iTHEMS Theoretical Physics Seminar @ RIKEN 2021/8/16

Application of AdS/CFT to nonequilibrium phenomena in external electric fields

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Contents

- AdS/CFT correspondence
- D3/D7 brane system
- Application1: electric field quench

JHEP 09 (2014) 126 [arXiv:1407.0798]
Phys.Lett.B 746 (2015) 311-314 [arXiv:1408.6293]
Nucl.Phys.B 896 (2015) 738-762 [arXiv:1412.4964]
K.Hashimoto, SK, K.Murata, T.Oka

- Application2: rotating electric field

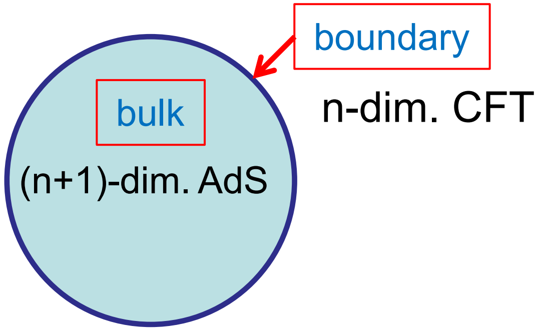
JHEP 05 (2017) 127 [arXiv:1611.03702]
K.Hashimoto, SK, K.Murata, T.Oka
JHEP 06 (2018) 096 [arXiv:1712.06786]
SK, K.Murata, T.Oka

ADS/CFT CORRESPONDENCE

AdS/CFT correspondence

- A duality relating a classical gravity in (n+1)-dim. anti-de Sitter (AdS) space and a strongly correlated conformal field theory (CFT) in n-dim.
  - Holography, the gauge/gravity duality

Maldacena (1998)



- The classical dynamics of gravity corresponds to the quantum physics of strongly correlated gauge theory
  - General relativity could describe strongly corrected quantum systems (with finite temperature), which are too difficult to solve.
  - QCD, Quark-gluon plasma(QGP), condensed matter physics, …

Anti-de Sitter (AdS) space

- Vacuum solution of the Einstein eq. with negative cosmological constant
- At infinity there is timelike boundary

bulk

$$\mathrm{d}s_{n+1}^2 = \frac{dr^2}{r^2} + r^2 \eta_{\mu\nu} dx^\mu dx^\nu$$

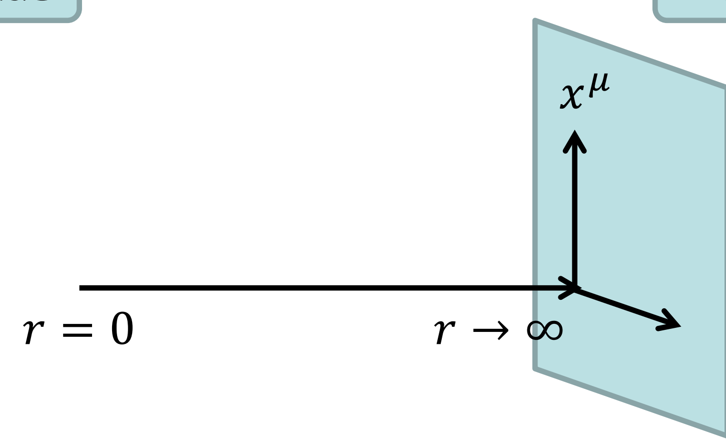
$r \to \infty$

boundary

$$\mathrm{d}s_n^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$(n+1)$-dim. AdS

n-dim. Minkowski



- Mathematical statement

$$Z_{\text{boundary}}[J] = \exp(iS_{\text{bulk}}[\Phi_{\text{cl}}])$$

$$J(x) = \Phi_{\text{cl}}(r,x)|_{r=\infty}$$

Partition function of CFT: $Z_{\text{boundary}}[J] \equiv \int \mathcal{D}\phi \exp\left[iS_{\text{CFT}} + i\int \mathrm{d}^4 x J\mathcal{O}\right]$

$$\langle \mathcal{O}(x) \rangle = -i\frac{\delta}{\delta J(x)} \log Z[J] = \frac{\delta S_{\text{bulk}}[\Phi_{\text{cl}}]}{\delta \Phi_{\text{cl}}|_{r=\infty}}$$

Ex. Gravitational field

$$\langle T_{\mu\nu}^{\text{CFT}} \rangle = \frac{\delta S_{\text{bulk}}[g_{\mu\nu}^{\text{cl}}]}{\delta g_{\mu\nu}^{\text{cl}}|_{r=\infty}}$$

field theory gravity

$$g_{\mu\nu}(r,x) = \eta_{\mu\nu} + \frac{1}{r^4}\langle T_{\mu\nu}^{\text{CFT}} \rangle + \cdots$$

$$\mathrm{d}s^2 = \frac{dr^2}{r^2} + r^2 g_{\mu\nu}(r,x)\mathrm{d}x^\mu \mathrm{d}x^\nu$$

Brief manual

- Field theory

$$J(x)$$

External source

- Classical gravity

$$\Phi(r,x)|_{r=\infty} = J(x)$$

Boundary condition of bulk field

Solve classical EoM

$$\Phi(r,x)$$

Bulk solution

$$\langle O(x) \rangle_J$$

Expectation value of response

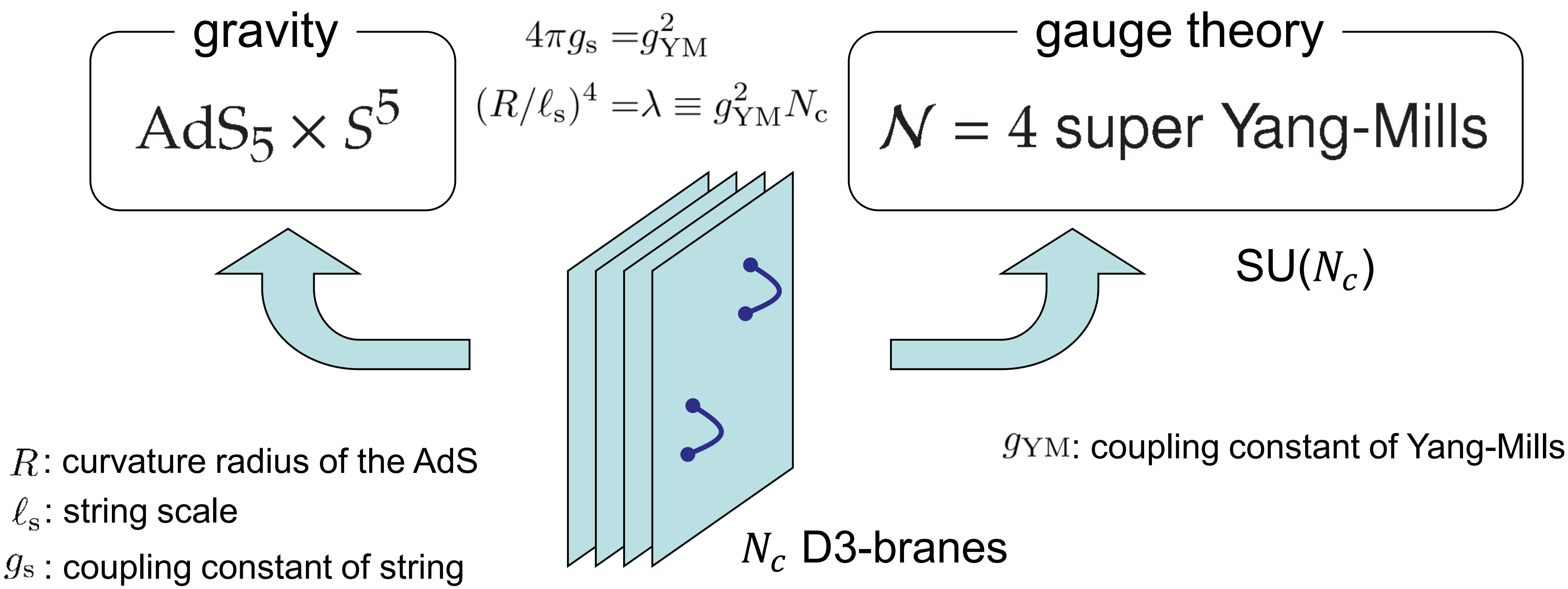
Expand it around AdS boundary ($r = \infty$)

$$\Phi(r,x) = J(x) + \frac{\langle O(x) \rangle_J}{r^\Delta} + \cdots$$

D3/D7 BRANE SYSTEM

String theoretical viewpoint

- $\mathrm{AdS}_5/\mathrm{CFT}_4$



Strongly coupled gauge theory corresponds to classical gravity

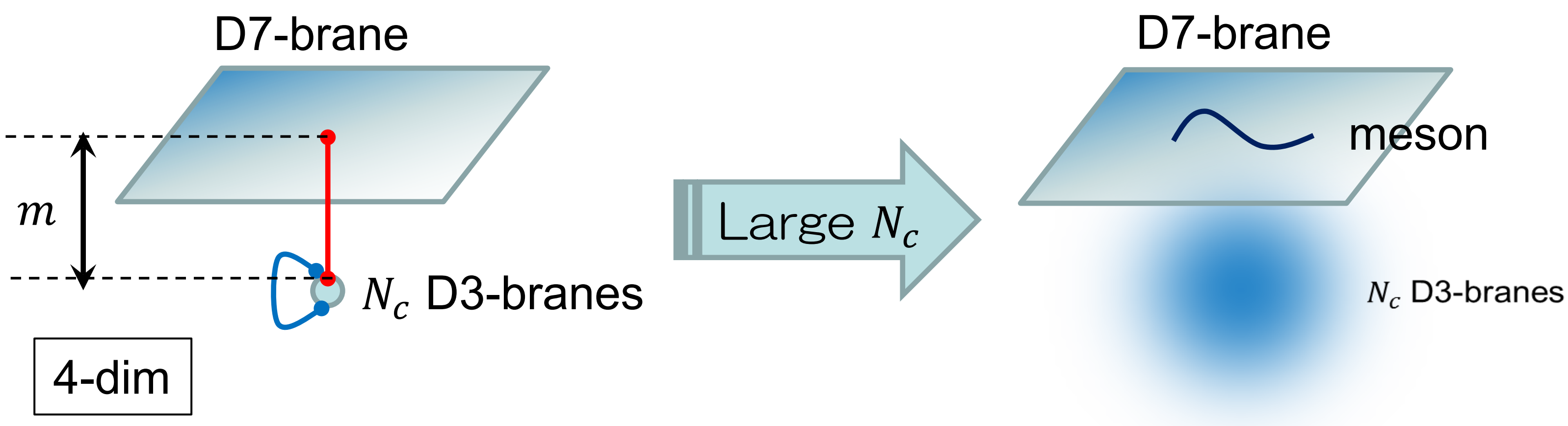
A new type brane will be added for quark degrees of freedom

