Is the Standard Model in the Swampland? Consistency Requirement from Gravitational Scattering

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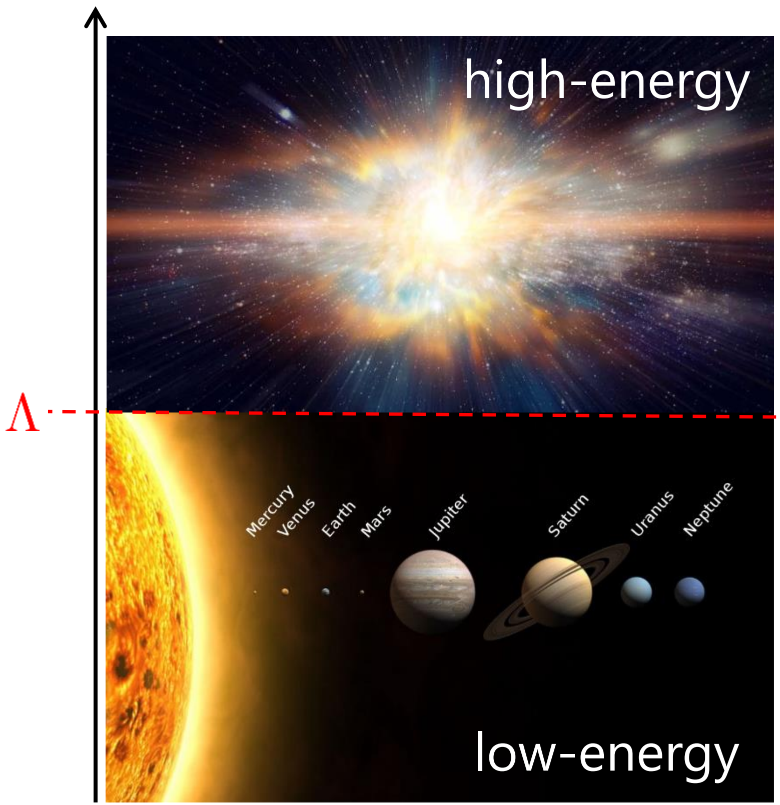
KA, T.Q. Loc, T. Noumi, and J. Tokuda, PRL 127 (2021) 9, 091602.

J. Tokuda, KA, and S. Hirano, JHEP 11 (2020) 054.

Seminar@iTHEMS, 2nd Nov. 2021

EFT is ubiquitous

- IR physics must be insensitive to UV physics.



Fundamental theory

Effective description here

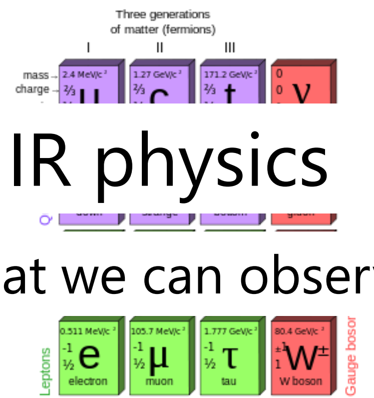
= low-energy effective (field) theory

Consistency between IR and UV

- ❑ IR physics must be insensitive to UV physics.

but, not totally independent!

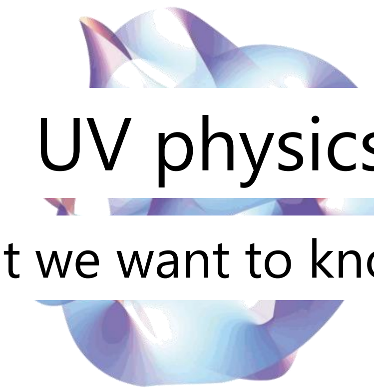
- ❑ There are general consistency relations b/w IR and UV.



IR physics

What we can observe

↔



UV physics

What we want to know finally

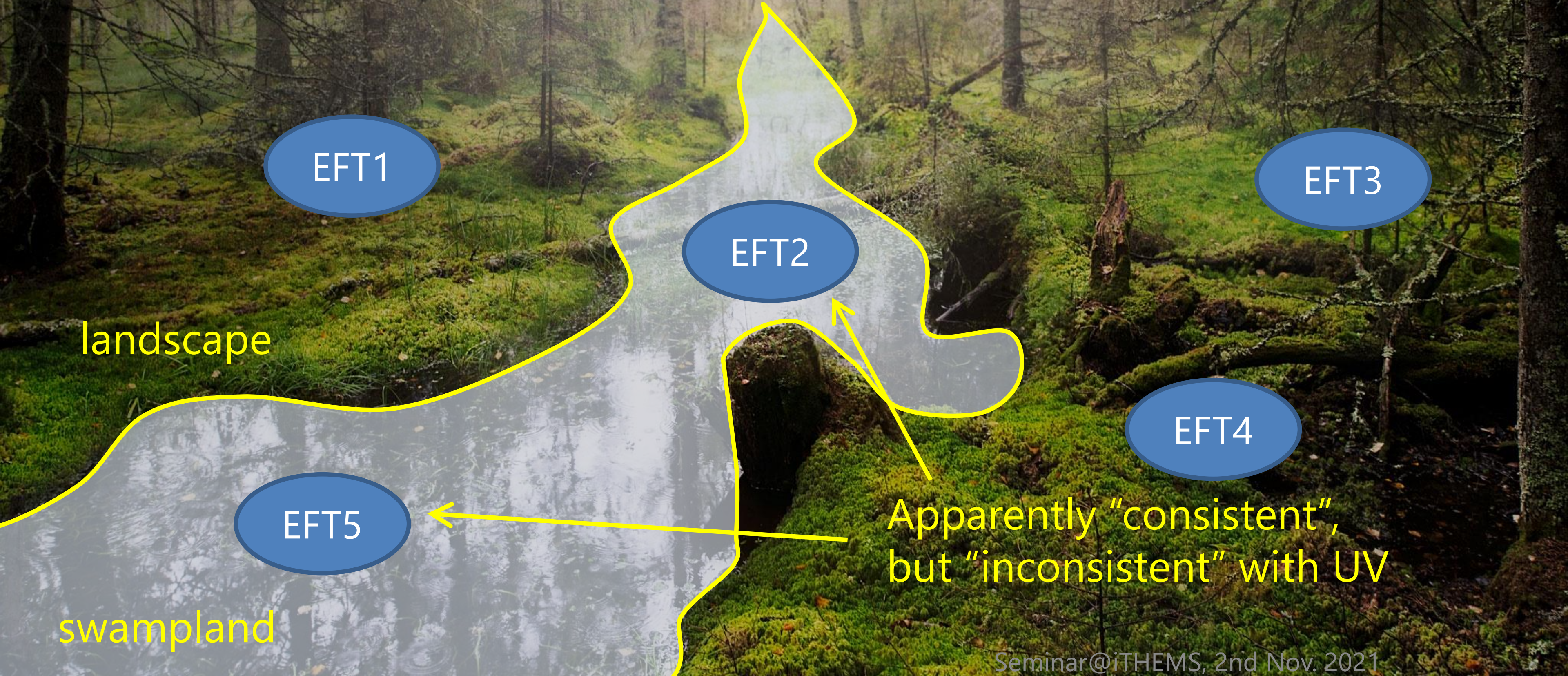
- ❑ The consistency relations may be used to
  - ✓ make predictions on IR physics,
  - ✓ or, extract information about the UV physics.

