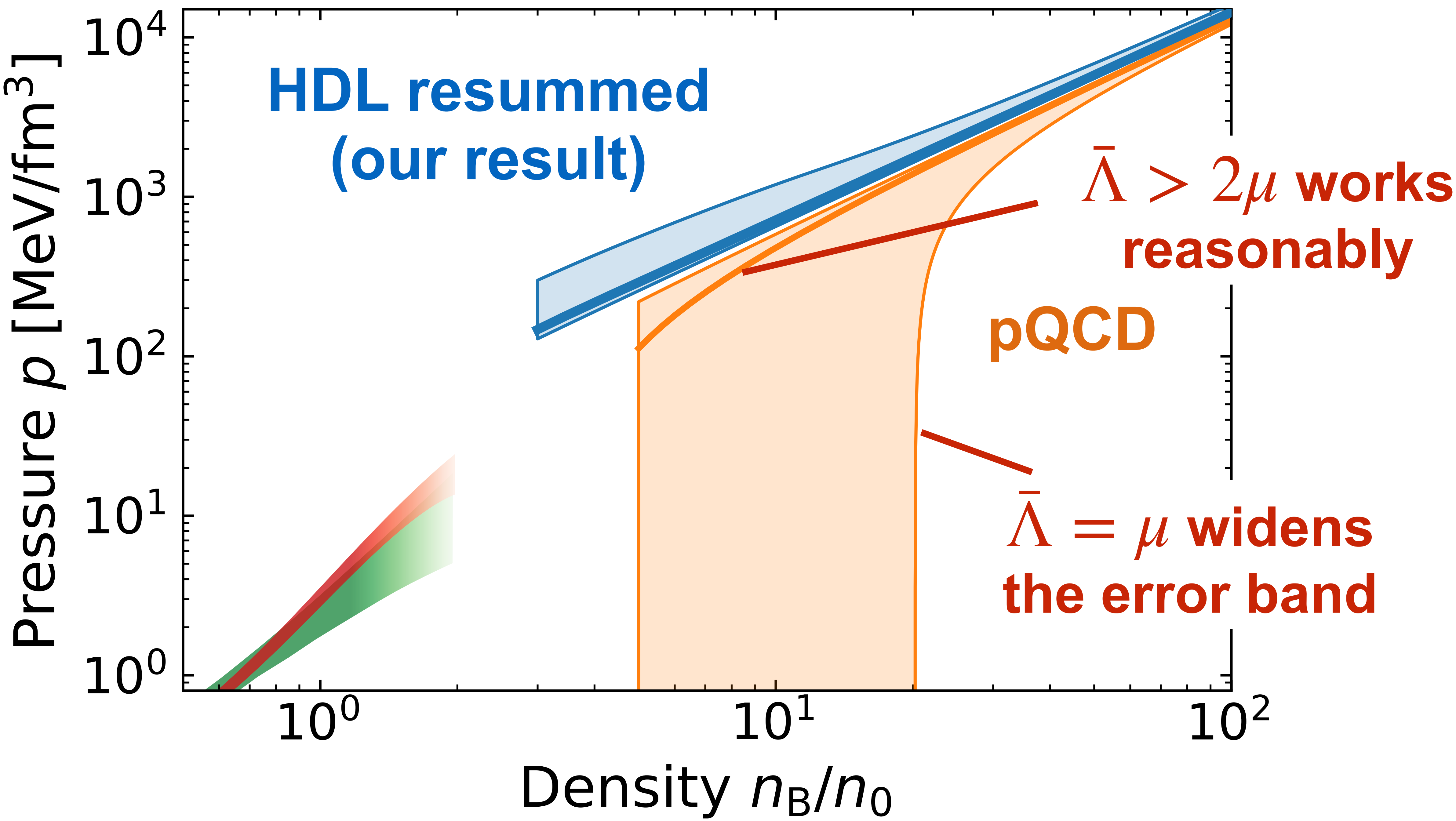
No convergence problem from the beginning.
Uncertainty seems not much improving…

Result from the HDL resummed QCD

Fujimoto, Fukushima: 2011.10891 (2020)

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \log(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}{\log(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}\right]$$

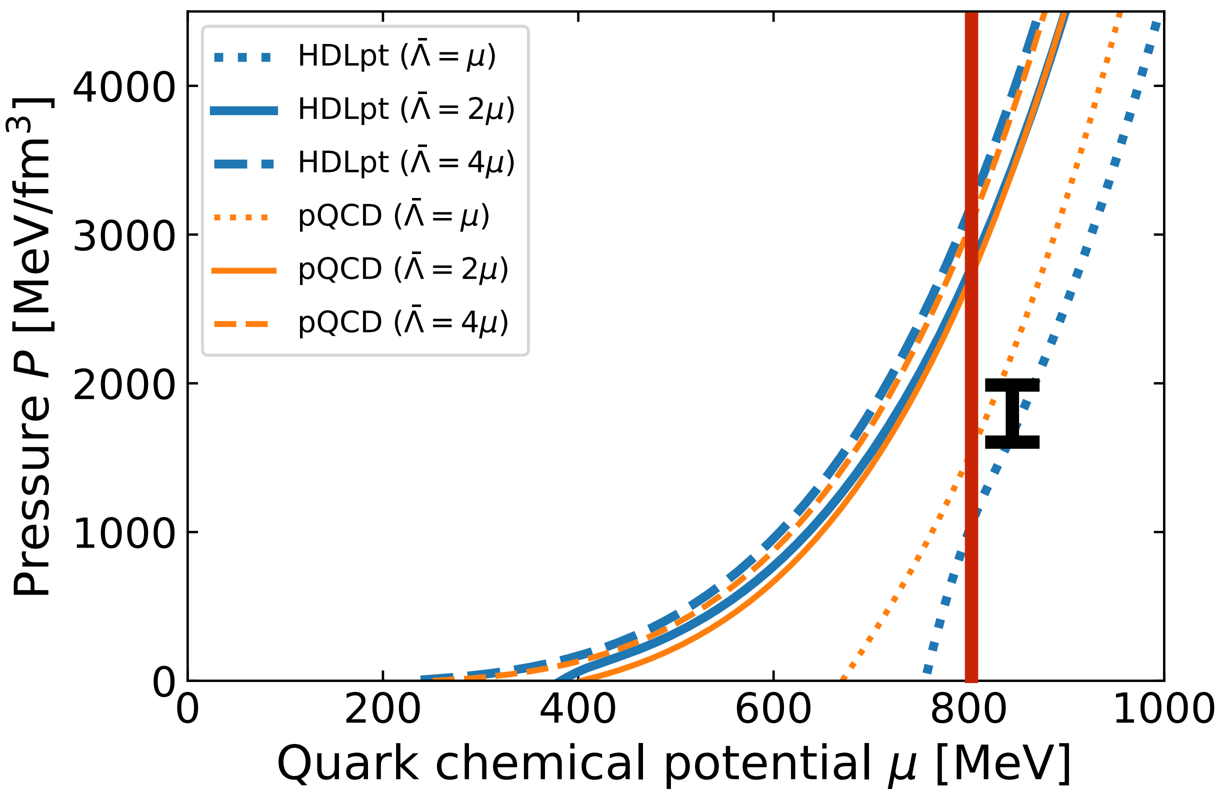
uncertainty: originate from scale variation $\bar{\Lambda} = \mu,\ 2\mu,\ 4\mu$