Holographic QCD constructed by D3/D7

Karch, Katz (2002), Grana, Polchinski (2002), Bertolini *et al.* (2002)

10-dim. flat spacetime

10-dim. $AdS_5 \times S^5$



N=4 super Yang-Mills + N=2 quark multiplet

A probe D7-brane is embedded in $AdS_5 \times S^5$

$$ds^2 = r^2[-dt^2 + d\vec{x}_3^2] + \frac{1}{r^2}[d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + d\bar{w}^2]$$

DBI action: $S_{D7} = -\mu_7 \int d^8 y \sqrt{-\det\left(g_{ab} + 2\pi\alpha' F_{ab}\right)}$

Embedding function
$w(\rho) = m\,(\text{const.})$

quark mass

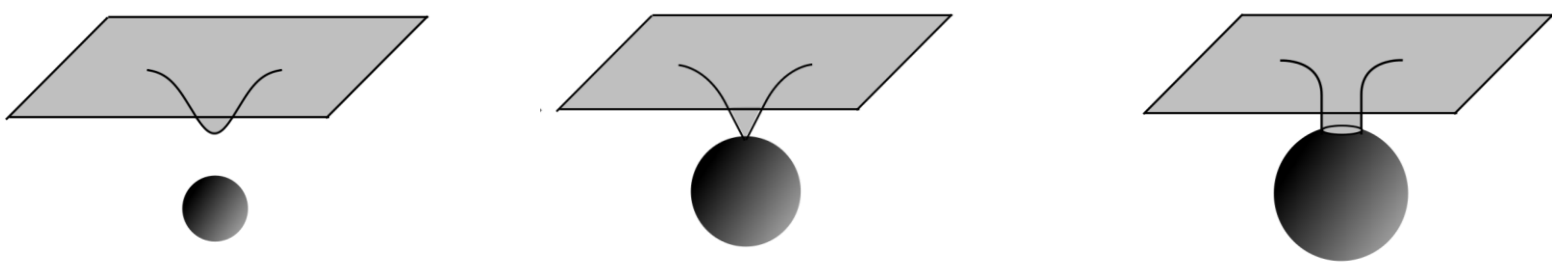
string between D3-branes and D7-brane ⇔ “quark”
fluctuations of D7-brane ⇔ “meson”

Phase transition in the D3/D7 system

- If we introduce a worldvolume gauge field living on the D7 brane or a black hole in bulk spacetime, topology of the brane configuration can change

Electric field or black hole become larger

Minkowski embedding | Critical embedding | Black hole embedding



Fluctuations are normal modes with real frequencies (confined)

Fluctuations are quasi-normal modes with complex frequencies (absorbed into a horizon)

Confinement/deconfinement in the meson sector

Mateos, Myers, Thomson (2006, 2007)

- If one includes electric field or finite temperature in the system, the phase transition occurs

Gravity side	Gauge theory side
Black hole in the bulk or Gauge field on the brane	Finite temperature in the gluon sector or Finite electric field
The brane is bending	Expectation values change

$$w(\rho) \sim m + \frac{c}{\rho^2} + \cdots ,$$

$$a_x(\rho) \sim -E_x t + \frac{j_x}{2\rho^2} + \cdots ,$$

$\langle \bar{\psi}\psi \rangle \propto c$: quark condensate

$\langle \bar{\psi}\gamma_\mu\psi \rangle \propto j_\mu$: electric current

m:quark mass, E_x:electric field

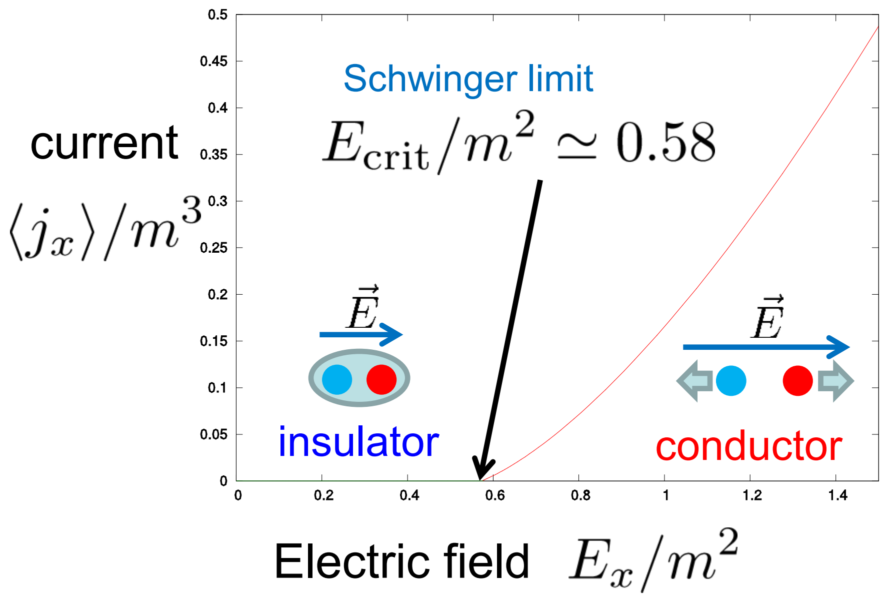
The brane intersects a horizon or not	Deconfinement or confinement
The fluctuations dissipate or are confined (quasi-normal modes or normal modes)	Mesons are unstable or stable

Electric field case

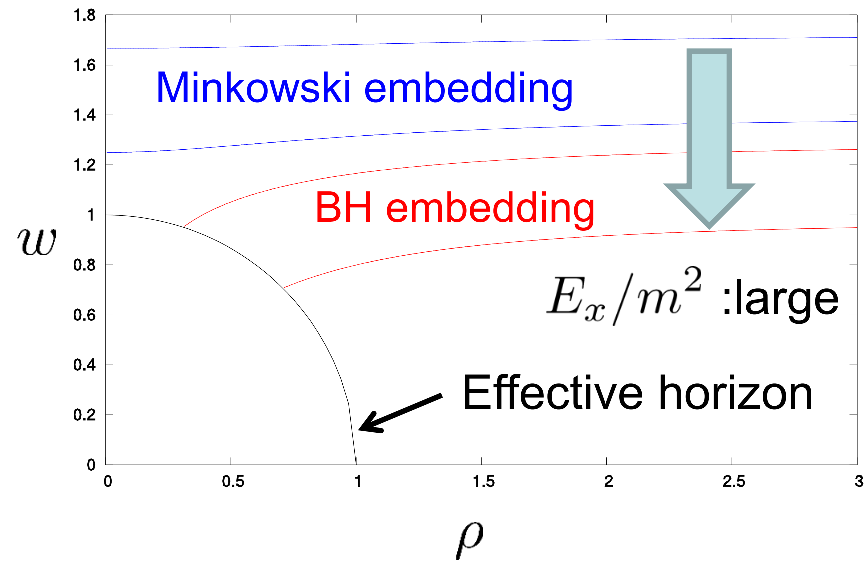
Karch, O'Bannon (2007)
Erdmenger, Meyer, Shock (2007)
Albash, Filev, Johnson, Kundu (2007)

- Schwinger effect

Electric field vs current in the boundary



Profile of the brane in the bulk



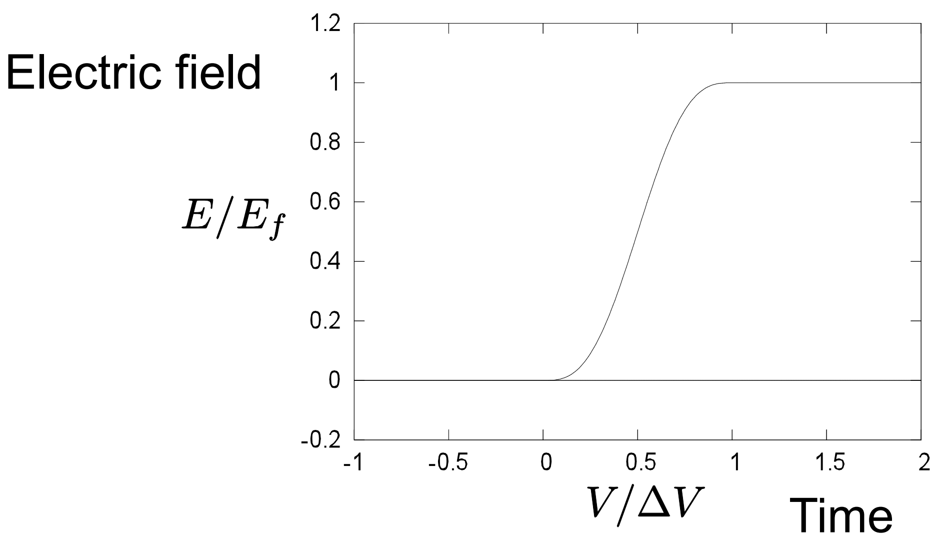
$$ds^2 = r^2[-dt^2 + d\vec{x}_3^2] + \frac{1}{r^2}[d\rho^2 + \rho^2 d\Omega_3^2 + dw^2 + d\bar{w}^2]$$