Swampland program

Vafa, 2005.

The swampland refers to low-energy EFT which is not compatible with quantum gravity.

Vafa, 2005



“Consistency” conditions

- There would be no definite swampland criteria because we don’t know UV.

  However, we have some expectations (or “plausible” assumptions) on UV.
  → Any self-consistency conditions from UV assumptions?

- **Scattering amplitudes do a job!**

  General assumptions are enough to find strong constraints.

  ||

  Poincare invariance, Unitarity, Analyticity, Boundedness.

  The last two are inferred from causality & locality.
  Sometimes called “axiomatic” S-matrix theory.

  A textbook example: partial wave unitarity ⇒ $|\mathcal{M}(s,\theta)| < 1$

  $s = E_{\mathrm{CM}}^2$ and θ: scattering angle

- The constraints on EFTs are often called positivity bounds. Adams, et al. 2006

Is the Standard Model in the Swampland?

- ❑ **SM + GR have to pass <u>swampland criteria</u> at low-energies.**

  = positivity bounds

↔ **SM + GR may fall into swampland if extrapolated to high-energy.**

⇒ the cutoff of SM + GR = new physics is required as a consistency.

c.f. Theory of massive weak boson violates perturbative unitarity at TeV.
⇒ The Higgs boson should be needed below TeV. Lee et al. 1977.

- ❑ **Gravity plays an essential role.**

Standard Model alone: UV complete
⇒ Positivity bounds are trivially satisfied at any scales (as it should be).

Standard Model + General Relativity
⇒ New physics (= quantum gravity?) is needed at or below 10^{16}GeV.

KA, T.Q. Loc, T. Noumi, and J. Tokuda, 2104.09682.

Contents

1. General properties of scattering amplitudes
2. Gravitational positivity bounds
3. The cutoff of SM + GR
4. Summary and discussions

“Axiomatic” S-matrix theory

- Our assumptions or “principles” on S-matrix:
  Poincare invariance, Unitarity, Analyticity, Boundedness.

  I explain what they are and what are implications from them.

  Ref: *The analytic S-matrix*, R. J. Eden et al, 1966

  *Analyticity and boundedness are well-understood in gapped system, while they are still less known in gapless system including gravity.

- Assume massless particles (photon&graviton) are weakly coupled so that these properties are inferred from perturbative calculations.

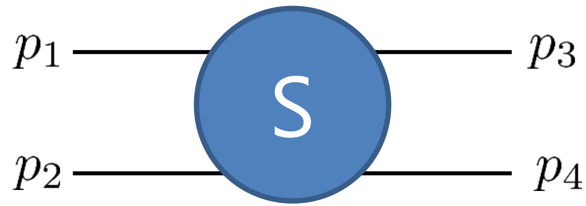
  ✓ Photon → OK because $\alpha \simeq 1/137 \ll 1$.
  ✓ Graviton → OK if UV completed below $M_{\rm pl}$.

  Super-Planckian physics is subtle…

- For simplicity, we focus on scattering of identical spin-0 particles here.

Translation + Lorentz (Poincare)

- Suppose the translation invariance
  ⇒ The asymptotic states are labelled by four momenta p.



$$= \langle p_3, p_4 | S | p_1, p_2 \rangle$$

Initial states Final states Probability

The S-matrix may be separated into trivial part and others. $S = 1 + iR$

$$\langle p_3, p_4 | R | p_1, p_2 \rangle = (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) \mathcal{M}(p_1, p_2, p_3, p_4)$$

conservation amplitude

- Lorentz invariance → $\mathcal{M}(p_1, p_2, p_3, p_4) = \mathcal{M}(s, t, u)$

$$s := -(p_1 + p_2)^2, \ t := -(p_1 - p_3)^2, \ u := -(p_1 - p_4)^2$$

$$s + t + u = \sum m_i^2 = 4m^2$$

Unitarity

- We suppose that the probability is conserved:

$$SS^\dagger = S^\dagger S = 1 \Leftrightarrow R - R^\dagger = iR^\dagger R$$

$$\Rightarrow 2\mathrm{Im}\langle p_3, p_4|R|p_1, p_2\rangle = \sum_n \langle p_3, p_4|R|n\rangle\langle p_1, p_2|R|n\rangle^*$$

The completeness relation: $\sum_n |n\rangle\langle n| = 1$

This is very simple but provide two important consequences.

1. If $p_1 = p_3$ $(p_2 = p_4) \Leftrightarrow t \equiv -(p_1 - p_3)^2 = 0$

$$\Rightarrow 2\mathrm{Im}\langle p_1, p_2|R|p_1, p_2\rangle = \sum_n |\langle p_1, p_2|R|n\rangle|^2$$

Optical theorem

In particular, we get $\mathrm{Im}\mathcal{M}(s \geq 4m^2, t \to 0) > 0$

2. The unitarity equation implies the existence of singularities of $\mathcal{M}(s, t)$

Analyticity

$$2\mathrm{Im}\langle p_3, p_4|R|p_1, p_2\rangle = \sum_n \langle p_3, p_4|R|n\rangle\langle p_1, p_2|R|n\rangle^*$$

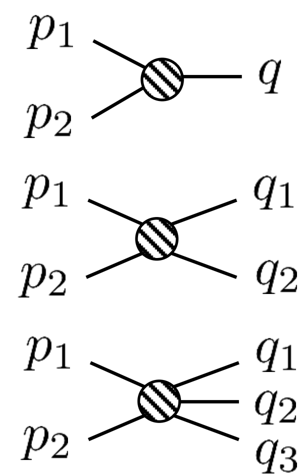
$s = E_{\mathrm{CM}}^2$

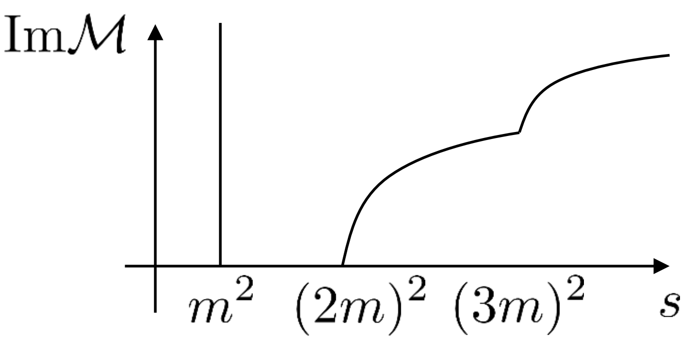
- **Singularities (imaginary parts) from internal on-shell states.**