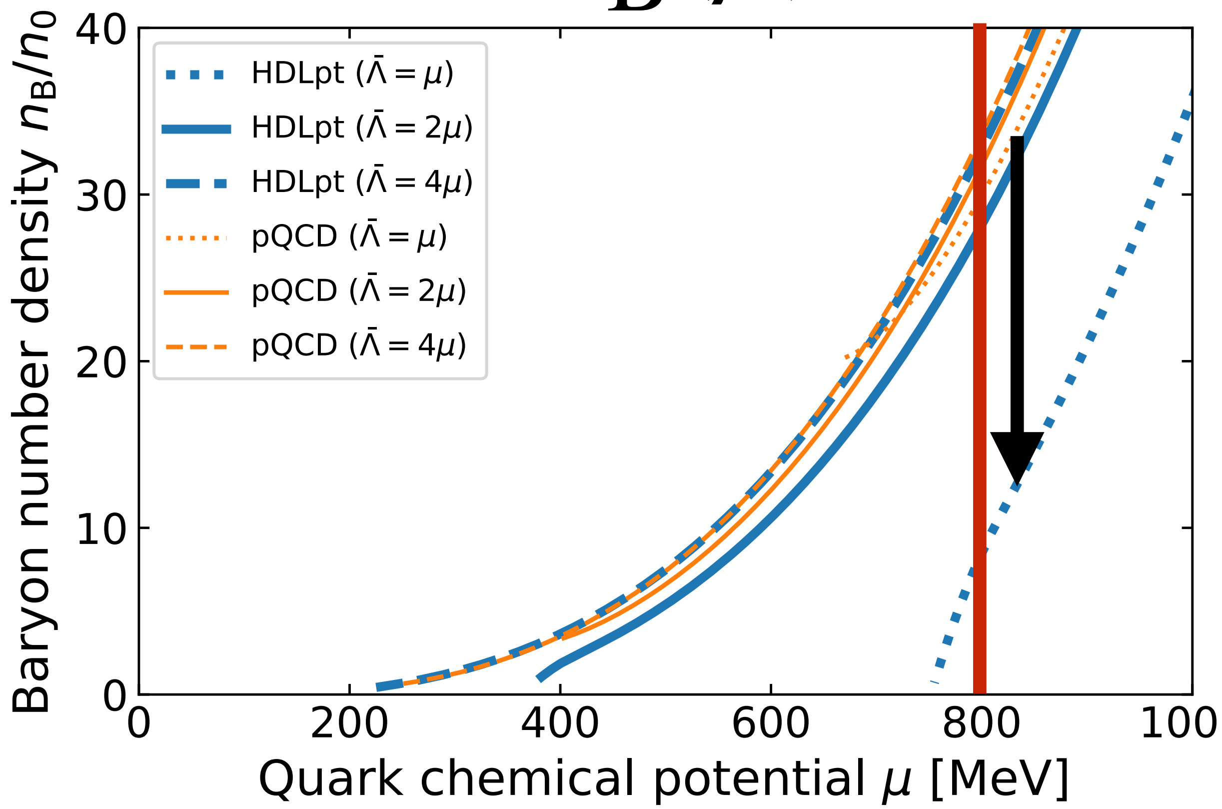
Heuristic argument

Fujimoto, Fukushima: 2011.10891 (2020)

$p(\mu)$



$n_\mathrm{B}(\mu)$



Pressure does not differ at constant μ

Density is screened in HDL resummation at constant μ

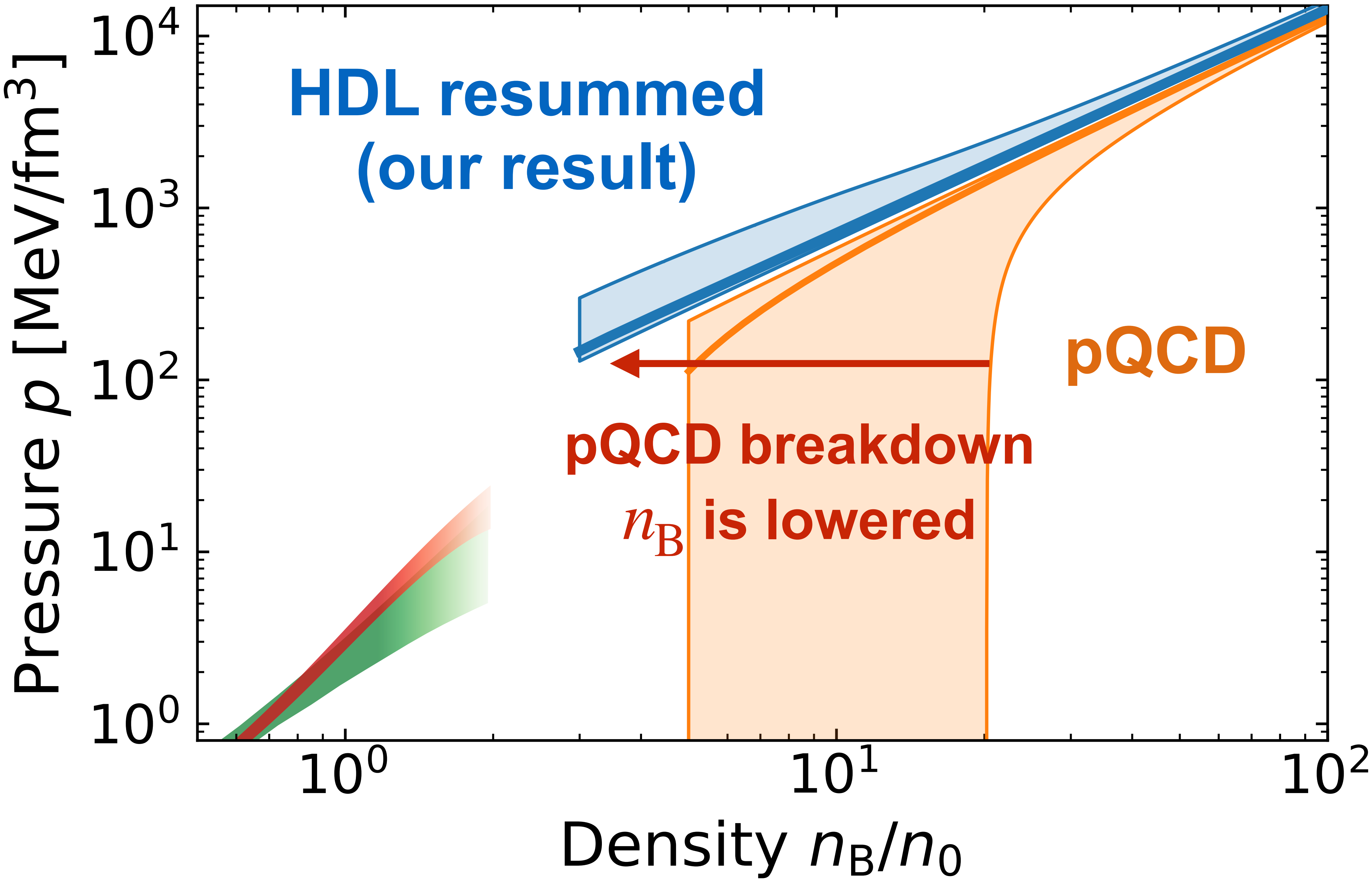
$\rightarrow$ in HDL resummation,
the same value of p realizes at lower n_B
especially for $\bar{\Lambda} = \mu$