Beyond the critical electric field, an effective horizon emerges on the brane
The electric current becomes non-zero value = Schwinger effect

APPLICATION1: ELECTRIC FILED QUENCH

Time-dependent phenomena

- We want to consider non-equilibrium phenomena beyond steady sates
  - Suddenly applied electric fields



E_f: electric field in the final state
ΔV: duration of the ramp

We should solve EoM of the D7-brane imposing time-dependent boundary conditions

Our setup: time-dependent electric field

Hashimoto, SK, Murata, Oka JHEP 09 (2014) 126

- Bulk spacetime
  - AdS$_5\times S^5$

$$ds^2 = \frac{1}{z^2}[-dV^2 - 2dVdz + dx^2 + d\vec{x}_2^2] + d\phi^2 + \cos^2\phi d\Omega_3^2 + \sin^2\phi d\psi^2$$

- D7-brane

The brane is symmetric in $\vec{x}_3, \Omega_3$-directions

  - Embedding function :

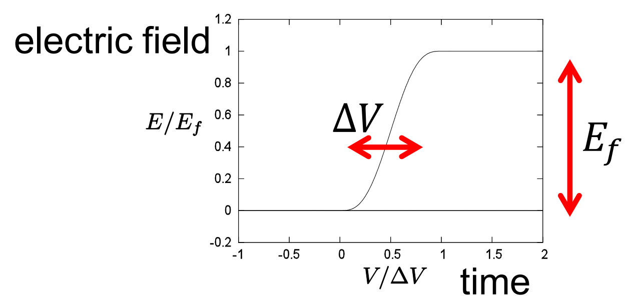
$$V = V(u,v), \quad z = Z(u,v), \quad \phi = \Phi(u,v), \quad \psi = 0$$

  - Gauge field : $2\pi\alpha' A_a dy^a = a_x(u,v)dx$

Boundary conditions of a_x at the AdS boundary = electric field in the boundary theory

$$a_x(Z=0,V) \equiv -\int^V dV' E(V')$$

$$E(V) = \begin{cases} 0 & (V<0) \\ E_f[V - \frac{\Delta V}{2\pi}\sin(2\pi V/\Delta V)]/\Delta V & (0 \le V \le \Delta V) \\ E_f & (V > \Delta V) \end{cases}$$



Equations of motion of the brane

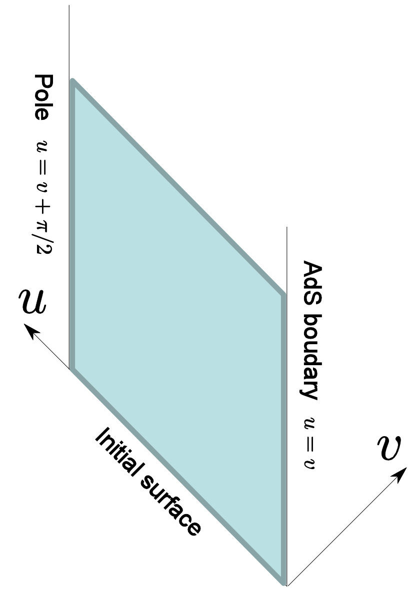
- DBI action

$$S[V,Z,\Phi,a_x] = -\int d^8\sigma \sqrt{-\det(h_{ab}+f_{ab})} \propto \int dudv \frac{\cos^2\Phi}{Z^3}\sqrt{\xi},$$

$$\xi \equiv (h_{uv} + Z^2\partial_u a_x \partial_v a_x)^2 - (h_{uu} + Z^2(\partial_u a_x)^2)(h_{vv} + Z^2(\partial_v a_x)^2)$$

$$h_{uv} = -Z^{-2}(V_{,u}V_{,v} + V_{,u}Z_{,v} + Z_{,u}Z_{,v}) + \Phi_{,u}\Phi_{,v},$$

$$h_{uu} = -Z^{-2}V_{,u}(V_{,u} + 2Z_{,u}) + \Phi_{,u}^2, \quad h_{vv} = -Z^{-2}V_{,v}(V_{,v} + 2Z_{,v}) + \Phi_{,v}^2$$



Numerical domain

Coordinate conditions: $\boxed{C_1 \equiv h_{uu} + Z^2(\partial_u a_x)^2 = 0,}$

Double-null coordinates $\boxed{C_2 \equiv h_{vv} + Z^2(\partial_v a_x)^2 = 0}$

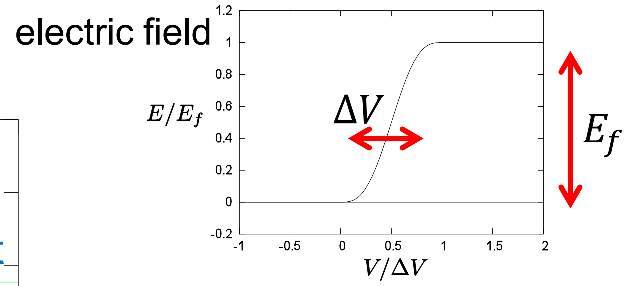
$$\partial_u\partial_v \mathbf{X} + \mathbf{f}(\mathbf{X}, \partial_u\mathbf{X}, \partial_v\mathbf{X}) = 0 \qquad \mathbf{X} = (V, Z, \Phi, a_x)$$

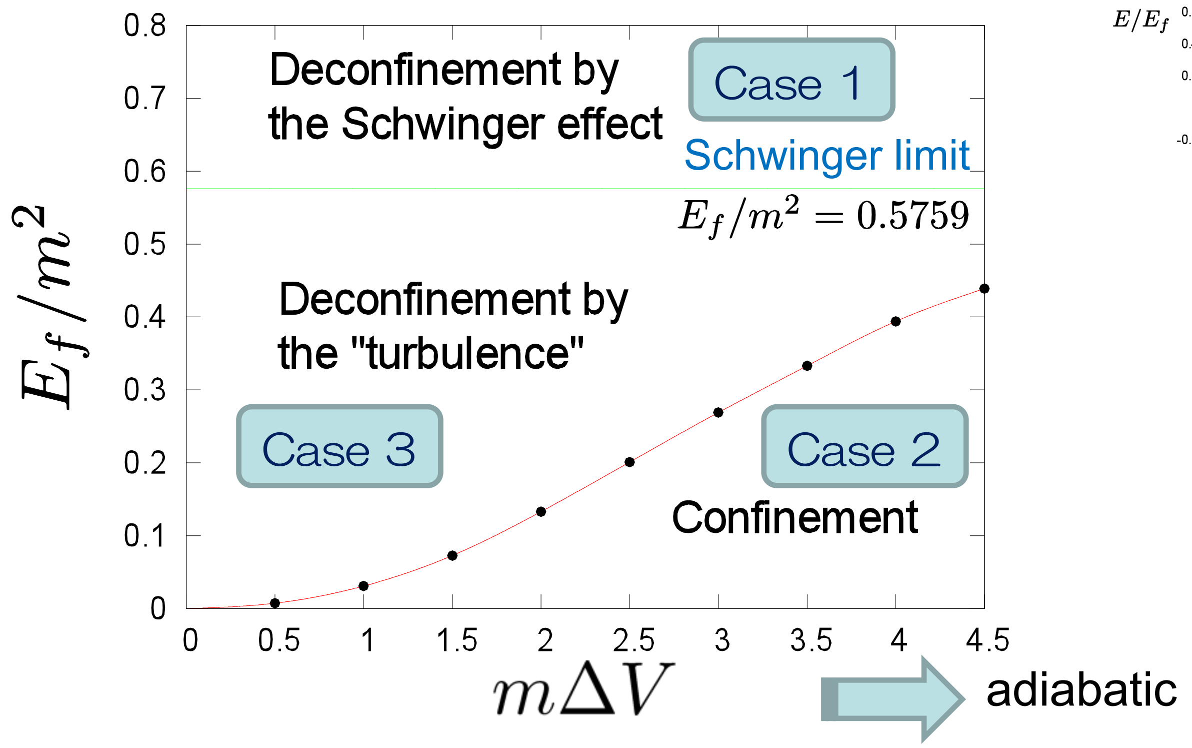
2-dimensional non-linear wave equations on an effective metric

Effective metric: $\gamma_{ab} \equiv h_{ab} + f_{ac}f_{bd}h^{cd}$

Numerical Results

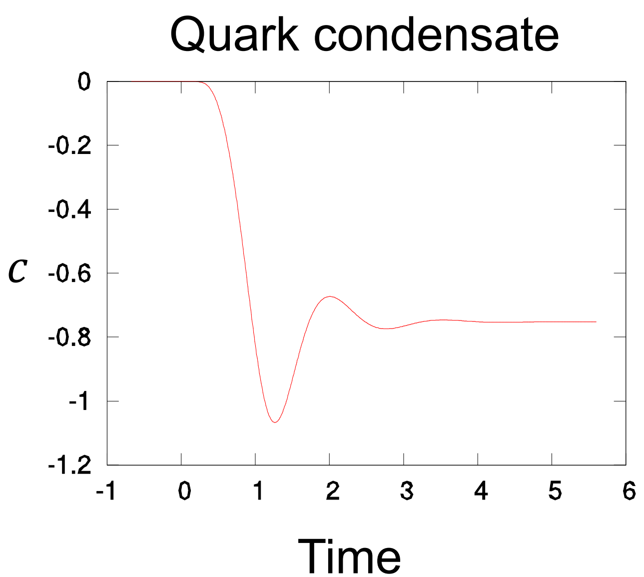
- Dynamical phase diagram
  - We found three cases depending on two model parameters: E_f and ΔV