$s < (2m)^2$: only the on-shell state is one particle state

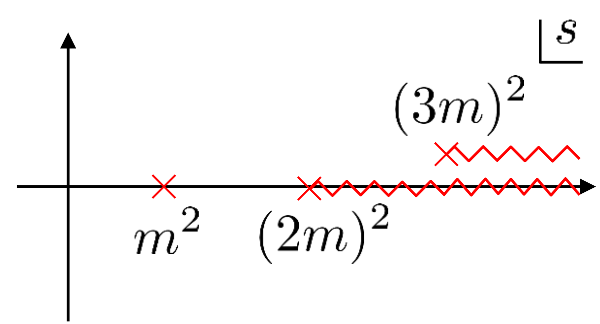
$(2m)^2 \leq s < (3m)^2$: two particle states

$(3m)^2 \leq s < (4m)^2$: two + three particle states





$\Rightarrow$

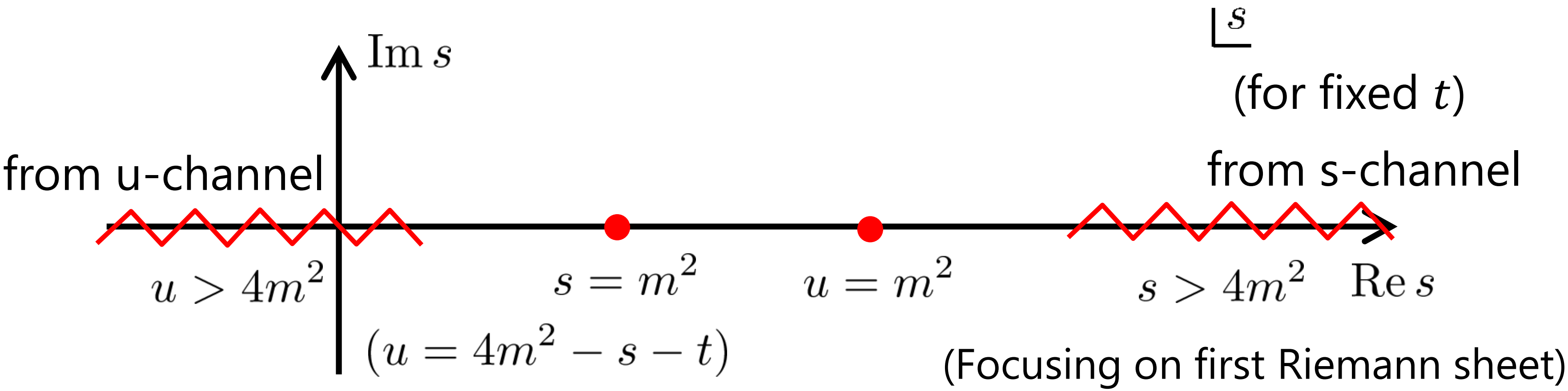


Pole at $s = m^2$

Branch cut along $s \geq 4m^2$

Analyticity

- The analytic structure of amplitude for a fixed t:



*The t-singularities do not arise because t is fixed away from singularity.

There are no other singularities (analyticity assumption).

Singularities from massless particles (photon&graviton) could be subtle.
But, they are weakly coupled below $M_{\rm pl}$ and just give poles at leading order.

Boundedness and dispersion relation

- **We need to know asymptotic behaviour to go to the nest step.**
- **Froissart bound** Froissart, 1961; Martin, 1963.

  For a gapped system, all the mentioned properties + locality implies

  $$|\mathcal{M}(s,t=0)| < |s^2| \quad \text{as} \quad |s| \to \infty$$

  *The Froissart bound itself is slightly stronger than this.

- **Boundedness in gravity?**

  No rigorous discussion, but the similar boundedness is widely accepted thanks to the knowledge of perturbative string

  $$\mathcal{M}_{\mathrm{GR}} \sim -\frac{s^2}{M_{\mathrm{pl}}^2 t} \to \mathcal{M}_{\mathrm{Regge,gra}} \sim -\frac{s^{2+\alpha' t+\cdots}}{M_{\mathrm{pl}}^2 t} < |s^2| \quad \text{for} \quad t<0$$

  Quantum gravity would soften the graviton exchange.

Regge behaviour

= exchanging of a family of particles

- Some theories exhibit the behaviour

$$\mathcal{M}(s,t) \sim \frac{\beta(t)}{\sin \pi\alpha(t)} \left(\frac{s}{-t}\right)^{\alpha(t)} \qquad \mathrm{Re}\,\alpha \geq L : \text{finite}$$

in the Regge limit $s \to \infty$ while t is kept finite. Typically, $\alpha(t) = \alpha_0 + \alpha' t$

See e.g. *The analytic S-matrix*, R. J. Eden et al.
The Theory of Complex Angular Momenta, V. N. Gribov.

- What does it mean?

✓ $\mathcal{M}$ has singularities at $\alpha(t) = J \in$ integers
✓ Spin-J particle exchange $\propto s^J$

⇒ $\mathcal{M}$ represents a sum of higher spin particle exchanges.

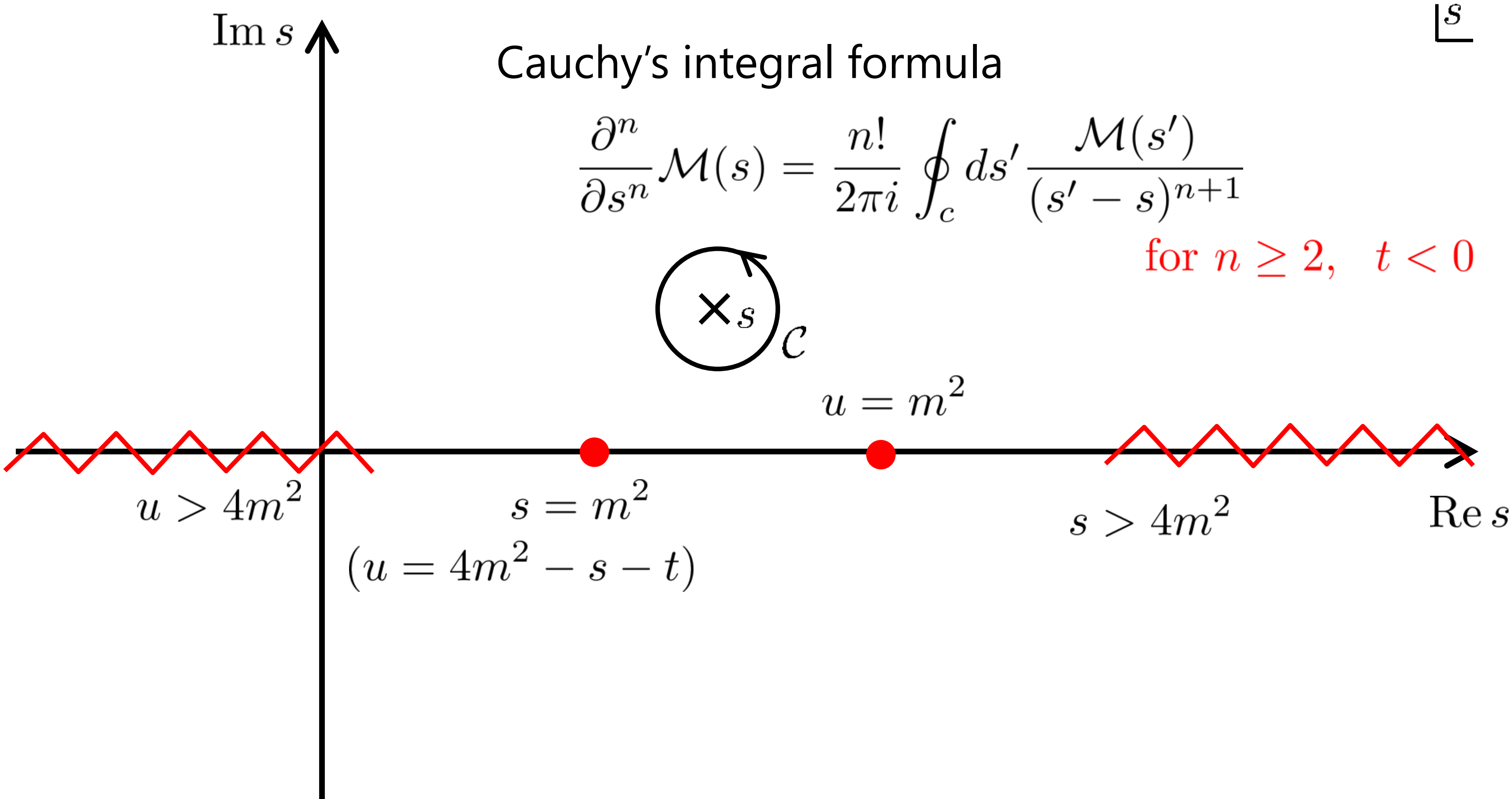
In QCD: higher spin particles = called Reggeon and Pomeron
In string: higher spin particles = string excitations ⇒ soften the amplitude