Result from the HDL resummed QCD

Fujimoto, Fukushima: 2011.10891 (2020)

$$\alpha_s(\bar\Lambda) = \frac{4\pi}{\beta_0 \log(\bar\Lambda^2/\Lambda^2_{\overline{\rm MS}})}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log^2(\bar\Lambda^2/\Lambda^2_{\overline{\rm MS}})}{\log(\bar\Lambda^2/\Lambda^2_{\overline{\rm MS}})}\right]$$

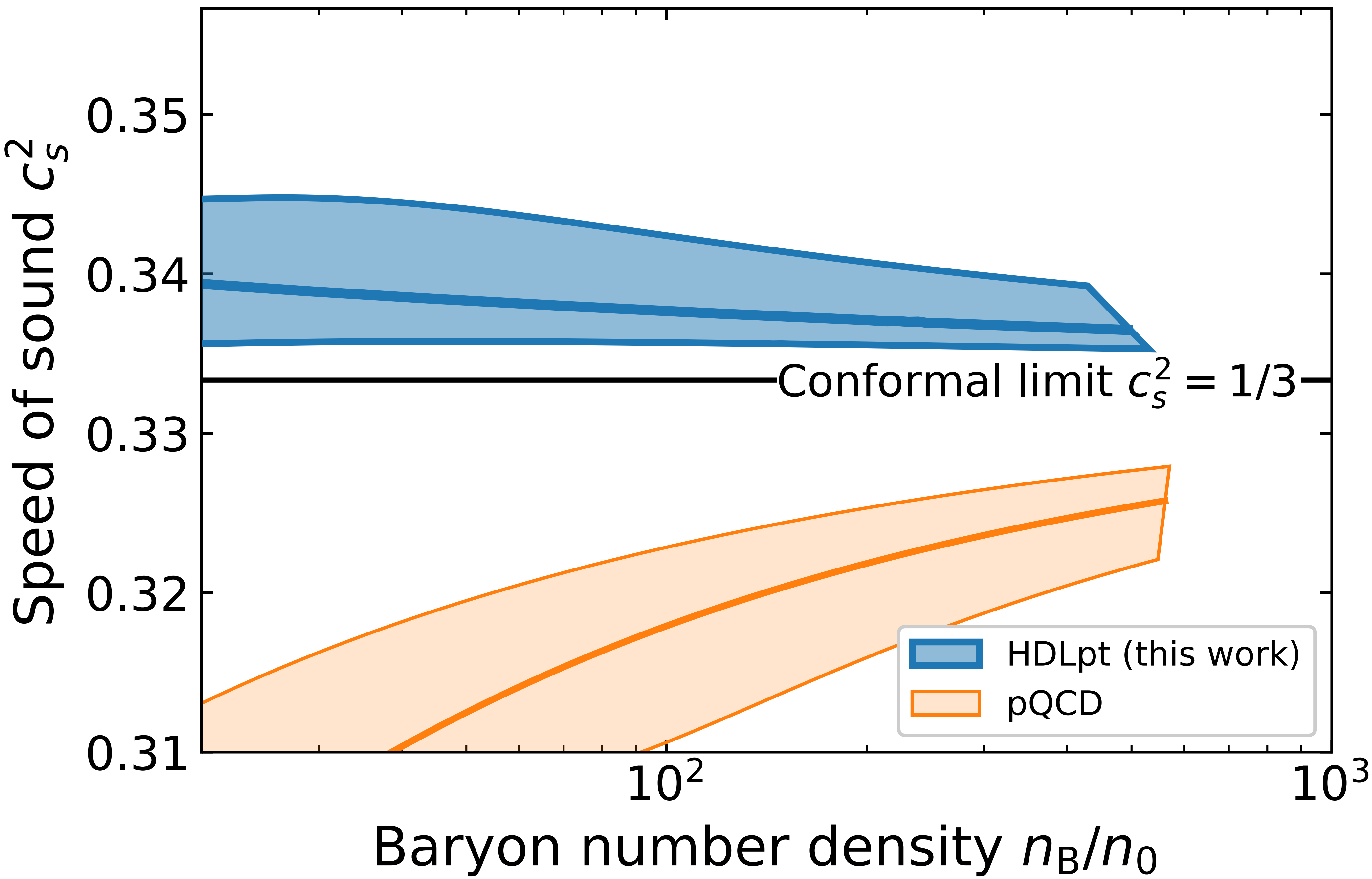
uncertainty: originate from scale variation $\bar\Lambda = \mu,\ 2\mu,\ 4\mu$



Speed of sound

Fujimoto, Fukushima: 2011.10891 (2020)