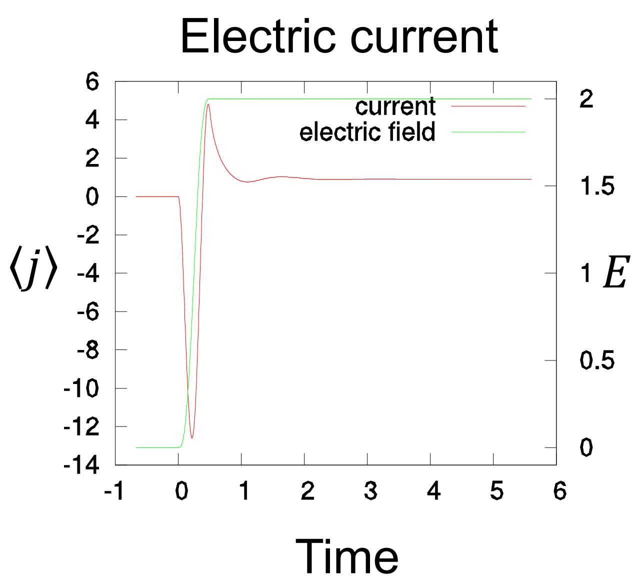


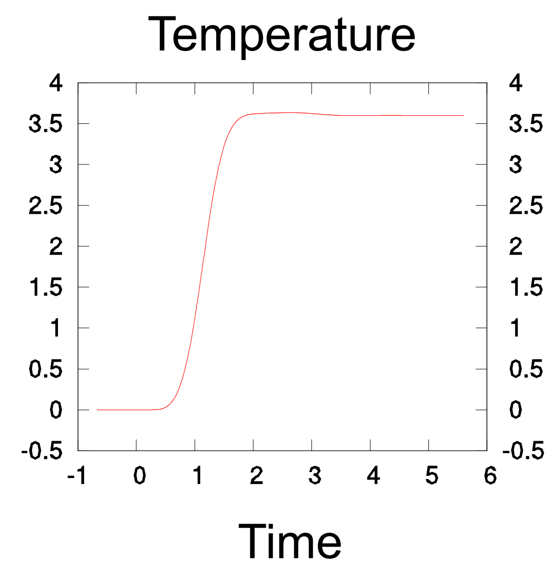


Case 1
super-Schwinger-limit

- Strong electric field $(E_f = 2.0, \Delta V = 0.50)$





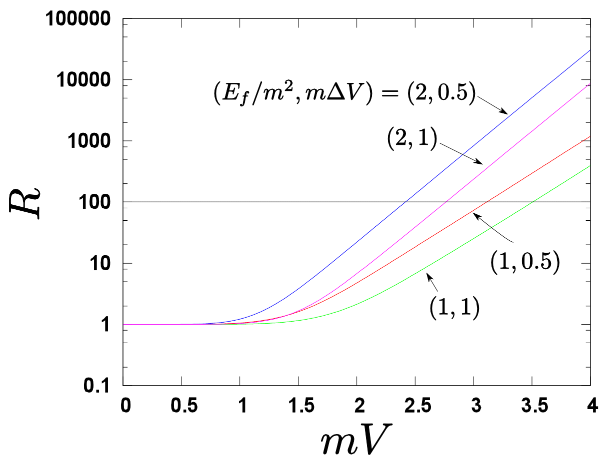


- The meson sector is deconfined by the Schwinger effect
- The system has been relaxed and thermalized
    - The effective horizon emerges on the worldvolume

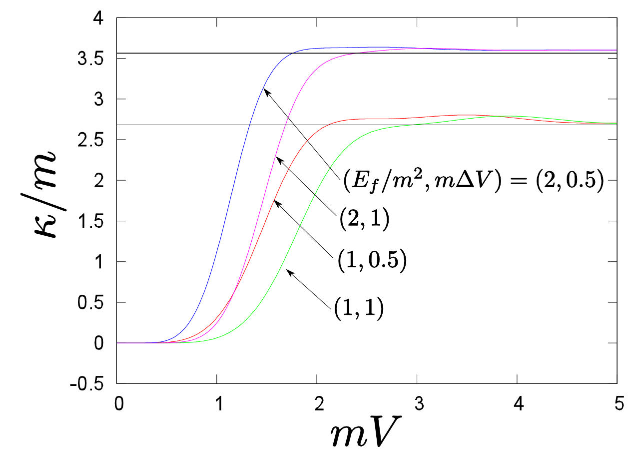
Thermalization time

- We can estimate thermalization time explicitly

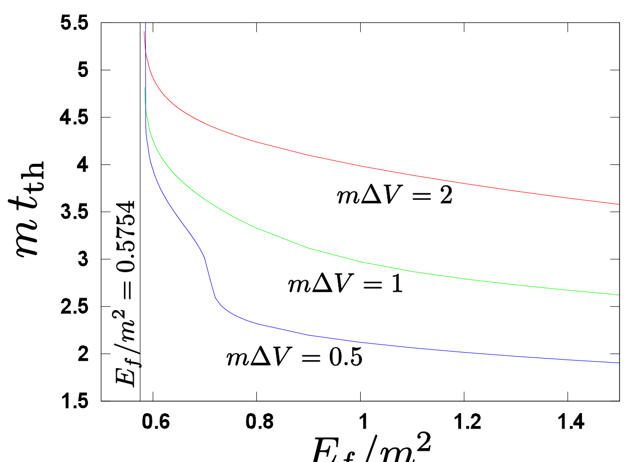
Time evolution of the redshift factor



Time evolution of the surface gravity



thermaliztion time

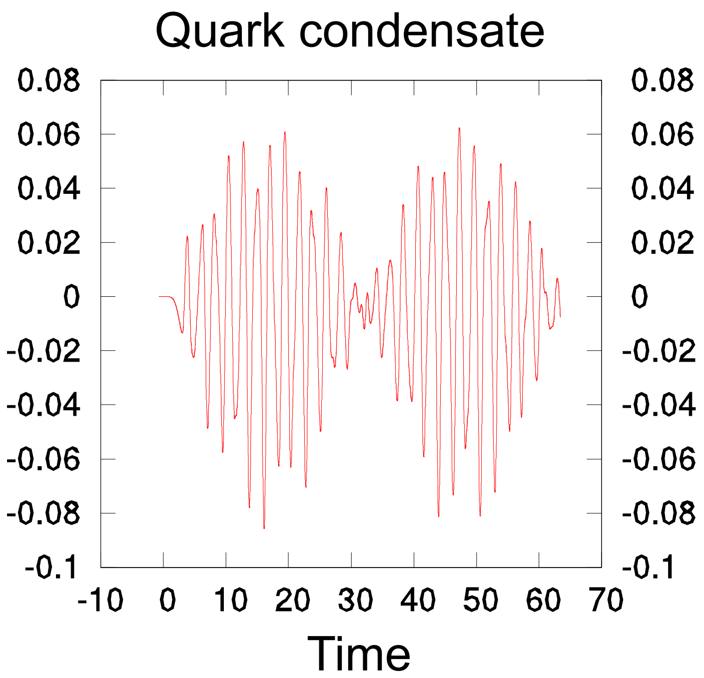


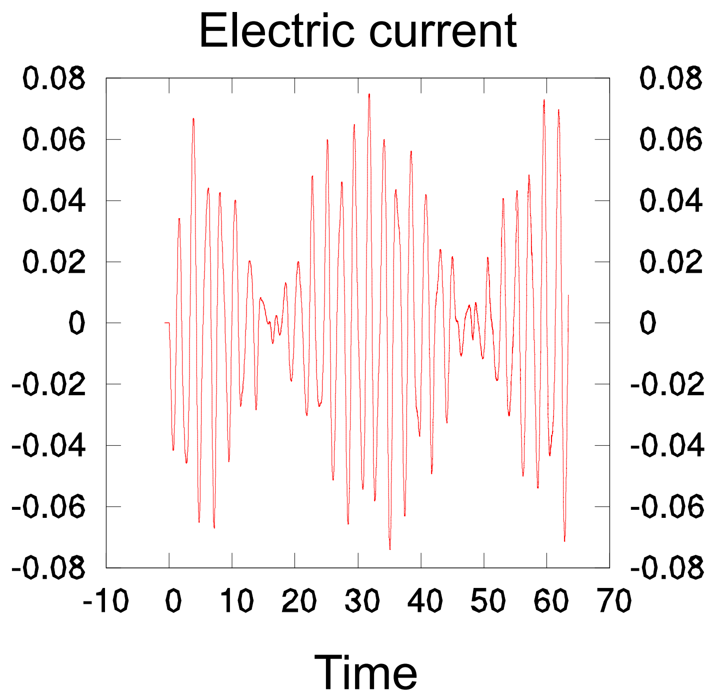
$$\boxed{t_{\rm th} \sim 1/\kappa \sim 1/\sqrt{E_f}}$$