Usage of Analyticity + Boundedness

Cauchy's integral formula

$$\frac{\partial^n}{\partial s^n}\mathcal{M}(s) = \frac{n!}{2\pi i}\oint_c ds' \frac{\mathcal{M}(s')}{(s'-s)^{n+1}}$$

for $n \geq 2, \quad t < 0$



Usage of Analyticity + Boundedness

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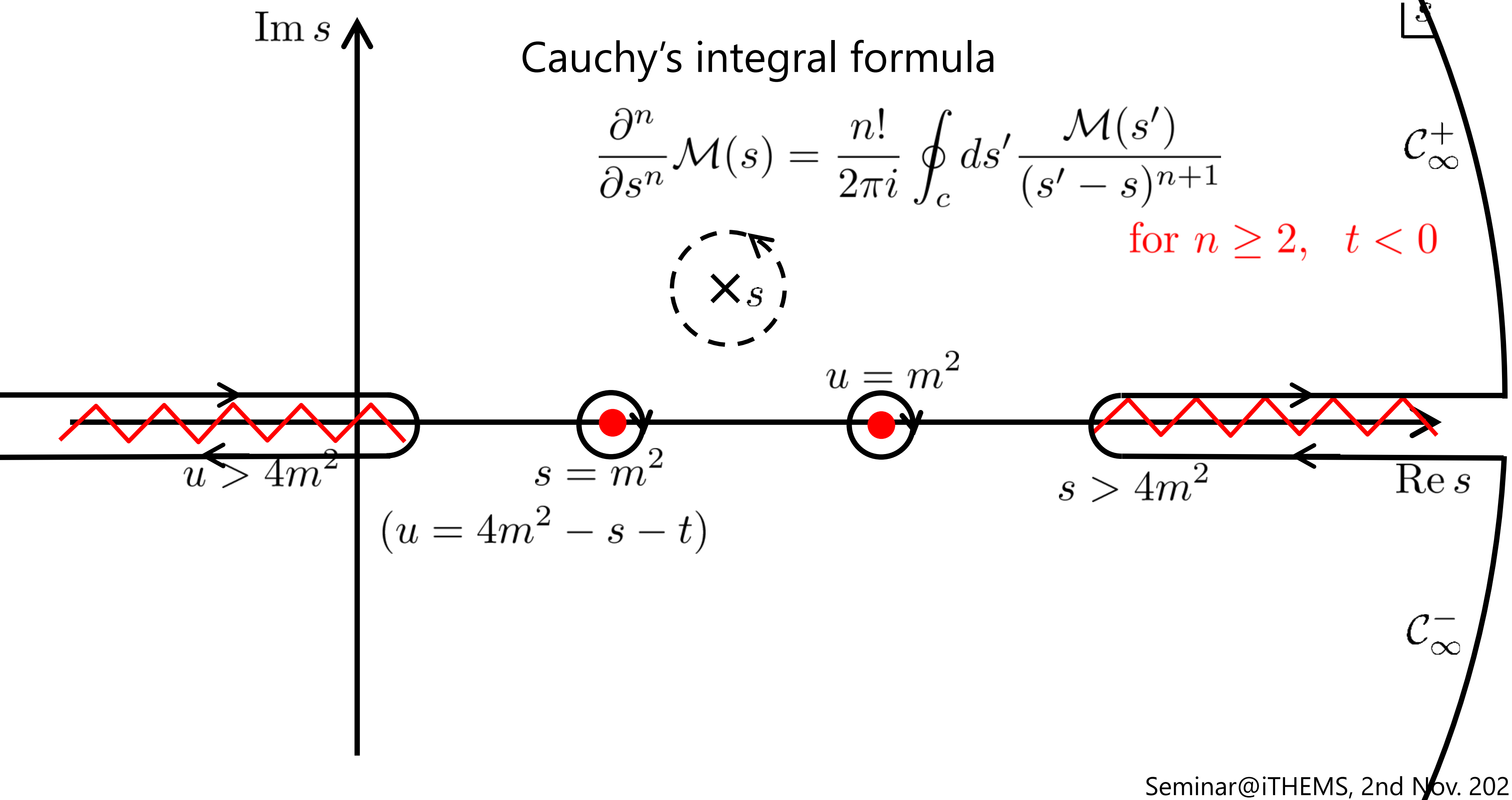


Dispersion relation

- ❑ s^n part of amplitude are determined by singularities (= imaginary part).

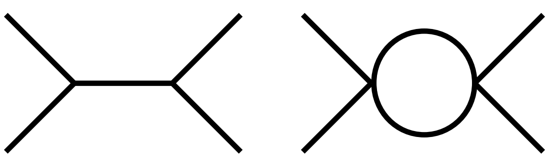
$$\frac{\partial^n}{\partial s^n}\mathcal{M}(s,t_0) = \frac{n!}{2\pi i}\oint_c ds' \frac{\mathcal{M}(s',t_0)}{(s'-s)^{n+1}}$$

$$= \frac{n!}{\pi}\int_{4m^2}^{\infty} ds' \frac{2\mathrm{Im}\mathcal{M}(s',t_0)}{(s'-s)^{n+1}} \text{ + u-channel + poles} \quad (n \geq 2)$$

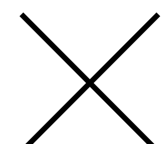
= s,u-poles and s,u-branch cuts

= exchanging particles

⇒ s^n $(n \geq 2)$ part of amplitude are originated from exchanging particles rather than a contact diagram.