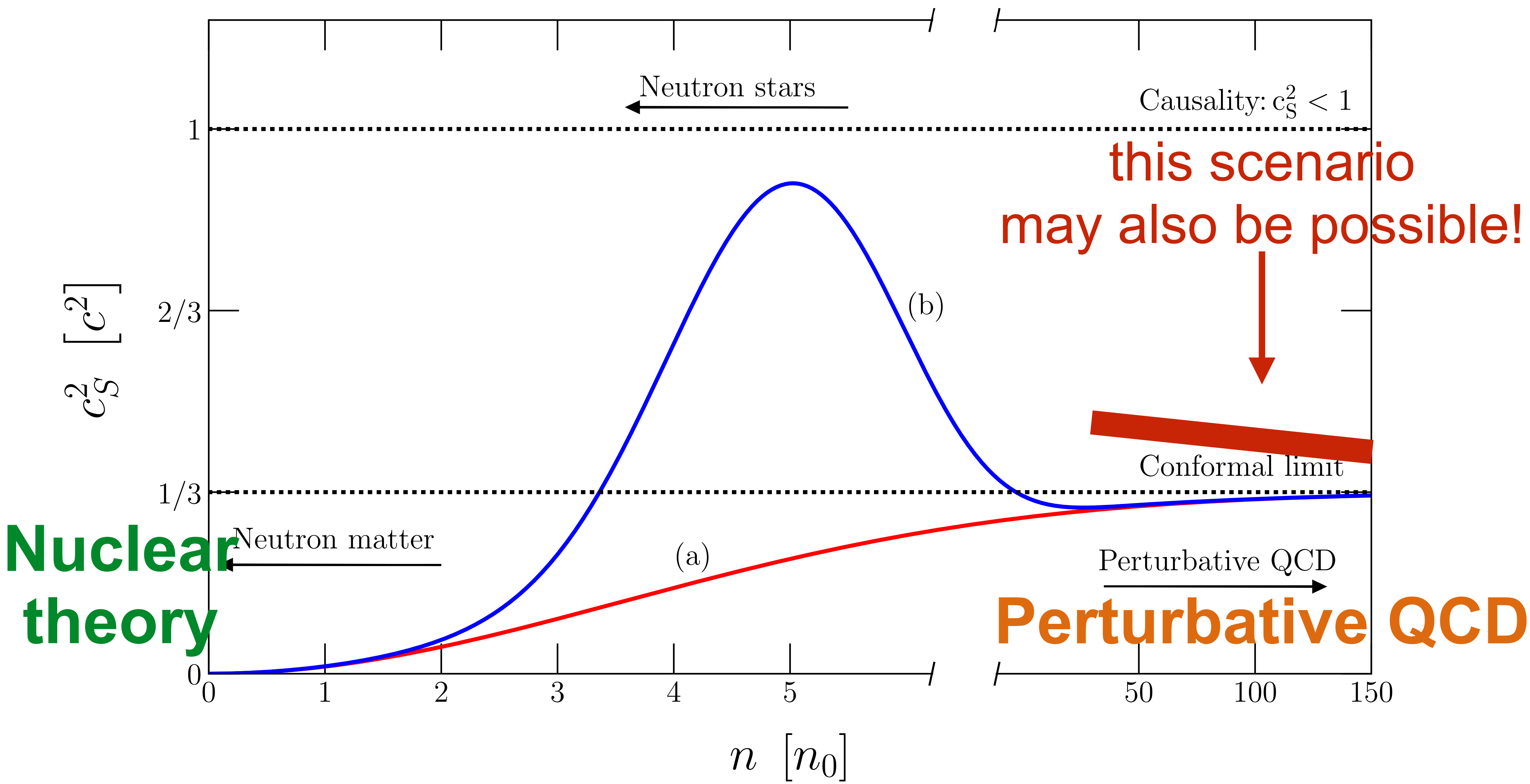
- Speed of sound: $c_s^2 = \partial p / \partial \varepsilon$;
  asymptotic freedom: $c_s^2 \to 1/3$ (**"conformal limit"**)



Speed of sound

Bedaque,Steiner (2015)
Tews,Carlson,Gandolfi,Reddy (2018)