As the electric field is larger,
it becomes typical thermal scale

Case 2
sub-Schwinger-limit

- Weak electric field $(E_f = 0.10, \Delta V = 2.0)$

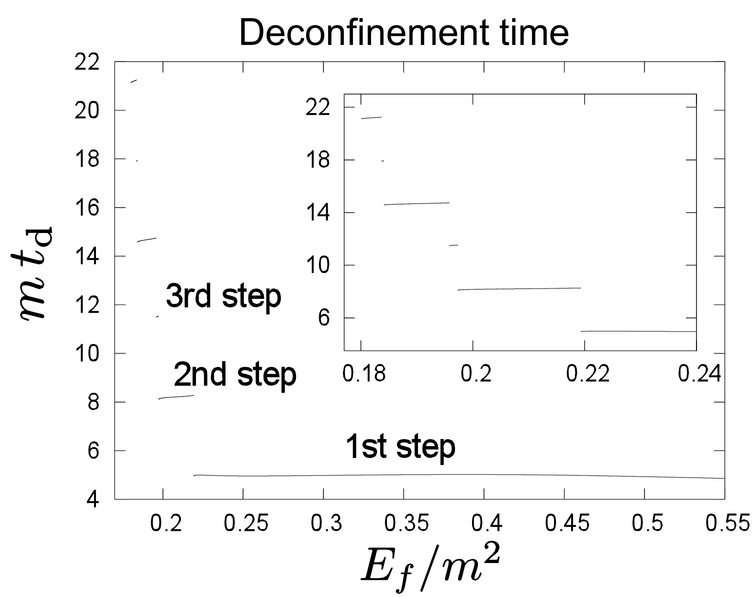


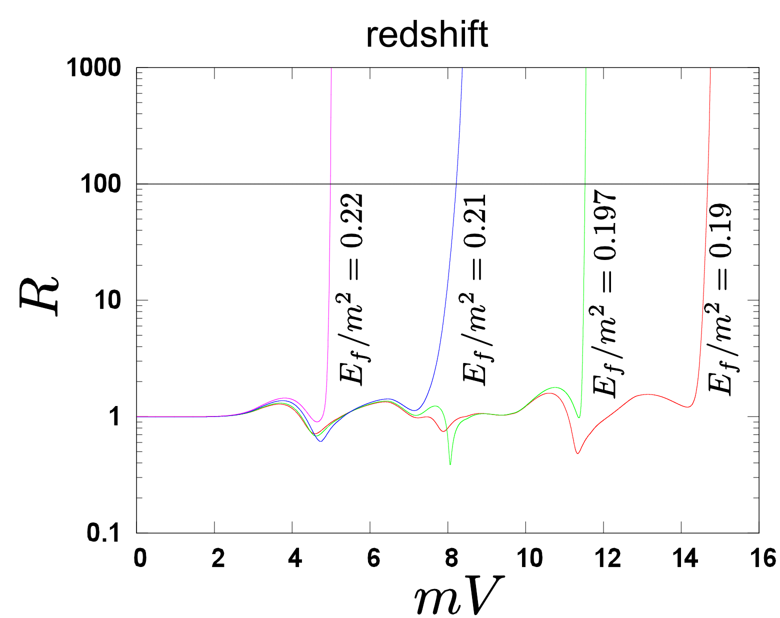


- Normal modes of fluctuations of the brane and the gauge field
  ⇔ discrete spectrum of meson
- "beat" ⇔ meson mixing
  - The Stark effect leads to splitting of degenerate mass spectrum

Case 3
Non-equilibrium deconfinement

- We found that sufficiently rapid quench causes the deconfinement transition even below the critical electric field
  - Notice that deconfinement is defined by divergence of redshift factor measured at the AdS boundary

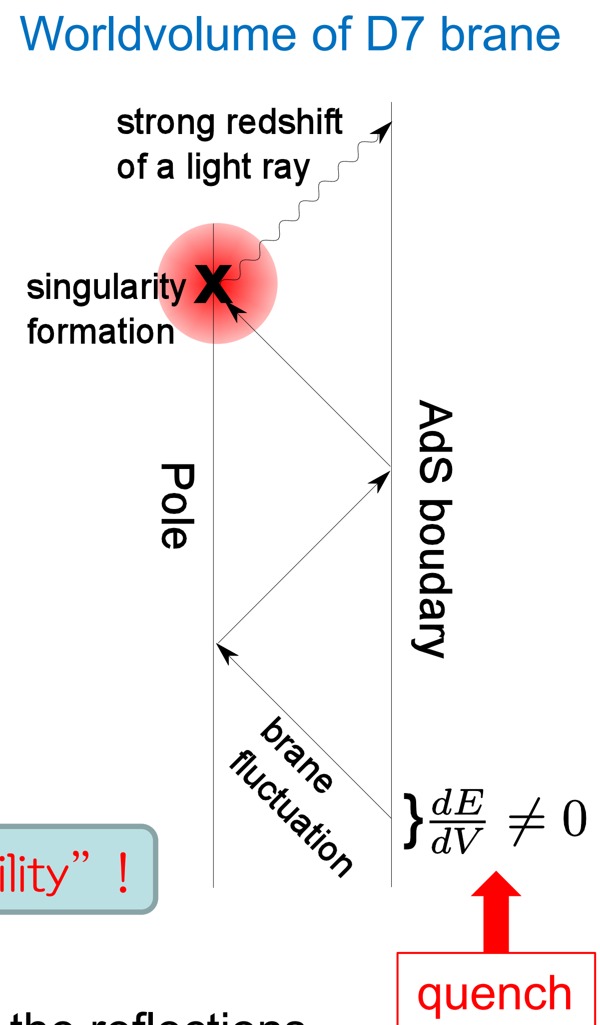




The deconfinement time becomes discretely longer as the electric field is weaker

“Turbulence” on the brane

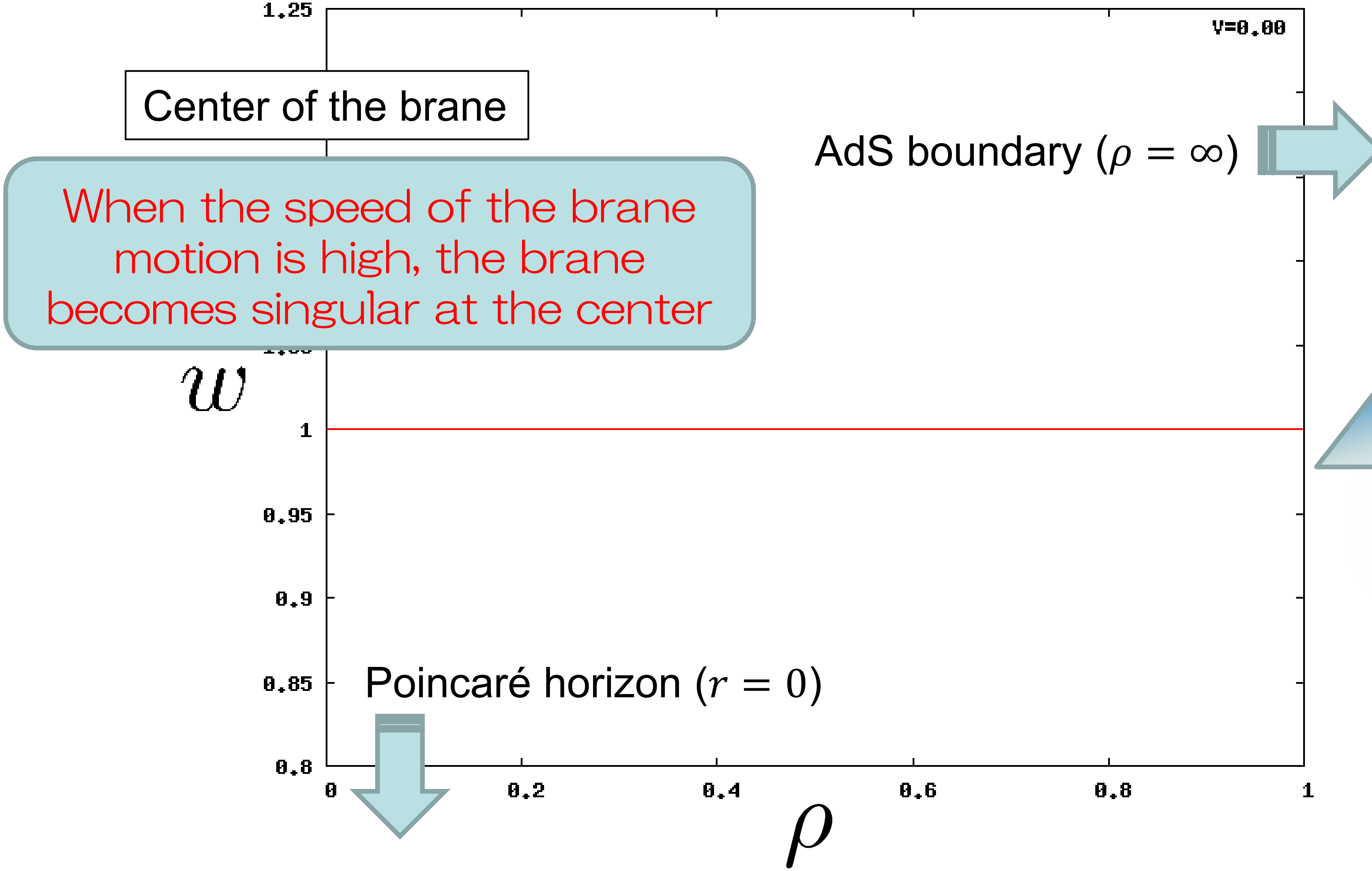
- What is happening on the brane?
  - The fluctuation caused by the quench at the boundary is amplified during coming and going on the brane
  - After reflecting several times, a strongly red-shifted region emerges at the center and then a naked-singularity will form on the brane

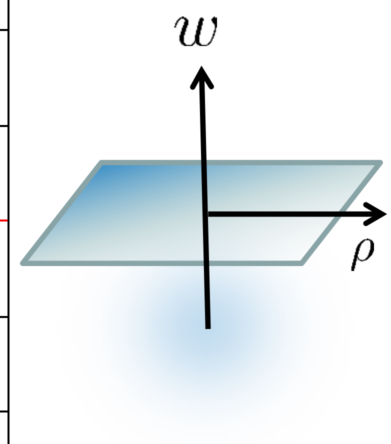


It seems to be similar to “AdS turbulent instability”!