⇒ nonpolynomial in s



⇒ s^n $(n < 2)$

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Positivity bounds

- The dispersion relation

$$\frac{\partial^n}{\partial s^n}\mathcal{M}(s,t<0)=\frac{n!}{\pi}\int_{4m^2}^{\infty}ds'\frac{\mathrm{Im}\mathcal{M}(s',t<0)}{(s'-s)^{n+1}}\ \text{+ u-channel + s,u-poles}$$

$(n \geq 2)$

It is useful to introduce the pole subtracted amplitude

$$\widetilde{\mathcal{M}}:=\mathcal{M}-\text{(s,t,u)-poles}$$

$$\Rightarrow\frac{\partial^n}{\partial s^n}\widetilde{\mathcal{M}}(s,t<0)=\frac{n!}{\pi}\int_{4m^2}^{\infty}ds'\frac{\mathrm{Im}\mathcal{M}(s',t<0)}{(s'-s)^{n+1}}\ \text{+ u-channel + t-poles}$$

- t-pole $\propto \frac{\partial^n}{\partial s^n}\frac{s^J}{t-m_J^2}=0$ for $n>J$ where J is the spin. $(n \geq 2)$

Note that poles arise from stable particles (= light particles).
$\Rightarrow$ Only the graviton is essential for the t-pole term.

Positivity bounds without gravity

- We evaluate the dispersion relation at $s = 2m^2 - t/2$.

$$\partial_s^n \widetilde{\mathcal{M}}|_{s=2m^2-t/2} = \frac{2n!}{\pi}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\mathcal{M}(s', t<0)}{(s' - 2m^2 + t/2)^{n+1}} + \cancel{\text{t-poles}} \quad (n \geq 2)$$

where $s \leftrightarrow u$ crossing is used.

- LHS can be computed by EFT since $\mathcal{M}_{\mathrm{EFT}}(s,t) = \mathcal{M}(s,t)$ at $s, t < \Lambda^2$

- RHS involves UV part but we know optical theorem:

$$\mathrm{Im}\mathcal{M}(s \geq 4m^2, t \to 0) > 0$$

$\Rightarrow$ The positivity bounds $\partial_s^n \widetilde{\mathcal{M}}_{\mathrm{EFT}}|_{s=2m^2-t/2, t=0} > 0 \quad (n \geq 2)$

A. Adams et al. 2006 (for $n = 2$).

EFT predicting a negative sign is "inconsistent" or in swampland.

Simple interpretation

- EFT usually accompanies with derivative interactions.

$$\mathcal{L}_{\rm EFT} = -\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 + \frac{c_1}{M^4}(\partial\phi)^4 + \cdots$$

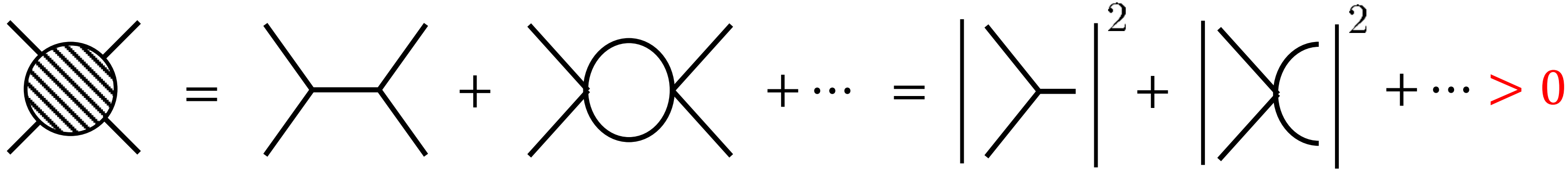
$$\mathcal{M}_{\rm EFT}(s,t) \sim \quad s^0 \quad s^2$$

The derivative interactions provide s^n part of $\widetilde{\mathcal{M}}_{\rm EFT} \Rightarrow c_1 > 0$

A. Adams et al. 2006

- The bounds can be diagrammatically understood.

$$\partial_s^n \widetilde{\mathcal{M}}_{\rm EFT}|_{s=2m^2-t/2,t=0} =$$ singularities = particle exchange



EFT operators are obtained by integrating out internal states.

Improved positivity

C. de Rham et al. 2017; B. Bellazzzini, 2017

- **EFF can compute some of RHS.**

$$\partial_s^n \widetilde{\mathcal{M}}|_{s=2m^2-t/2} = \frac{2n!}{\pi}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\mathcal{M}(s',t<0)}{(s'-2m^2+t/2)^{n+1}} + \cancel{\text{t-poles}} \qquad (n \geq 2)$$

- We have $\mathcal{M}_{\mathrm{EFT}}(s,t) = \mathcal{M}(s,t)$ at $s, t < \Lambda^2$.

  EFT can compute the quantity

$$B^{(n)}(t,\Lambda) := \partial_s^n \widetilde{\mathcal{M}}|_{s=2m^2-t/2} - \frac{2n!}{\pi}\int_{4m^2}^{\Lambda^2} ds' \frac{\mathrm{Im}\mathcal{M}(s',t)}{(s'-2m^2+t/2)^{n+1}}$$

$\Rightarrow$ Improved positivity bounds = bounds on cutoff

$$B^{(n)}(t \to -0,\Lambda) = \frac{2n!}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\mathcal{M}(s',t\to -0)}{(s'-2m^2)^{n+1}} > 0$$

Note that the cutoff Λ would be unknown parameter.
Increasing Λ means extrapolating EFT to a higher scale.

Gravitational positivity

J. Tokuda, KA, and S. Hirano, JHEP 11 (2020) 054.

- **Graviton provides the t-pole term at $n=2$.**

$$B^{(2)}(t<0,\Lambda)=\frac{2n!}{\pi}\int_{\Lambda^2}^{\infty}ds'\,\frac{\mathrm{Im}\mathcal{M}(s',t<0)}{(s'-2m^2+t/2)^3}$$ + graviton t-pole

where graviton t-pole $\propto \dfrac{1}{M_{\mathrm{pl}}^2 t}$

- The forward limit $t\to -0$ is subtle because $\dfrac{1}{M_{\mathrm{pl}}^2 t}\to-\infty \Rightarrow B^{(2)}>-\infty???$

- **Note that $B^{(2)}$ is the regularised amplitude and has no t-pole.**

$$B^{(2)}(t\to-0,\Lambda):\text{finite}\Rightarrow\int_{\Lambda^2}^{\infty}ds'\to+\infty\ \text{as}\ t\to-0$$

We must have "$\infty-\infty$" in RHS. How can we evaluate the finite part?