- The commonly shown graph for speed of sound

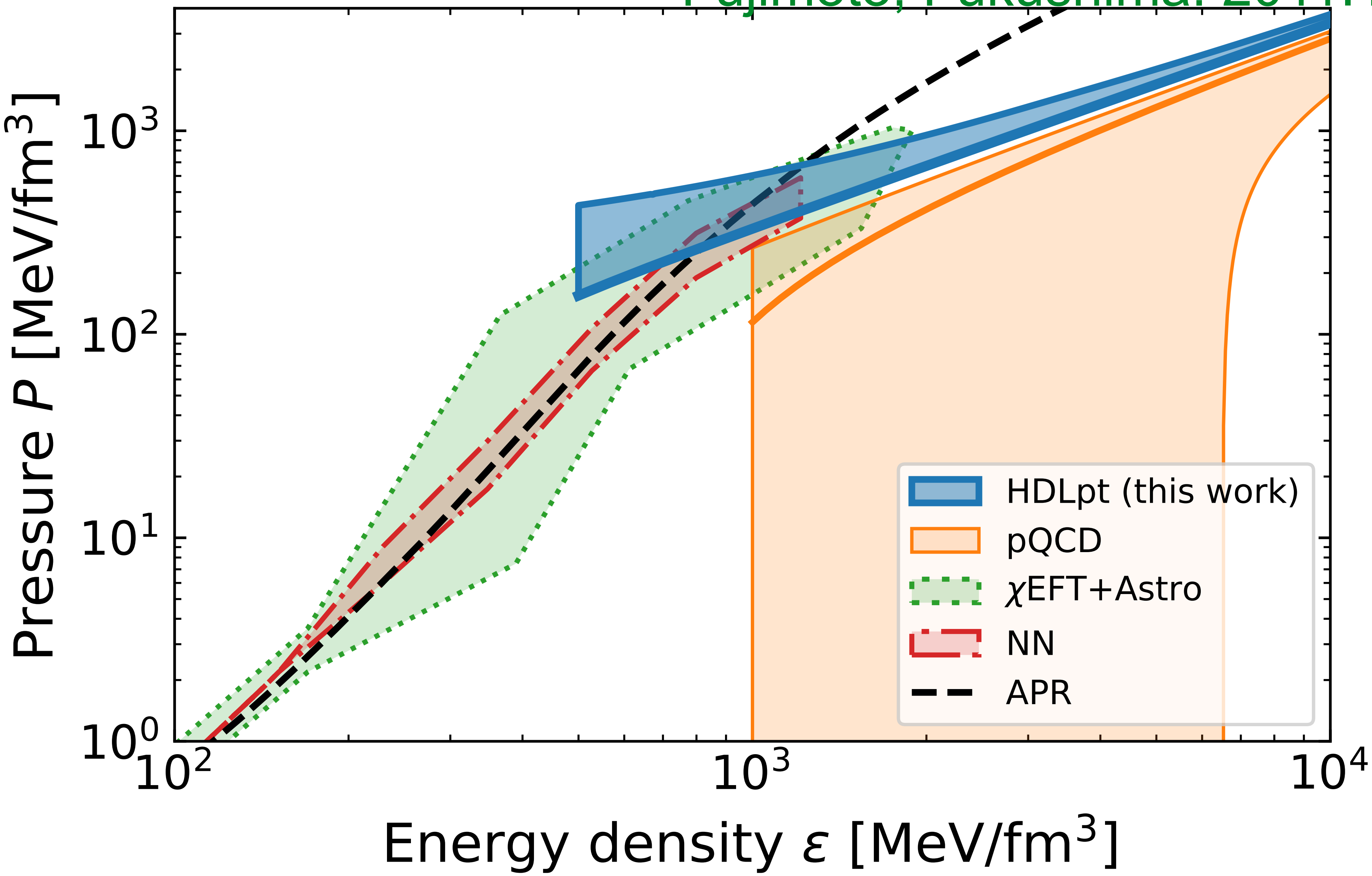


If $c_s^2 > 1/3$ at high density, then there must be a peak

Many discussions, e.g., Hippert,Fraga,Noronha (2021)

Smooth matching to the nuclear EoS?

Fujimoto, Fukushima: 2011.10891 (2020)



NN EoS extracted from X-ray NS data using neural network

Fujimoto,Fukushima,Murase (2017-)

χEFT+Astro $n_B \leq n_0$: ChEFT EoS

$n_B > n_0$: extrapolation w/ the $2M_\odot$ pulsar constraint

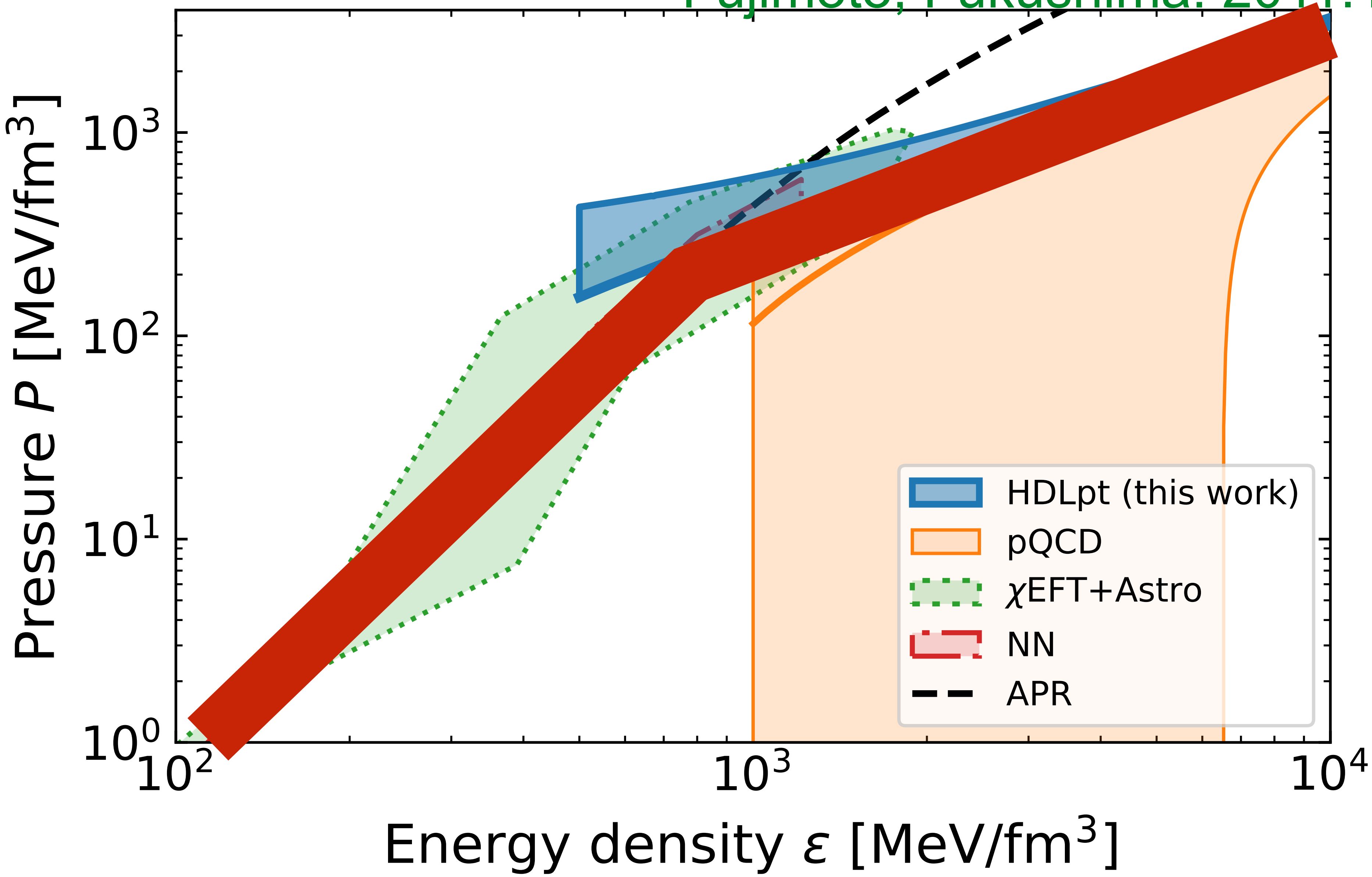
Hebeler,Lattimer,Pethick,Schwenk (2013)

APR conventional nuclear EoS

Akmal,Pandharipande,Ravenhall (1998)

Smooth matching to the nuclear EoS?