Discreteness of the deconfinement time = number of the reflections
Deconfine (divergence of the redshift factor) = singularity formation

Profile of the D7-brane in the bulk





Bulk spacetime: $\mathrm{d}s^2 = r^2[-\mathrm{d}t^2 + \mathrm{d}\vec{x}_3^2] + \frac{1}{r^2}[\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\Omega_3^2 + \mathrm{d}w^2 + \mathrm{d}\bar{w}^2]$

$(r^2 \equiv \rho^2 + w^2)$

Spectral analysis

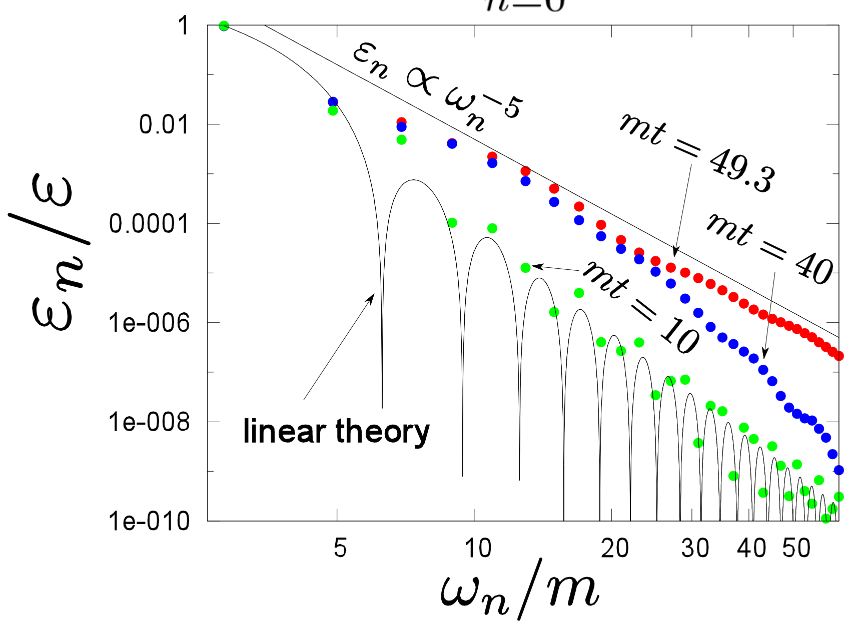
Hashimoto, SK, Murata, Oka arXiv:1408.6293, 1412.4964

- We decompose the brane fluctuations into the mode functions in linear perturbations
  - fluctuations of the brane and gauge field:

$$\chi(t,z) = \sum_{n=0}^{\infty} c_n(t) e_n(z)$$

$$e_n(z) \equiv N_n z^2 F(n+3, -n, 2; z^2)$$

Mode functions



Meson mass

$$S_{\rm D7} \propto \int d^4x \int_0^1 dz \frac{1-z^2}{2z}[(\partial_t\chi)^2 - m^2(1-z^2)(\partial_z\chi)^2] + \mathcal{O}(\chi^3)$$
$$= \frac{1}{2}\int d^4x \sum_{n=0}^{\infty}[\dot{c}_n^2 - \omega_n^2 c_n^2] + \text{interaction}$$

Energy of each mode

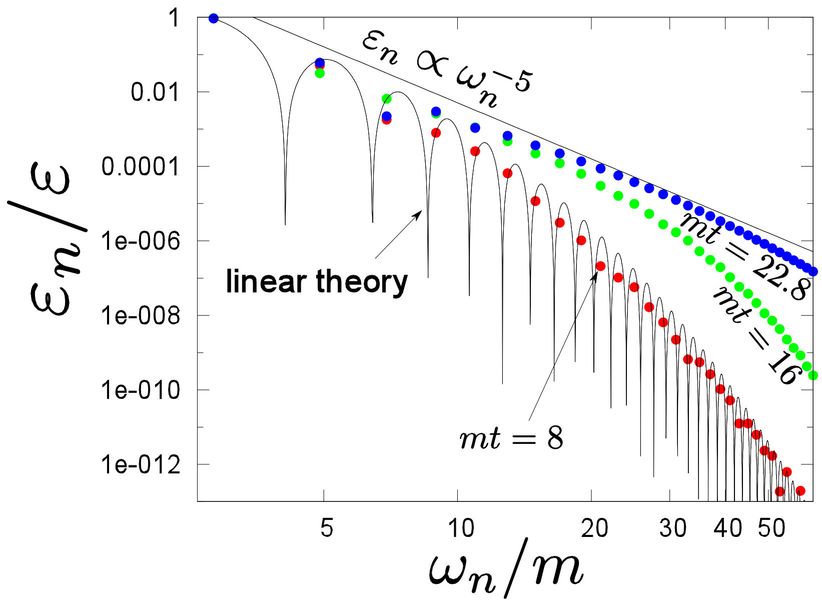
$$\varepsilon = \sum_{n=0}^{\infty} \varepsilon_n = \sum_{n=0}^{\infty} \frac{1}{2}[\dot{c}_n^2(t) + \omega_n^2 c_n^2(t)]$$

As time goes, the energy is transferred to higher modes

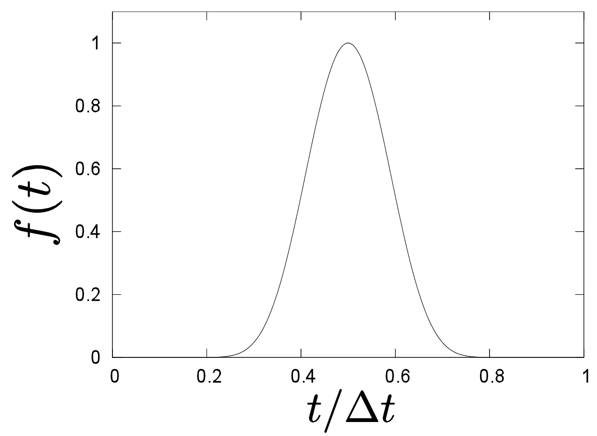
Universality

- Turbulence in quark mass quench
  - Time-dependent quark mass: $m(t) = m + \delta m f(t)$

Energy spectrum



Shape of the quench



The energy spectrum approaches $\varepsilon_n \propto \omega_n^{-5}$ as time goes

Features of the probe brane turbulence

- The worldvolume gauge field or the quench induced by the electric field is NOT essential for the turbulence
  - We can observe the turbulence for other triggers of excitations
- Non-linearity is important
  - The initial energy spectrum excited by the quenches can be well described in linear theory
  - Even if the input is perturbative, the non-linearity will cause energy cascade from lower modes to higher modes in the time evolutions

- Wave turbulence in non-linear wave

Summary 1

- We studied time-dependent phenomena in holographic QCD constructed by the D3/D7 system
  - We numerically solved EOMs of the probe D7-brane under time-dependent boundary conditions
- We found non-equilibrium deconfinement transition even for sub-Schwinger electric fields
  - Energy cascade from lower meson modes to higher ones
  - "turbulent meson condensation"
- We found universalities of the turbulence on the brane
  - The turbulence can occur for various excitations on the brane
  - Final energy spectrum for meson modes obeys a power law

APPLICATION2:
ROTATING ELECTRIC FIELD

Time periodic phenomena

- Recently, time periodic states have been of great interest in condensed matter physics
  - Floquet states

- Naively, holographic realization is not so easy
  - At least, two directions are inhomogeneous; EoMs are hyperbolic PDE

Rotating electric field

- Phase transition by AC electric fields
- Time dependent system
  - Floquet states were investigated in CMP   Oka, Aoki (2009)
- We can reduce EoMs to ODE in a holographic setup

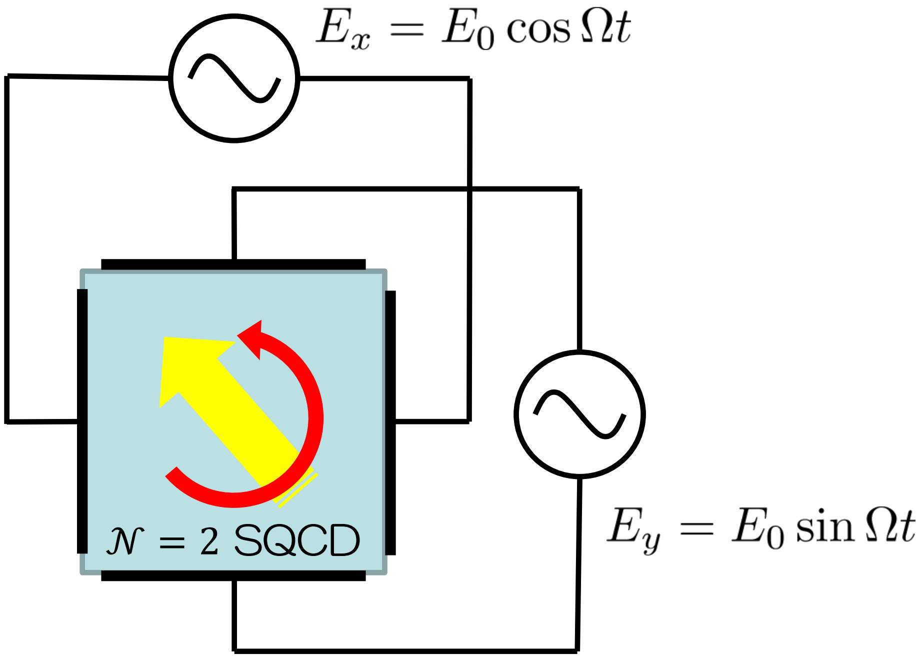
Rotating electric field at the boundary:

$$A_x + iA_y = iE_0 e^{i\Omega t}$$

Worldvolume gauge field on D7:

$$a_x(t,\rho) + ia_y(t,\rho) = b(\rho)e^{i(\Omega t - \chi(\rho))}$$

Inspired by Oka-san



Setup

Hashimoto, SK, Murata, Oka JHEP 05 (2017) 127
SK, Murata, Oka JHEP 06 (2018) 096

- DBI-action

$$S_{\mathrm{D7}} = -\int d^8\sigma \sqrt{-\det(h_{ab}+F_{ab})}$$

The action becomes time independent!