Gravitational positivity

- **This is crucially related to the Regge behaviour = UV completion**

$$B^{(2)}(t<0,\Lambda) = \frac{2n!}{\pi}\int_{\Lambda^2}^{\infty} ds' \frac{\mathrm{Im}\mathcal{M}(s',t<0)}{(s'-2m^2+t/2)^3} \quad + \text{graviton t-pole}$$

The integrand $\mathrm{Im}\mathcal{M}(s',t<0)$: physical part ⇒ should be finite.

The divergence must arise because of the non-convergence of integral.

- **The Regge behaviour**

$$\mathrm{Im}\mathcal{M}_{\mathrm{gra,Regge}} = f(t)s^{2+\alpha' t+\cdots}, \quad \alpha' > 0$$

⇒ The integral converges at $t<0$ while diverges as $t\to -0$.

The graviton t-pole should be cancelled by the graviton s,u-Regge states!

This is indeed the case in the perturbative string: t-pole = ∑ s,u-Regge

Gravitational positivity

- The explicit computation can be found in J. Tokuda, KA, and S. Hirano, JHEP 11 (2020) 054. but, the idea is very simple.

$$\mathrm{RHS} = \int_{\Lambda^2}^{\infty} ds' + t\text{-pole} = \int_{\Lambda}^{M_s^2} ds' + \int_{M_s^2}^{\infty} ds' + t\text{-pole}$$

where M_s is the scale when graviton is Reggeized, say the string scale.

We compute $\int_{M_s^2}^{\infty} ds' + \text{t-pole}$ at a finite negative t and then take $t \to -0$.

$$\int_{M_s^2}^{\infty} ds' + t\text{-pole} = -\frac{1}{M_{\rm pl}^2 t} + \frac{1}{M_{\rm pl}^2 t} + \text{finite} \to \text{finite as } t \to -0$$

- The finite part depends on the details of the Regge behaviour. However, if the physics of Regge is determined by the single scale M_s

$$\int_{M_s^2}^{\infty} ds' + t\text{-pole} = \pm \mathcal{O}(M_{\rm pl}^{-2} M_s^{-2})$$

Gravitational positivity

- ❑ We thus find the inequality

$$B^{(2)}(t \to -0, \Lambda) > -\mathcal{O}(M_{\rm pl}^{-2} M_s^{-2}) \qquad \text{for } \Lambda < M_s.$$

*Currently, there is no rigorous argument about the value of the finite residual although it can be computed once UV is assumed. $O(M_{\rm pl}^{-2} M_s^{-2})$ is our working assumption.

See also Y. Hamada+ 2019, B. Bellazzini+ 2019, L. Alberte+ 2020, M. Herrero-Valea+ 2020, N. Arkani-Hamed+ 2020, S. Caron-Huot+ 2021.

- ❑ Without knowledge (or assumption) about quantum gravity

→ Graviton t-pole "trivialises" the positivity. $B^{(2)} > -\infty???$

*It may be *suppressed* by non-forward limit amplitudes. S. Caron-Huot+ 2021.

- ❑ With knowledge (assumption) about quantum gravity

→ The tree-level gravity contribution is subtracted as a consistency with UV. We can find non-trivial constraints on gravitational EFTs from loops.

Contents

QFT is EFT

L. Alberte+2020 for QED+GR; KA+, 2104.09682 for SM+GR.

- Let us consider a renormalizable QFT coupled to GR.

$$\mathcal{M} = \mathcal{M}_{\rm QFT} + \mathcal{M}_{\rm GR} \to \mathcal{M}_{\rm QFT} \quad \text{as} \quad M_{\rm pl} \to \infty$$

$$B^{(2)} = B^{(2)}_{\rm QFT} + B^{(2)}_{\rm GR}$$

- QFT alone is supposed to be UV complete (= satisfy general properties)
  ⇒ trivially satisfies the dispersion relation.

$$B^{(2)}_{\rm QFT}(\Lambda) \equiv \frac{2n!}{\pi} \int_{\Lambda^2}^{\infty} ds' \frac{{\rm Im}\mathcal{M}_{\rm QFT}(s', t \to -0)}{(s' - 2m^2)^3}$$

  concluding the behaviour $B^{(2)}_{\rm QFT}(\Lambda \to \infty) = 0 \quad (B^{(2)}_{\rm QFT}(\Lambda) > 0 \text{ for finite } \Lambda)$

- Gravitational interactions are not UV complete within GR
  ⇒ They do not need to follow the dispersion relation at UV (GR = EFT).

$$B^{(2)}_{\rm GR}(\Lambda \to \infty) \neq 0$$

Cutoff of renormalizable QFT

- The positivity bound conclude

$$B^{(2)}(\Lambda) > -\mathcal{O}(M_{\rm pl}^{-2}M_s^{-2}) \qquad \text{for } \Lambda < M_s.$$

where $B^{(2)} = B^{(2)}_{\rm QFT} + B^{(2)}_{\rm GR}$

- We have the behaviours

$$B^{(2)}_{\rm QFT}(\Lambda \to \infty) = 0, \qquad B^{(2)}_{\rm GR}(\Lambda \to \infty) \neq 0$$

- GR provides a negative contribution in some scattering processes.

e.g. C. Cheung+2014, S. Andriolo+2018, L. Alberte+2020.

$$B^{(2)}_{\rm GR}(\Lambda \to \infty) = \text{negative constant} \qquad |B^{(2)}_{\rm GR}| \gg M_{\rm pl}^{-2}M_s^{-2}$$

$\Rightarrow$ Gravity eventually dominates over the particle interactions, violating the positivity bound at UV.

QFT + GR cannot be extrapolated into arbitrary high-energy scale!
As it should be because quantum gravity is needed.

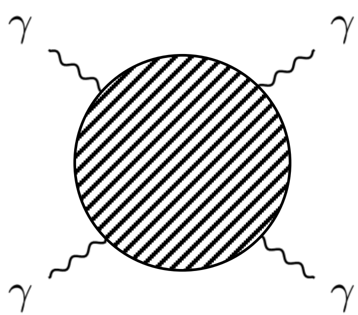
Cutoff of Standard Model

- What we need is to compute scattering amplitudes in SM+GR and to show when the inequality is saturated.

$$B^{(2)}(\Lambda) \simeq 0 \Rightarrow \Lambda = \Lambda_{\text{cut}}$$

where $B^{(2)} = B^{(2)}_{\text{SM}} + B^{(2)}_{\text{GR}}$ Recall $B^{(2)}(\Lambda) \gtrsim 0$ and $B^{(2)}(\Lambda \to \infty) < 0$

- We consider the light-by-light scattering ($\gamma\gamma \to \gamma\gamma$).