Fujimoto, Fukushima: 2011.10891 (2020)



NN EoS extracted from X-ray NS data using neural network

Fujimoto,Fukushima,Murase (2017-)

χEFT+Astro $n_B \leq n_0$: ChEFT EoS

$n_B > n_0$: extrapolation w/ the $2M_\odot$ pulsar constraint

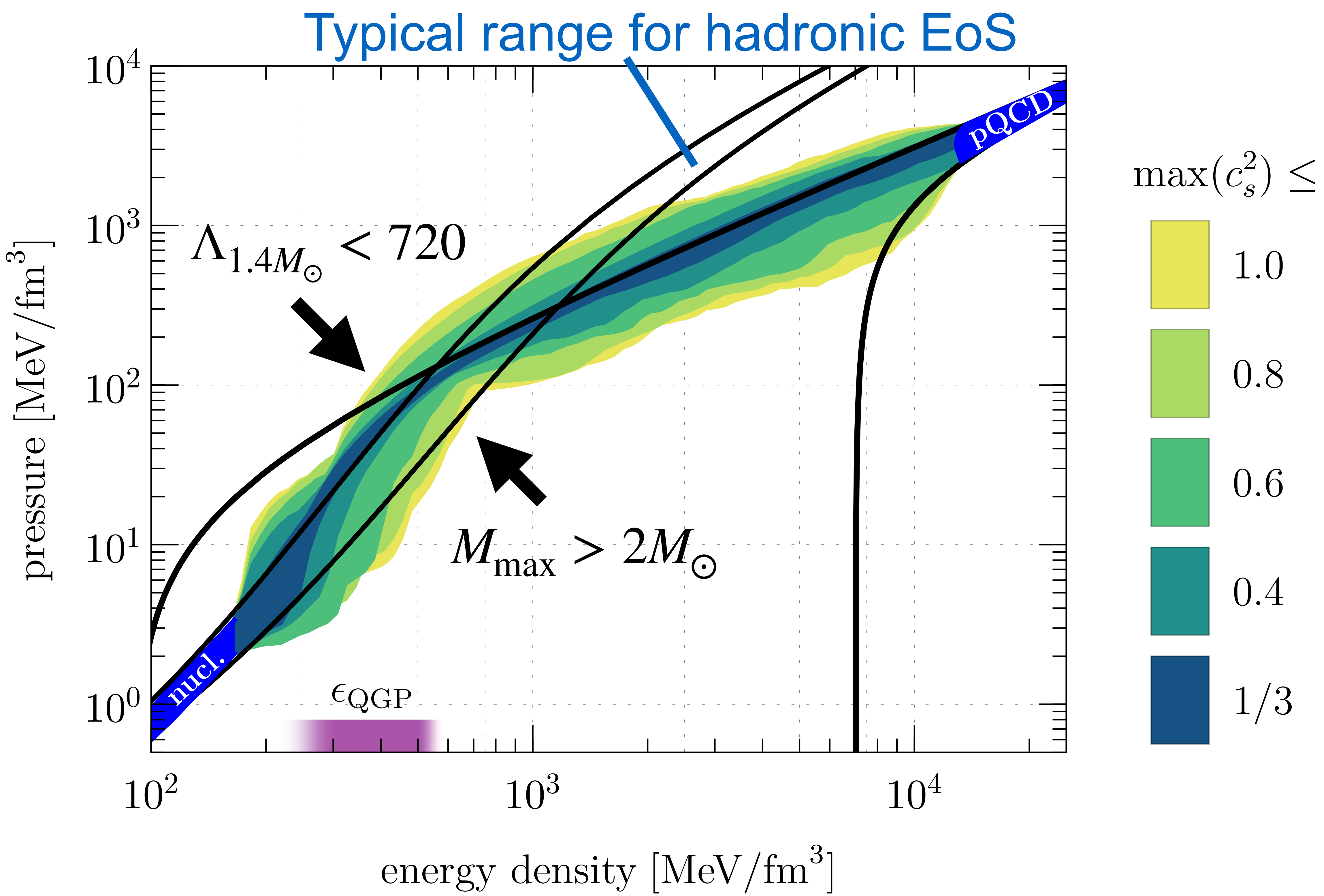
Hebeler,Lattimer,Pethick,Schwenk (2013)

APR conventional nuclear EoS

Akmal,Pandharipande,Ravenhall (1998)

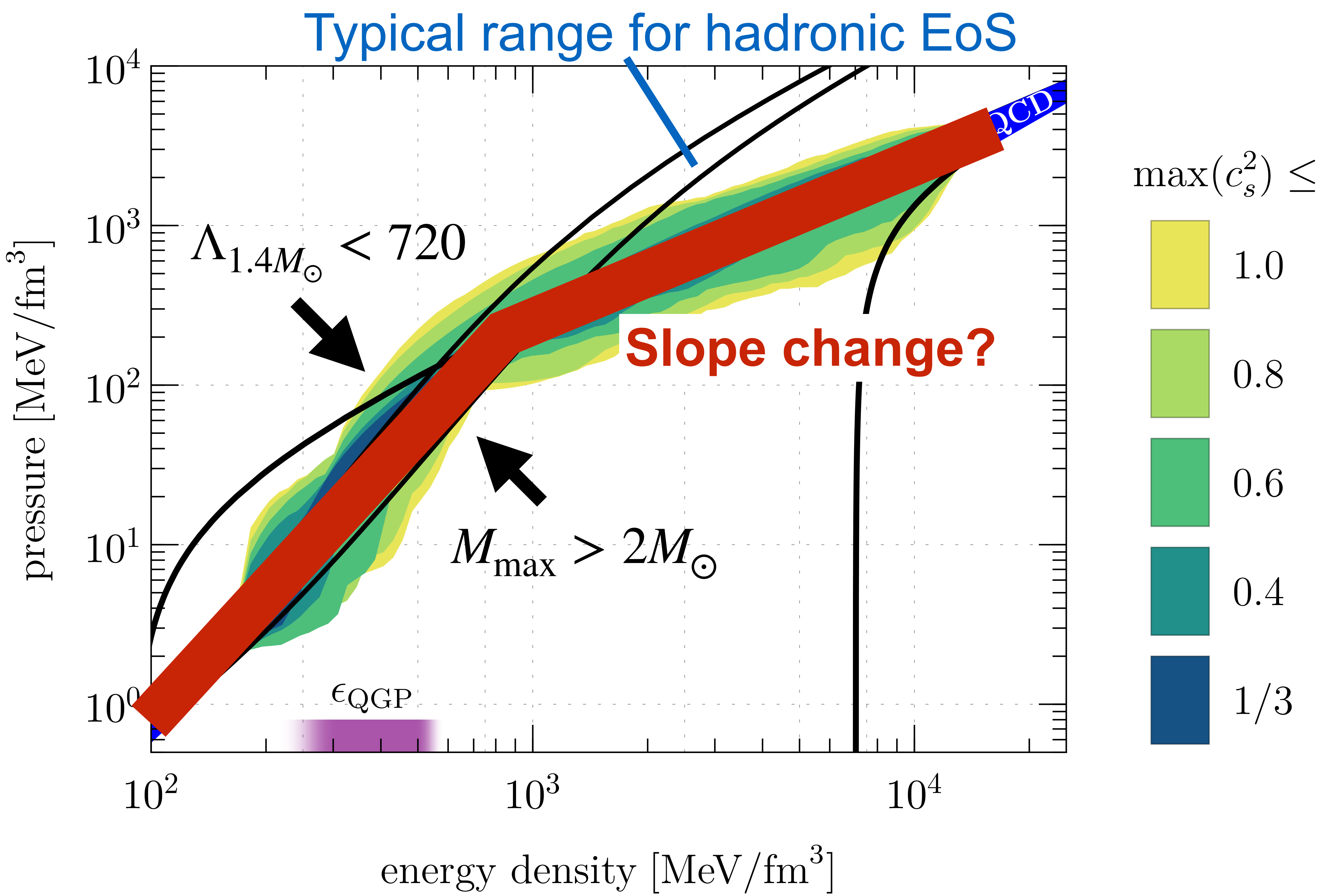
Observational constraint

Annala,Gorda,Kurkela,Nattila,Vuorinen (2019)