$$\propto -\int d\rho \frac{\rho^3}{(w^2+\rho^2)} \sqrt{[(\rho^2+w^2)^2 - \Omega^2 b^2](1+w'^2+b'^2) + (\rho^2+w^2)^2 b^2 \chi'^2}$$

static $\boxed{\int d\rho \frac{\rho^3}{(w^2+\rho^2)} \sqrt{[(\rho^2+w^2)^2 - E^2](1+w'^2) + (\rho^2+w^2)^2 a'^2}}$

Embedding function: $w(\rho)$

Worldvolume gauge field: $a_x + i a_y = b(\rho) \exp i(\Omega t + \chi(\rho))$

- Boundary conditions at the AdS boundary

$$b|_{\rho=\infty} = \frac{E}{\Omega}, \quad \vec{E} = E \begin{pmatrix} \cos\Omega(t-t_0) \\ \sin\Omega(t-t_0) \end{pmatrix}$$

Rotating electric field

$$w(\rho) = m + \frac{c}{\rho^2} + \cdots, \quad \vec{a}(t,\rho) = -\int^t dt' \vec{E}(t') + \frac{1}{2\rho^2}\vec{j}(t) + \cdots$$

Symmetry

Static electric field

- Time translation: yes
- Rotation: no
- Helical: no

Rotating electric field

- Time translation: no
- Rotation: no
- Helical: yes

generator

$$\frac{\partial}{\partial t}, \quad \cancel{\frac{\partial}{\partial \phi} \equiv x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}}, \quad \cancel{\frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial \phi}}$$

$$\cancel{\frac{\partial}{\partial t}}, \quad \cancel{\frac{\partial}{\partial \phi} \equiv x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}}, \quad \frac{\partial}{\partial t} + \Omega\frac{\partial}{\partial \phi}$$

Time translation in static cases changes to helical (corotating) translation in rotating cases

- EoM

$$b'' = \frac{1}{\rho(w^2+\rho^2)\boxed{\left[(w^2+\rho^2)^2-\Omega^2 b^2\right]}}\Big\{ -3(w^2+\rho^2)^3 b'(1+w'^2+b'^2)$$
$$-\Omega^2 b(1+w'^2+b'^2)\left[\rho(w^2+\rho^2)-bb'(3w^2+\rho^2)\right]+b(w^2+\rho^2)^3(\rho-3bb')\chi'^2\Big\}\ ,$$

$$\chi'' = \frac{1}{b\rho(w^2+\rho^2)\boxed{\left[(w^2+\rho^2)^2-\Omega^2 b^2\right]}}\Big\{ -3b^3$$
$$-\left[2\rho b'(w^2+\rho^2)^3+3b(w^2+\rho^2)^3(1+$$
$$-\Omega^2 b^3\left(4\rho^2+4\rho w w'+(\rho^2+3w^2)(1+w'^2+b'^2)\right)\right]\chi'\Big\}\ ,$$

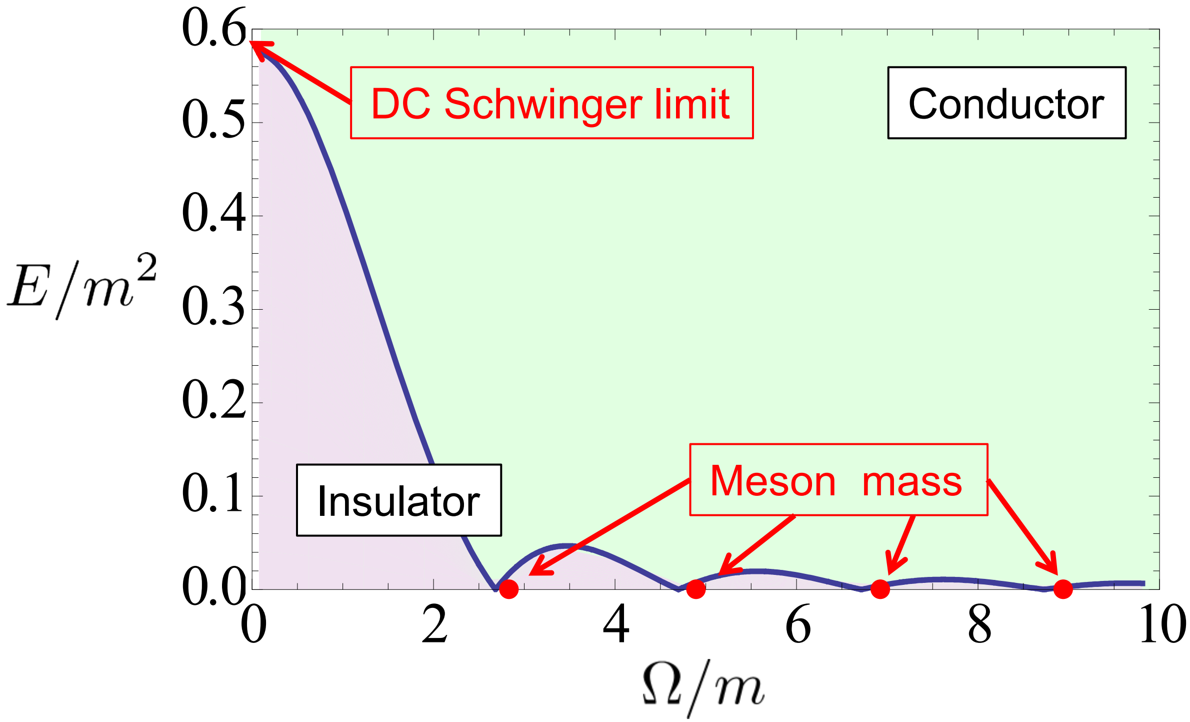
$$w'' = \frac{1}{\rho(w^2+\rho^2)\boxed{\left\{(w^2+\rho^2)^2-\Omega^2 b^2\right\}}}\Big\{ -3(w^2+\rho^2)^3(1+w'^2+b'^2)w'$$
$$+\Omega^2 b^2\left[2\rho w+(\rho^2+3w^2)w'\right](1+w'^2+b'^2)-3b^2(w^2+\rho^2)^3 w'\chi'^2\Big\}\ .$$

Denominator will vanish ⇒ effective horizon

We can obtain solutions in the same way as in static cases

Phase diagram

- Global structure



$$\vec{E} = E \begin{pmatrix} \cos \Omega t \\ \sin \Omega t \end{pmatrix}$$

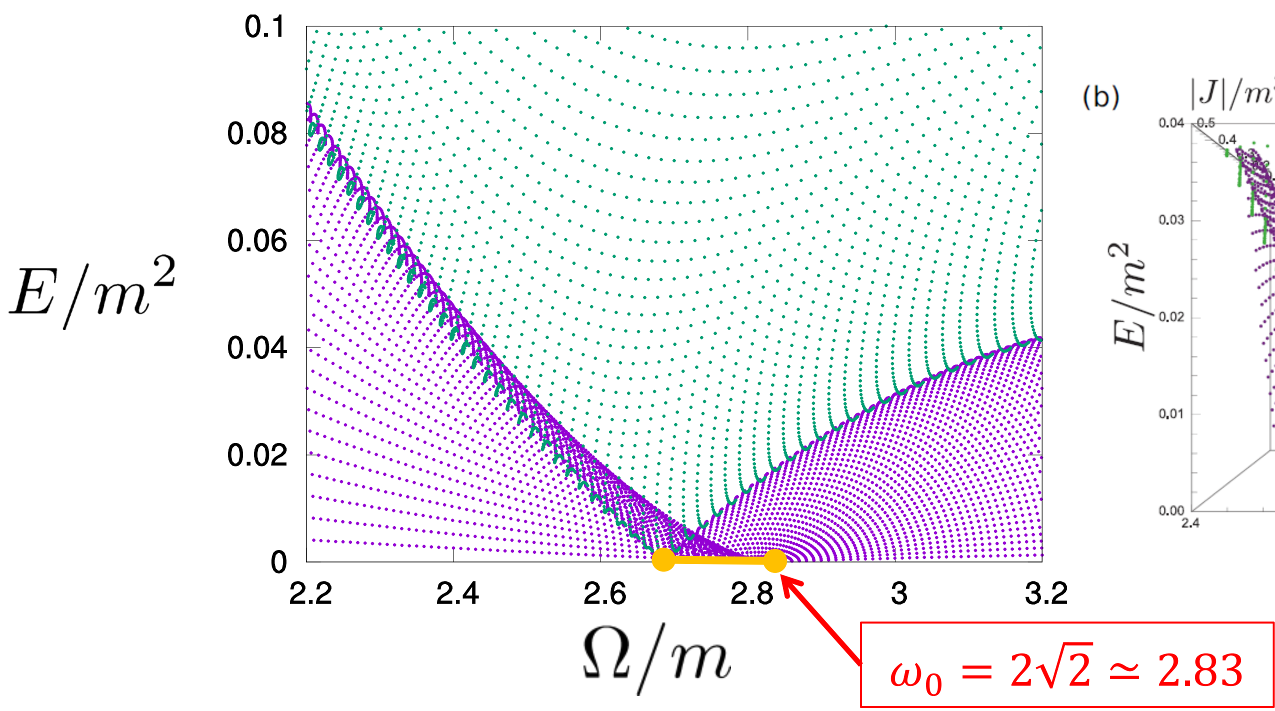
Mass spectrum of mesonic excitations:

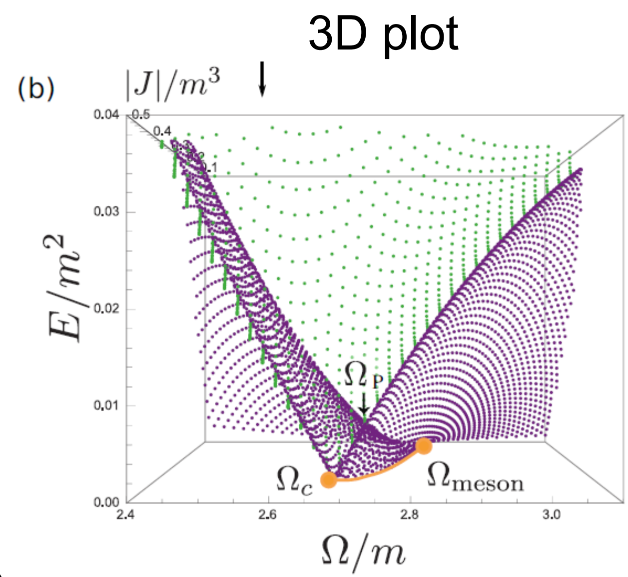
$$\omega_n = 2\sqrt{(n+1)(n+2)}$$

- As the frequency increases, the critical value becomes smaller
- Near frequencies of the meson masses, the critical value is infinitesimal

Phase diagram

- Fine structure around the lowest mode





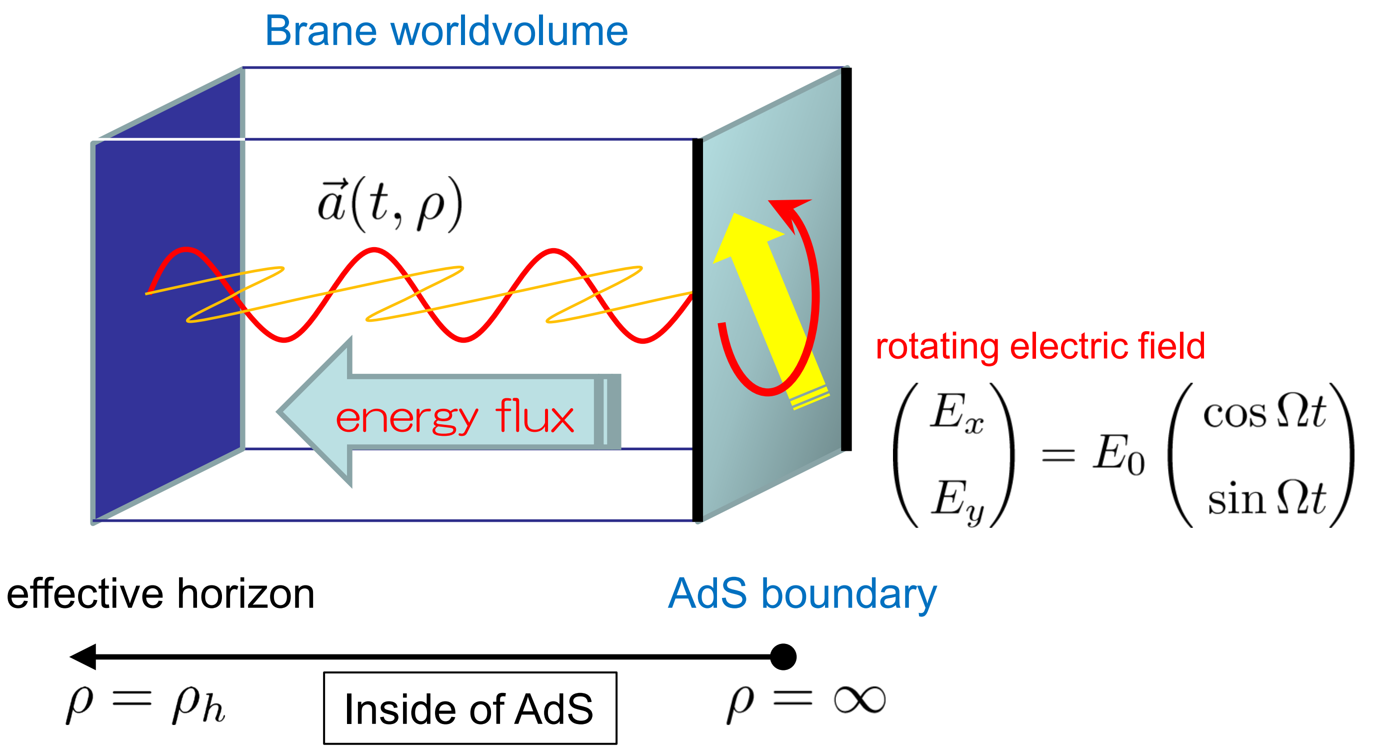
- Multiple phases of insulator and conductor states coexist in vicinity of the critical curve
- Even in infinitesimal electric fields, there are non-trivial states other than the vacuum state

Summary 2

- The rotating electric fields can provide a new axis of the phase diagram
  - AC Schwinger limit is smoothly connected with DC Schwinger limit in static electric field
- Frequencies of the rotating field can reduce the critical amplitude to cause dielectric breakdown
  - The AC Schwinger limit periodically becomes infinitesimal near the frequencies corresponding to the meson mass spectrum
- Even in $E = 0$, non-trivial states emerge other than the vacuum state
  - Non-linear excitation of vector meson, Floquet condensation of vector meson

APPENDIX

Sketch from bulk viewpoint

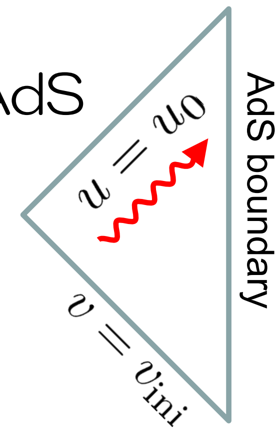


Revisit of deconfinement and thermalization

- If static, both of deconfinement and thermalization are given by the same condition in gravity side: the horizon exists or not
  - Quasi-normal modes = unstable mesons
  - Hawking temperature = thermalization
- In time-dependent cases, this definition is not so useful
  - When does the horizon form for temporal observers at the AdS boundary?
    - By definition, "horizon" is invisible from the AdS boundary (=null infinity) forever.
  - Since we have no preferred time-slice in gravity, the formation time is ambiguous

Redshift factor and surface gravity

- Redshift factor
  - The ratio between the energy observed on the AdS boundary and the initial surface (static region)



$$R(u_0) = \frac{k^a \xi_a|_{v=v_{\rm ini}}}{k^a \xi_a|_{v=u_0}} = -\frac{m}{2}\frac{V_{,v}(u_0,u_0)}{\Phi_{,u}(u_0,v_{\rm ini})}$$

- Surface gravity
  - Relation between times to define the Initial state and the final state

$$\kappa(u_0) = \frac{d}{du}\log R(u_0)$$

If this quantity is almost constant, it becomes Hawking temperature observed at u_0

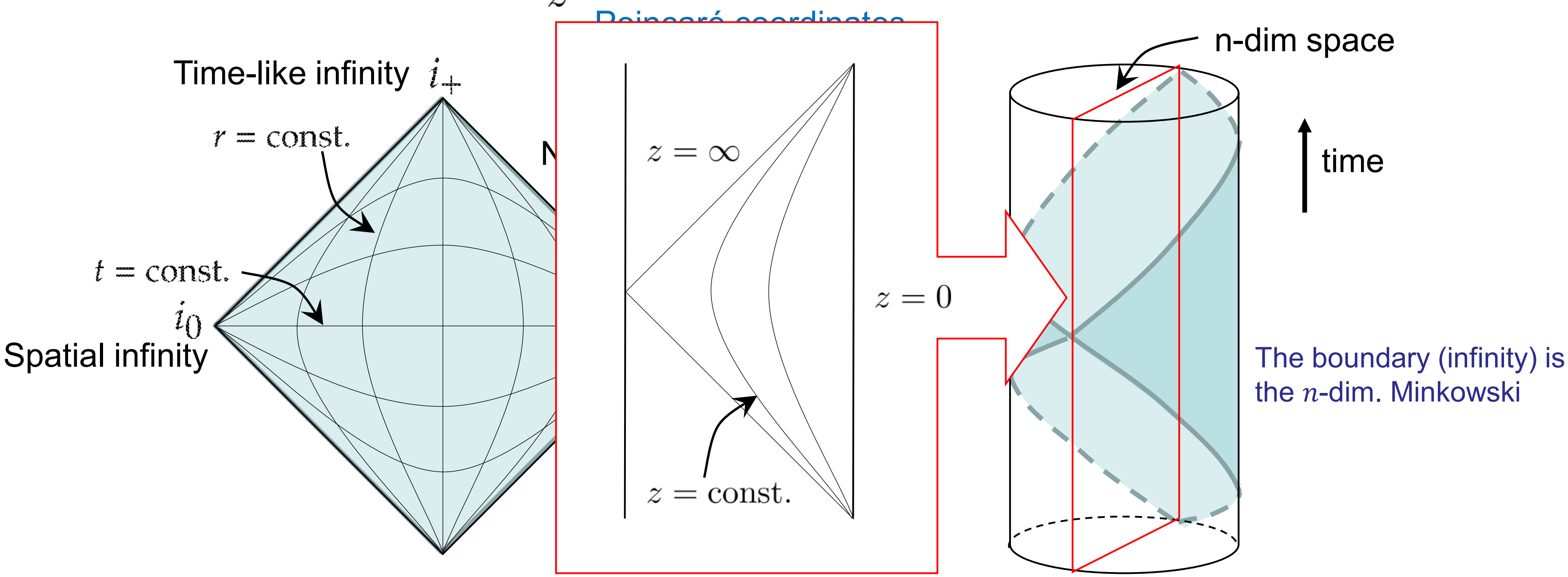
Redshift becomes too large ⇔ deconfinment
Surface gravity becomes constant ⇔ thermalization

We can define and calculate these only from the causal past of temporal observers

Anti-de Sitter (AdS) space

- Vacuum solution of the Einstein eq. with negative cosmological constant
  - metric: $ds^2_{n+1} = \frac{1}{z^2}[dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu]$

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\Lambda g_{\mu\nu} \quad (\Lambda < 0)$$



n-dim. Minkowski

$(n+1)$-dim. AdS