$$B^{(2)} = B^{(2)}_{\text{SM}} + B^{(2)}_{\text{GR}}$$

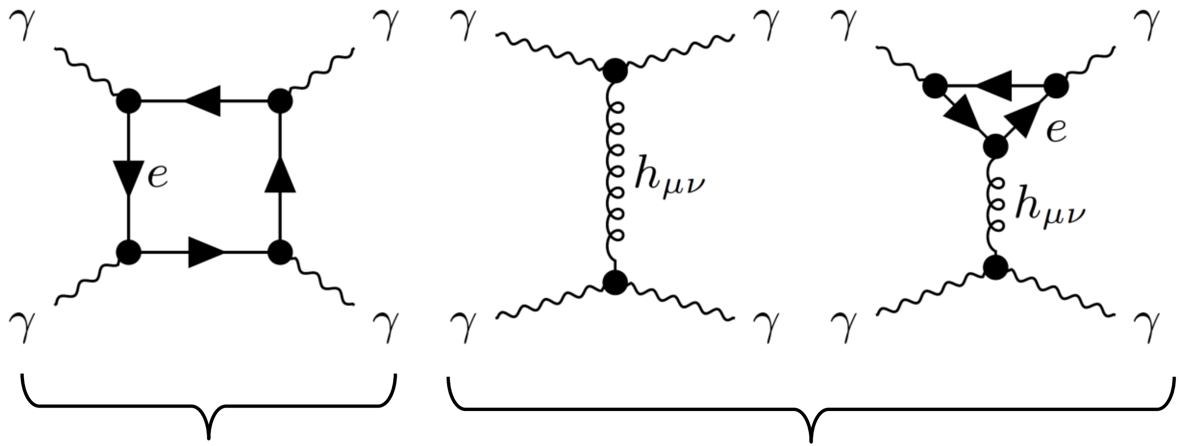
$$B^{(2)}_{\text{SM}} = B^{(2)}_{\text{QED}} + B^{(2)}_{\text{Weak}} + B^{(2)}_{\text{QCD}}$$

The helicity states are summed to manifest the $s \leftrightarrow u$ crossing.

QED and GR

See L. Alberte+2020.

- The leading contributions are electron loops



$$B^{(2)}_{\rm QED} \qquad\qquad B^{(2)}_{\rm GR}$$

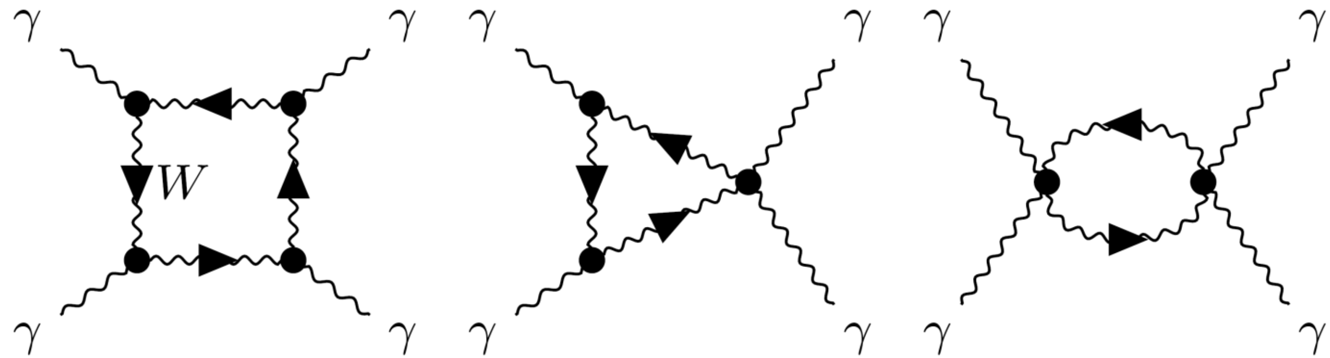
$$B^{(2)}_{\rm QED}(\Lambda) \approx \frac{64\alpha^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right), \quad B^{(2)}_{\rm GR}(\Lambda) \approx -\frac{22\alpha}{45\pi m_e^2 M_{\rm pl}^2}, \text{ at } \Lambda \gg m_e$$

Note that the tree-level graviton exchange does not contribute to $B^{(2)}_{\rm GR}$.

Also we should have $B^{(2)}_{\rm GR} \propto \frac{\alpha}{m^2 M_{\rm pl}^2}$ → lightest charged state is dominant.

Weak interaction

- The leading contributions are W boson loops



$$\underbrace{\hphantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}_{B^{(2)}_{\mathrm{Weak}}}$$

Leptons just provide the same contribution as electrons,
while W boson (spin-1) loops provide qualitatively different contribution.

$$B^{(2)}_{\mathrm{Weak}}(\Lambda) \approx \frac{128\alpha^2}{m_W^2 \Lambda^2} \quad \text{at } \Lambda \gg m_W$$

Electroweak + GR

- ✓ Spin-1/2 loop: $$B^{(2)}_{\mathrm{QED}}(\Lambda) \approx \frac{64\alpha^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m_e} - \frac{1}{4}\right) \propto \Lambda^{-4}$$
- ✓ Spin-1 loop: $$B^{(2)}_{\mathrm{Weak}}(\Lambda) \approx \frac{128\alpha^2}{m_W^2\Lambda^2} \propto \Lambda^{-2}$$
- ✓ c.f spin-0 loop: $$B^{(2)}_{\text{spin-0}} \approx \frac{4\alpha^2}{\Lambda^4} \propto \Lambda^{-4}$$ See L. Alberte+2020.
- ✓ GR contribution: $$B^{(2)}_{\mathrm{GR}}(\Lambda) \approx -\frac{22\alpha}{45\pi m_e^2 M_{\mathrm{pl}}^2} \propto \Lambda^0$$

The spin-1 is dominant and the mass appears in the denominator
⇒ the lightest charged particle (low-energy physics) determines $B^{(2)}(\Lambda)$, implying beyond SM (described within QFT) is irrelevant.

Our result must be universal against unknown new physics.

Electroweak + GR

- Ignoring QCD, we analytically find the maximum cutoff of EW + GR

$$\Lambda_{\rm EW} = \sqrt{\frac{2880\pi\alpha}{11}\frac{m_e M_{\rm pl}}{m_W}} \simeq 3.8\times 10^{13}\mathrm{GeV}$$

- However, the lightest charged spin-1 particle is not W boson.

EW scale ~ 100 GeV $\leftrightarrow$ QCD scale ~ 100 MeV

Also, QCD is strongly coupled and predicts higher spin mesons as well.

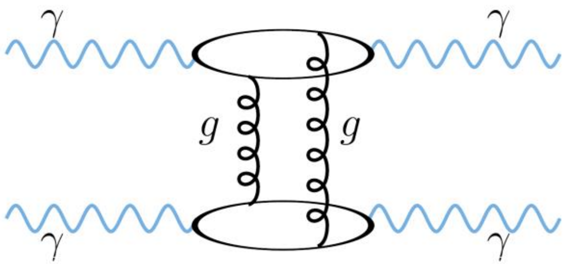
- We have to take into account QCD!

We cannot use perturbative calculations even at UV
since $B^{(2)}$ is computed by the forward limit amplitude $t \to 0$.