Observational constraint

Annala,Gorda,Kurkela,Nattila,Vuorinen (2019)



Discussion: conventional pQCD vs HDLpt

- mismatch already at the order of $\mathcal{O}(\alpha_s)$:

Baier,Redlich (1999)

$$\frac{p_{\rm HDLpt}}{p_{\rm ideal}} \simeq 1 - 2N_c\frac{m_q^2}{\mu^2} + \mathcal{O}\left(\frac{m_q^4}{\mu^4}\right) = 1 \boxed{- 4\frac{\alpha_s}{\pi}} + \mathcal{O}(\alpha_s^2)$$

$$\frac{p_{\rm pQCD}}{p_{\rm ideal}} = 1 \boxed{- 2\frac{\alpha_s}{\pi}} - \mathcal{O}(\alpha_s^2)$$

Resolving the $\mathcal{O}(\alpha_s)$ discrepancy

- mismatch already at the order of $\mathcal{O}(\alpha_s)$:

Baier,Redlich (1999)

$$\frac{p_{\rm HDLpt}}{p_{\rm ideal}} \simeq 1 - 2N_c\frac{m_q^2}{\mu^2} + \mathcal{O}\left(\frac{m_q^4}{\mu^4}\right) = 1 \boxed{- 4\frac{\alpha_s}{\pi}} + \mathcal{O}(\alpha_s^2)$$

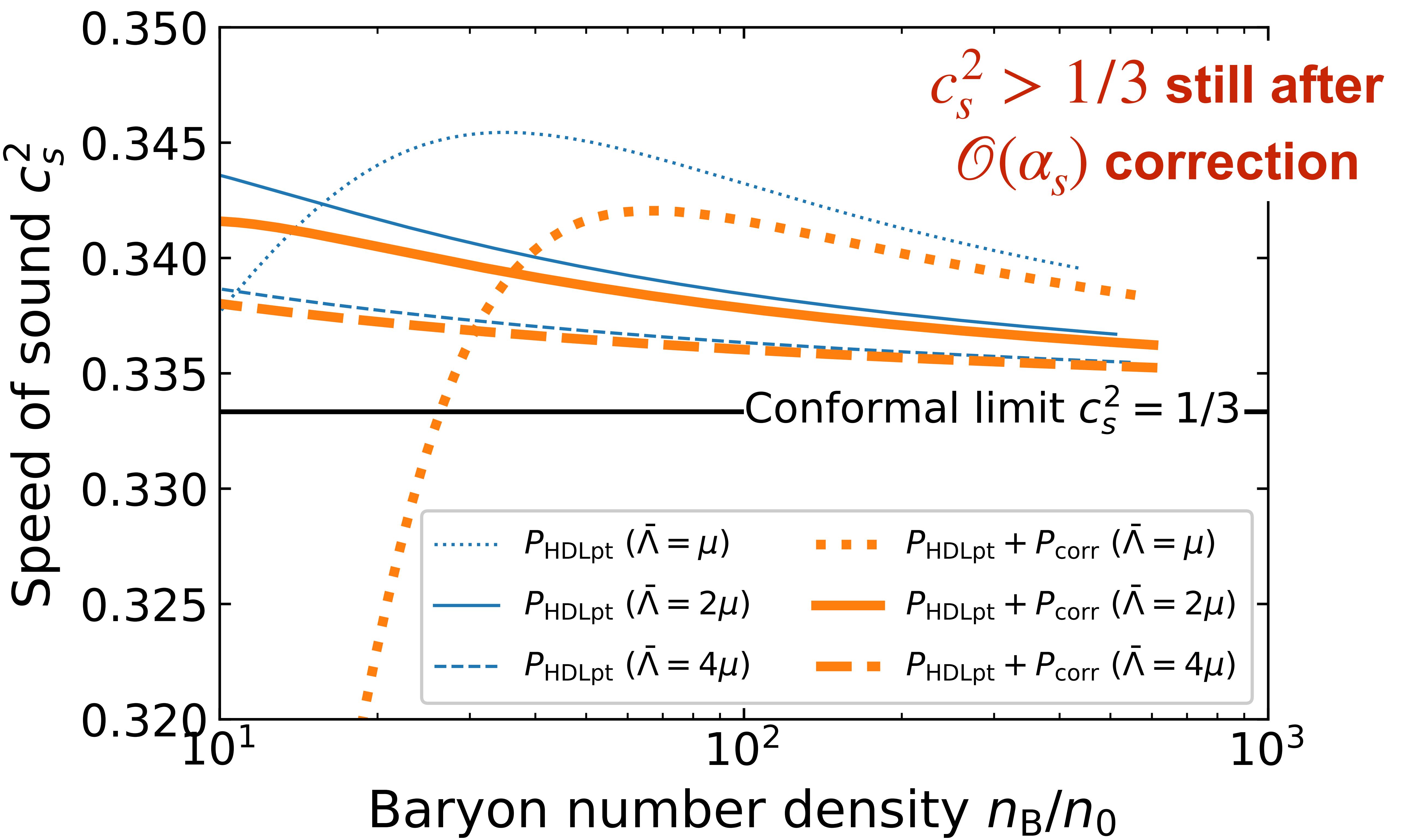
$$\frac{p_{\rm pQCD}}{p_{\rm ideal}} = 1 \boxed{- 2\frac{\alpha_s}{\pi}} - \mathcal{O}(\alpha_s^2)$$

- Add $p_{\rm corr} = 2\frac{\alpha_s}{\pi}p_{\rm ideal}$ to $p_{\rm HDLpt}$ to remedy the $\mathcal{O}(\alpha_s)$ discrepancy

- $c_s^2 < 1/3$ in the pQCD calculation is attributed to negative β-function at $\mathcal{O}(\alpha_s)$, so we care about $\mathcal{O}(\alpha_s)$ discrepancy

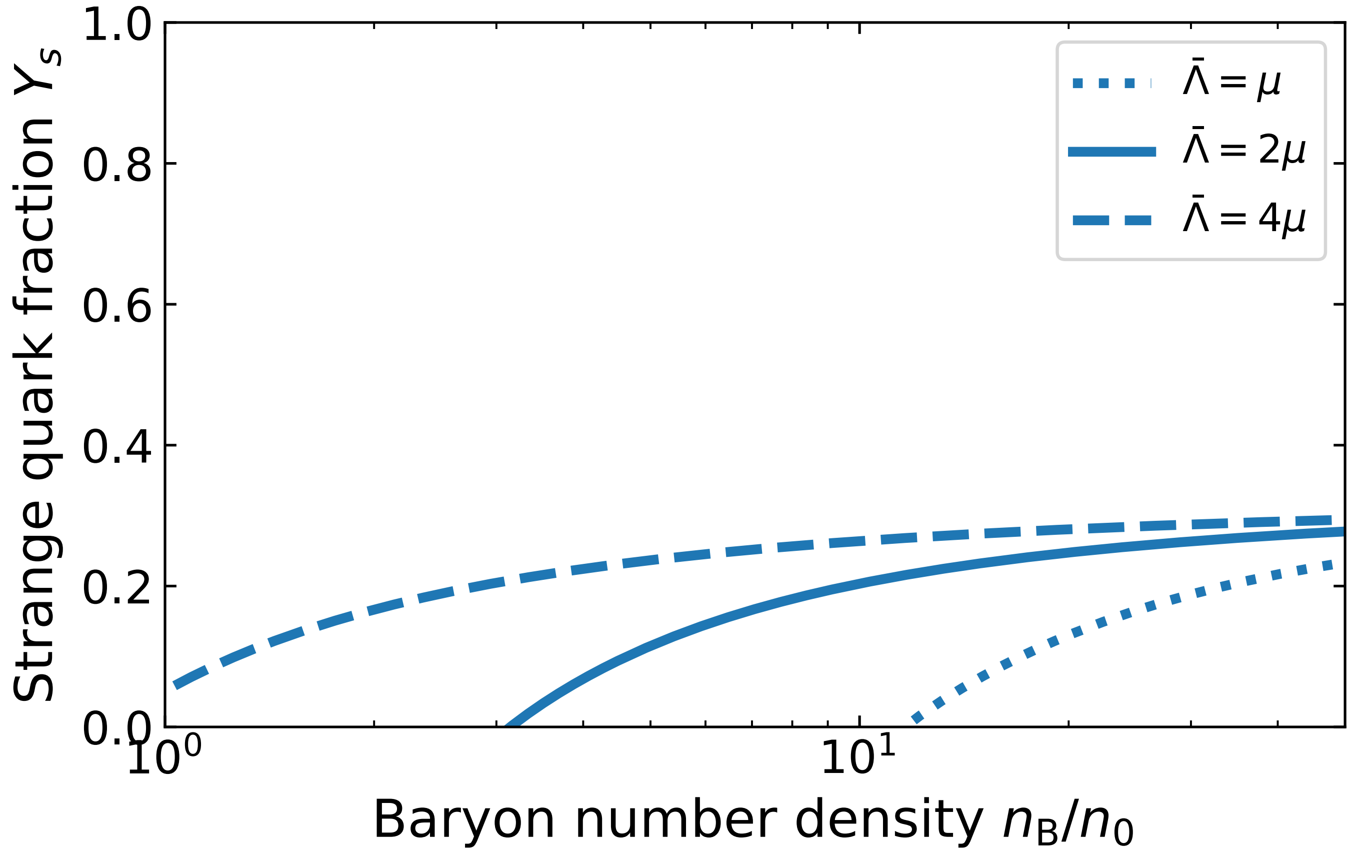
Resolving the $\mathcal{O}(\alpha_s)$ discrepancy

Fujimoto, Fukushima: 2011.10891 (2020)



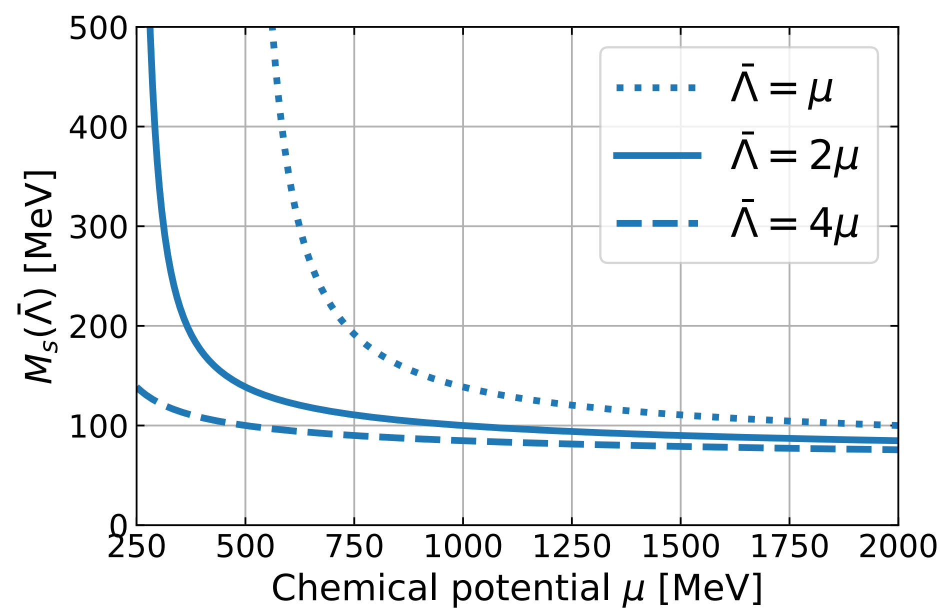
Discussion: strangeness content

- Fraction of the strange quarks:



large scale dependence…

- Threshold strongly depends on the running of the current mass of strange quarks:



Outline of the talk

- Introduction

- Hard Dense Loop (HDL) resummation and
  the setup of calculations

- The result for the EoS and
  its phenomenological implication

- **Summary**

Summary and outlook

- Resumming hard dense loops: systematic reorganization of perturbation theory, convergence improved

- The result turned out to extend the pQCD applicability down to the realistic density in neutron stars

- Several issues to be explored:
  * Deeper reason why uncertainty is smaller?
  * Evaluating higher order 2PI skeleton diagrams?
  * Multi-pronged *ab initio* approach to the EoS:
    QCD + ChEFT + NS observation + (QHC?) + …