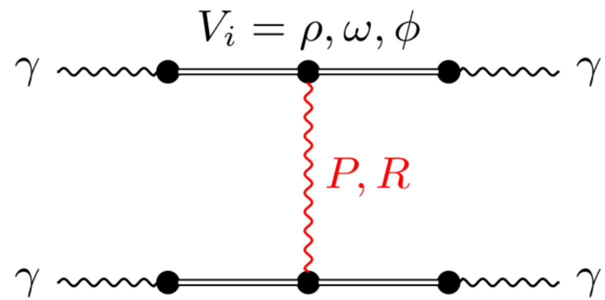
The region $s \gg \mathrm{GeV}^2, t \ll \mathrm{GeV}^2$ may be analysed by the Regge theory.

VDM-Regge model

M. Klusek-Gawenda+ 2016



$+\cdots=$



An elementary process

Resummation

Quark pairs $\simeq$ vector mesons

gluon exchanges $\simeq$ Reggeon&Pomeron exchanges

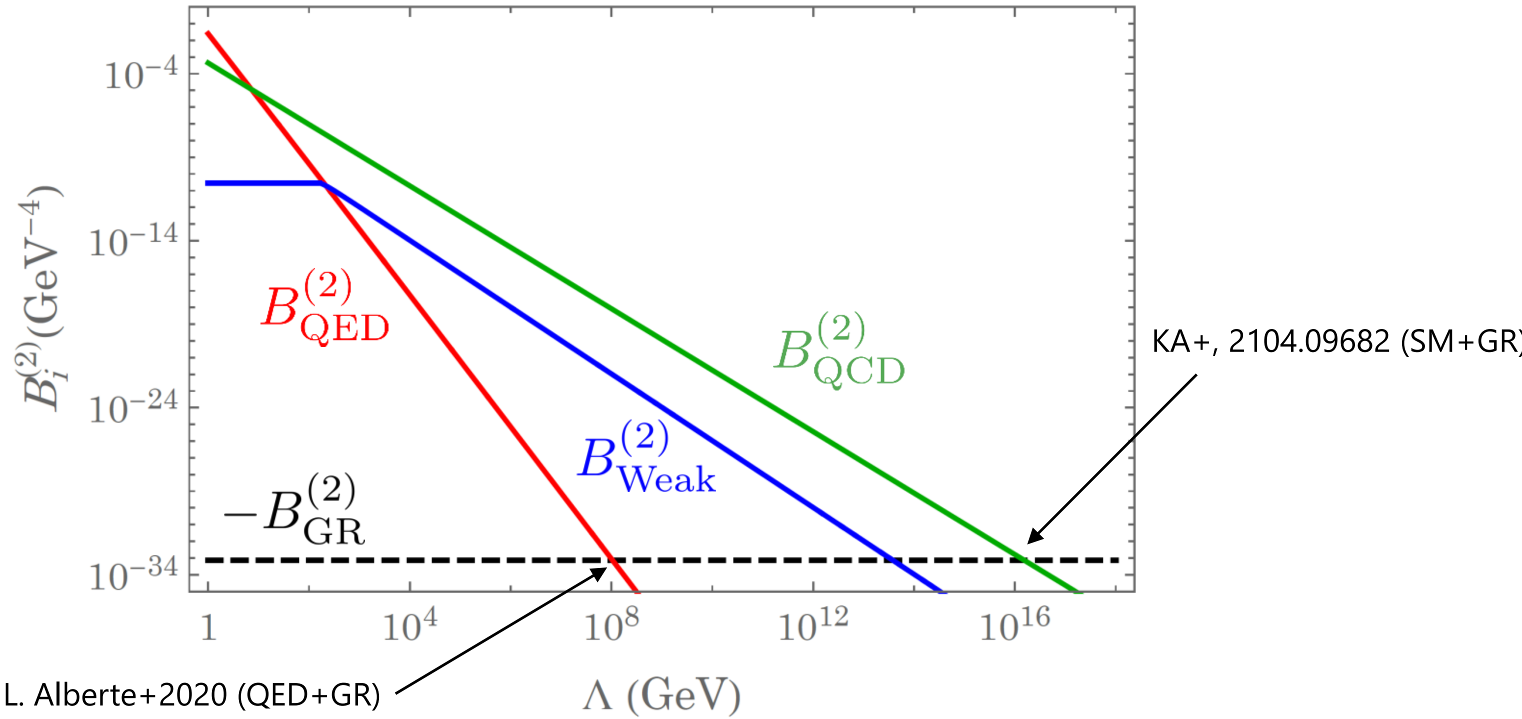
Photon transforms mesons and mesons undergoes hadronic interactions.

This would be a good model to describe QCD process in the Regge limit.

$s\to\infty$ while t is kept finite

We use the experimental values and extrapolate it to compute $B^{(2)}_{\mathrm{QCD}}$

Cutoff of Standard Model



$$B_{\mathrm{QED}}^{(2)} + B_{\mathrm{Weak}}^{(2)} + B_{\mathrm{QCD}}^{(2)} \simeq -B_{\mathrm{GR}}^{(2)} \Rightarrow \Lambda_{\mathrm{SM}} \simeq 3 \times 10^{16}\mathrm{GeV}$$

SM + GR (+ beyond SM) is “inconsistent” if it is extrapolated into 10^{16}GeV.

Summary

- ❑ Poincare invariance, unitarity, analyticity, and boundedness.

  ⇒ The positivity bounds: $B^{(2)}(\Lambda) > -\mathcal{O}(M_{\rm pl}^{-2} M_s^{-2})$ at $\Lambda < M_s$

  where M_s is the scale of quantum gravity.

- ❑ What we find: the inequality does not hold above

$$\Lambda_{\rm SM} \simeq 3 \times 10^{16} \mathrm{GeV}$$

- ❑ At least this implies we need a new physics at or below 10^{16}GeV.

  However, beyond SM (e.g. SUSY) may not help to push up Λ.

  ⇒ We probably need quantum gravity there!

  In fact, the positivity bound can be satisfied if $M_s < \Lambda_{\rm SM} \simeq 3 \times 10^{16} \mathrm{GeV}$

Discussions

- 10^{16}GeV is the expected GUT scale (unification of particle interactions). while our result is obtained by consistency between gravity and particle.

  Seems no relation, but is there a hidden reason? Or just coincidence?

- Our result suggests new swampland condition

$$\Lambda < \sqrt{\frac{2880\pi\alpha}{11}\frac{m_e M_{\rm pl}}{m_W}} \Leftrightarrow y_e \sin\theta_W > \sqrt{\frac{11}{1440}\frac{\Lambda}{M_{\rm pl}}} \quad \text{without QCD.}$$

  From other scatterings, say HH→HH etc. ⇒ $g_1, g_2, y_e \gtrsim \Lambda/M_{\rm pl}$???

  Particles must have coupling to other particles?

  Reminiscent of weak gravity conjecture.

- Finally, we need to understand more about the assumptions and S-matrix.

  IR physics ↔ UV properties of S-matrix

Crossing symmetry

- One amplitude represents three processes after analytic continuation.

$$\mathcal{M}(s,t,u)=\mathcal{M}(s,t)=\mathcal{M}(t,u)=\mathcal{M}(u,s) \qquad s+t+u=\sum m_i^2=4m^2$$